Unsteady Fluid Forces on a Blade in a Cross-Flow Turbine*

Junichiro FUKUTOMI**, Yoshiyuki NAKASE**,
Masashi ICHIMIYA** and Hirohito EBISU***

The internal flow in a cross-flow turbine is nonuniform because the water passes through only part of the runner. Therefore, the unsteady fluid forces act on a blade through rotation. Experimental and theoretical studies for determination of fluid forces on the blade in a cross-flow turbine are conducted. In the experiment, the tangential and radial forces are measured on a test blade using strain gauges and slip rings. On the other hand, in the theoretical study, they are calculated numerically using the unsteady momentum theory. The calculated results are compared with experimental data and good agreement is demonstrated. Furthermore, the maximum forces are found to occur immediately before the blade leaves the nozzle exit in both the experimental and theoretical results.

Key Words: Water Turbine, Fluid Force, Numerical Analysis, Unsteady Flow, Fluid Machinery, Turbomachinery, Cross-Flow Turbine

1. Introduction

Previous studies of the cross-flow turbine, which has come into wide use as a small hydropower turbine, have been generally directed at improving efficiency and developing low-cost turbines by investigation of the internal flow conditions⁽¹⁾⁻⁽⁶⁾. However investigations to determine the magnitude of the fluid forces acting on blades in the turbine have not been sufficiently carried out as yet. Dixhorn et al.⁽⁷⁾ investigated how forces act on blades in the cross-flow turbine, but in their theoretical study, they assumed the flow relative to the blades to be steady and uniform in the peripheral direction.

The internal flow in a cross-flow turbine is nonuniform because the fluid passes through only part

of the runner. Therefore, unsteady fluid forces act on a blade through rotation of the runner, so these forces become repeated loads. Estimation of these unsteady forces acting on the blade is important to determine the strength of material required and to avoid vibration of the blade. Therefore in this study, the fluid forces acting on the blade in the turbine are measured experimentally using strain gauges and slip rings, and calculated numerically using an unsteady momentum theory.

Nomenclature

B : width of runner
 D_1 : outside diameter of runner
 F_r : radial blade force
 F_r^* : nondimensional radial blade force
 $= F_r / (\rho \omega^2 B D_1^3)$
 F_t : tangential blade force
 F_t^* : nondimensional tangential blade force
 $= F_t / (\rho \omega^2 B D_1^3)$
 H : available head of water
 N : rotational speed
 N_{11} : unit rotational speed $= ND_1 / \sqrt{H}$
 p : static pressure

* Received 9th February, 1994. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 59, No. 561, B (1993), pp. 1640-1645. (Received 1st October, 1992)

** Department of Mechanical Engineering, The University of Tokushima, 2-1 Minamijosanjima, Tokushima 770, Japan

*** Tokushima Prefectural Office, 1-1 Bandaicho, Tokushima 770, Japan

θ : peripheral blade position (Fig. 1)

ρ : density

ω : angular velocity

Subscripts

0 : immediately before the blade inlet (nozzle exit)

1 : blade inlet

2 : blade outlet

3 : immediately behind blade outlet

2. Experimental Apparatus and Method

Figure 1 shows the test cross-flow water turbine. The working fluid, water, flows from the head tank whose water level is kept constant, through a duct to the nozzle from the upper right in Fig. 1, then enters the runner, does work and flows into the open channel which is used to measure the flow rate. The entry arc of the nozzle, δ , is 90° and the width of the throat of the nozzle, S_0 , is 64 mm. The rear wall shape of the nozzle is a circular arc. The nozzle region extends from $\theta=45^\circ$ to 135° , where θ is the peripheral position of the blade. $\theta=0^\circ$ means the top position and θ is measured in the direction of the runner rotation (clockwise direction) as shown in Fig. 1. At $\theta=45^\circ$, the blade meets the jet issuing from the nozzle and at $\theta=135^\circ$, escapes from it. The outside and inside diameter of the runner, D_1 and D_2 , are 315 and 214 mm, respectively and the runner width, B , is 100 mm. The blades consist of 26 pieces of iron with a circular arc profile whose thickness is 5.7 mm, and inlet and outlet angle, β_1 and β_2 , are 30° and 90° , respectively. The gap between the nozzle and runner is 1 mm.

The flow rate was measured with a rectangular weir in the discharge channel. The output of the turbine was calculated from the rotational speed of the runner measured with a magnetic pick-up mounted on the axis end and from the torque mea-

sured with a torque detector of torsion-bar type. The experiments were conducted with two available heads H , 1.48 and 1.25 m. In the present experiment, a draft tube was not used, so the total head from the position immediately before the nozzle to the center of the runner is taken as the available head. To represent the position of a blade relative to the nozzle, the end of the inner wall of the nozzle where the blade meets the jet issuing from the nozzle is called the nozzle inner end, and the end of the outer wall of the nozzle where the blade escapes from the jet is called the nozzle tip.

As indicated in Fig. 2, one side of the shaft through the center of one test blade is supported by the self-aligning bearing and another side is fastened to two radial and tangential aluminum links to which strain gauges are attached. The width and thickness of the links in the region where the gauge is attached are 5 and 0.5 mm, respectively. The gauges were sealed with a high-density wax to eliminate moisture problems. The strain on these links was transmitted through a slip ring mounted on the edge of the turbine shaft to a dynamic strain meter, and finally was recorded on a data recorder. The conversion of strains into the radial and tangential forces is carried out using the load-strain curve which was obtained beforehand by applying loads in radial and tangential directions independently. The effect of the load in one direction on the strain in another direction was small according to the preliminary examination. The effects of the blade weight and centrifugal forces acting on the blade were eliminated by subtracting values obtained when the runner was rotating in air.

3. Theoretical Analysis

The main purpose of this study is to determine the magnitude and position of the maximum fluid force on the blades. From the results of previous investigation⁽³⁾⁻⁽⁵⁾, the force is expected to become maximum in the first stage, so the theoretical analysis

Fig. 1 Schematic diagram of the test cross-flow turbine

Fig. 2 Test blade mounting

was carried out for the nozzle outlet region, $45^\circ \leq \theta \leq 135^\circ$ on the first stage. The first stage is the region where the water flows through the runner from outside to inside and the second stage is the region where the water flows from inside to outside.

In the present analysis, to estimate fluid force on a blade, an unsteady momentum theory is applied to the control surface abcd shown in Fig. 3 which includes one blade and half a blade passage on each side of the blade. The surfaces ab and cd are parallel to the inlet and outlet of the runner, respectively. The surfaces ad and bc are the central surfaces of the passage on each side of the blade. In the present analysis, the surface cd is assumed for simplicity to be a leading front of the fluid until water fills the passage, though the real front is represented by the labelled solid line. The surface cd is a free surface on which the pressure is equal to atmospheric pressure.

The assumptions in the present analysis are given below.

- (1) The flow is incompressible.
- (2) The absolute angle of the inlet flow to the runner, that is, the angle between absolute velocity and peripheral velocity, α_0 , is peripherally constant at 15° .
- (3) The flow is two-dimensional with respect to the passage between the blades.
- (4) The slip factor at the blade outlet $k = \Delta v_{u2}/U_2$ is peripherally constant at 0.3.

3.1 Fundamental equations

The resultant force acting on a moving control volume $\mathbf{F}$ is expressed from momentum theory as,

$$\mathbf{F} = \int_V \frac{\partial(\rho \mathbf{w})}{\partial t} dV + \int_A \rho \mathbf{w}(\mathbf{w} \cdot d\mathbf{A}) \quad (1)$$

where ρ is the density of the fluid, $\mathbf{w}$ is the relative velocity of the flow, V is the control volume, dV is the volume element in the control volume, A is the control surface and $d\mathbf{A}$ is an elemental area vector in the direction of the outward normal to the control surface. On the other hand, the force on the fluid in the control volume $\mathbf{F}$ consists of a surface force $\mathbf{F}_{cs}$, a body force $\mathbf{F}_{bf}$ and a reaction force $\mathbf{F}_{re}$,

$$\mathbf{F} = \mathbf{F}_{cs} + \mathbf{F}_{bf} + \mathbf{F}_{re}. \quad (2)$$

Fig. 3 Control surface

As the surface force, only the pressure is considered, so the surface force is

$$\mathbf{F}_{cs} = \int_A p d\mathbf{A} \quad (3)$$

The body force is written as

$$\mathbf{F}_{bf} = \int_V \rho \mathbf{x} dV \quad (4)$$

where

$$\mathbf{x} = \mathbf{r}\omega^2 - \boldsymbol{\omega} \times 2\mathbf{w} - \mathbf{g} \quad (5)$$

where $\mathbf{x}$ denotes body force per unit mass.

Therefore, the reaction force is written as

$$\begin{aligned} \mathbf{F}_{re} = & \int_V \frac{\partial(\rho \mathbf{w})}{\partial t} dV + \int_A \rho \mathbf{w}(\mathbf{w} \cdot d\mathbf{A}) \\ & - \int_A p d\mathbf{A} - \int_V \rho \mathbf{x} dV. \end{aligned} \quad (6)$$

$\mathbf{F}_{re}$ is the force with which the blade acts on the fluid, so the tangential and radial components of the force with which the fluid acts on the blade are expressed as

$$F_t = -\rho \int \frac{\partial W_t}{\partial t} dV - \int W_t dG + \int p dA_r + \rho \int X_t dV \quad (7)$$

$$F_r = \rho \int \frac{\partial W_r}{\partial t} dV + \int W_r dG + \int p dA_t + \rho \int X_r dV \quad (8)$$

where dG is mass flow rate, suffices t and r denote tangential and radial direction, respectively, and F_t and F_r have positive sign in the rotating and radial outer direction, respectively.

3.2 Inlet conditions and calculation methods

In the present analysis, the static pressure which has been obtained experimentally at the runner inlet, that is, the nozzle exit, is used as the inlet condition. Figure 4 shows the peripheral distribution of the pressure at the runner inlet at the point of maximum efficiency, $N_{11} = 42$ (rpm, m). The pressure was measured at a radial position $r = 1.03(D_1/2)$ using static pressure taps whose diameter was 1 mm and which were attached to the nozzle wall. Firstly, the absolute velocity across the surface ab at the runner inlet v_0 with little loss in the nozzle is calculated from

Fig. 4 Pressure distribution at nozzle exit

Bernoulli's equation as

$$v_0 = \sqrt{2gH \left\{ 1 - \frac{p_0}{\rho g H} \right\}}. \quad (9)$$

The relative velocity w_0 is given from the velocity triangle at the runner inlet with the absolute flow angle α_0 and the peripheral velocity of the runner u_1 as

$$w_0 = \sqrt{v_0^2 + u_1^2 - 2v_0 u_1 \cos \alpha_0}. \quad (10)$$

The effect of the variation of rotational speed on the flow condition at the runner inlet is considered to be as follows: as the rotational speed varies, the centrifugal force acting on the fluid varies, so the pressure at the runner inlet also varies. In the present investigation, according to the experimental values, the pressure at the runner inlet was assumed to be proportional to the unit rotational speed N_{11} on the basis of the pressure distribution at the point of maximum efficiency, $N_{11}=42$ as shown in Fig. 4. That is, the pressure at any N_{11} is approximated by the value at $N_{11}=42$ times $N_{11}/42$.

The relative velocity along the passage between the blades was estimated using the continuity equation along the passage with consideration of the geometrical shape and thickness of the blade.

The pressures at any point along the flow are given in turn as follows. Immediately behind the blade inlet, suffix 1, the pressure is given from Bernoulli's equation for the relative system by

$$p_1 = \frac{\rho(w_0^2 - w_1^2)}{2} + p_0. \quad (11)$$

Immediately before the blade outlet, suffix 2, the pressure is given from the one-dimensional energy equation with the head loss h_l along the passage between the blades by

$$p_2 = -\rho \left\{ L \frac{dw_2}{dt} + \frac{(w_2^2 - w_1^2)}{2} + \frac{(u_1^2 - u_2^2)}{2} \right\} + p_1 - \rho g h_l \quad (12)$$

$$L = \int_1^2 \frac{A_2}{A(s)} ds \quad (13)$$

where s is the distance along the passage, $A(s)$ is the cross-sectional area of the passage, A_2 is the area at the passage outlet and L denotes the equivalent passage length. h_l is estimated as a friction head loss when the passage is considered as a pipe with a friction factor $\lambda=0.02$.

Next, immediately behind the blade outlet, suffix 3, the pressure is given from the angle of the blade outlet, 90° and the slip factor of the blade, k by

$$p_3 = p_2 - \frac{\rho(ku_2)^2}{2}. \quad (14)$$

The pressures on the surfaces ab and cd in Fig. 3 are estimated as p_0 and p_3 , respectively, and on the surfaces ad and bc, they are taken as the mean of values immediately behind the blade inlet and immediately before the blade outlet, p_1 and p_2 . Moreover, the

passages between the blades outside the flow from the nozzle are assumed to be free from the water, so the pressure there is assumed to be atmospheric pressure.

The tangential and radial forces, F_t and F_r , are calculated from Eqs. (7) and (8) with the aforementioned velocity, pressure and geometrical shape of the blade.

4. Results and Discussion

4.1 Tangential force F_t

Figure 5 shows the variation of the tangential force F_t with the blade position θ at three values of N_{11} at the available head $H=1.48$ m. In the figures, the solid and broken lines show experimental and calculated values, respectively. As N_{11} decreases, F_t increases and becomes more uniform. This is because when the rotational speed decreases, the pressure at the runner inlet decreases, so the flow rate increases and the momentum which is introduced to the control volume increases and the variation of the tangential momentum between front and rear of the blade becomes large. In addition, with the decrease of the inlet pressure, the flow at the runner inlet becomes more uniform.

The variation of the experimental value with the blade position θ shows that as the blade enters the nozzle region at the nozzle inner end, it suddenly experiences a force, and the force increases almost linearly until water fills the passage. Then, it decreases near the center of the nozzle and again increases immediately before the nozzle tip. After the blade escapes from the nozzle, the force suddenly decreases and approaches zero, then increases again. The reason for the decrease of the force near the center of the nozzle is that the increase in static pressure immediately before the blade with the increase of θ , as shown in Fig. 4, results in decrease of fluid velocity, so the momentum which is introduced to the control volume decreases. The reason for the sudden increase of the force immediately before the nozzle tip is that the decrease of static pressure and the increase of fluid velocity cause large momentum change. Moreover, the large pressure difference acts on the blade because the static pressure in the nozzle area which acts on the pressure side is greater than the atmospheric pressure acting on the suction side of the blade. The force is maximum near the nozzle tip, $\theta=135^\circ$. The reason for the decrease in the force after the nozzle area is that the flow does not pass through the runner into the open inner space but returns halfway into the blade passage, so the fluid moves little in the blade passage. The region in which the force acts on the blade the second time corresponds to the region in which the flow passes through

(a) $N_{11}=49.2(\text{rpm, m})$ (b) $N_{11}=42.0(\text{rpm, m})$ ($\gamma_{\max}$ point) (c) $N_{11}=33.6(\text{rpm, m})$ Fig. 5 Variation of tangential blade force F_t with blade position θ ($H=1.48$ m)

from inside to outside the runner, that is, the flow which crossed the open internal space acts on the blades again in a second stage. The maximum value in this region is about 65~75% of that in the region near the nozzle tip.

The calculated value corresponds well to the experimental one and the calculated position of the maximum force is near the nozzle tip. The calculated

Fig. 6 Nondimensional tangential blade forces

value slightly leads the experimental one in the nozzle region, because the flow angle at the nozzle exit α_0 may be smaller than the assumed one, 15° , so the actual flow rate may be lower. Immediately before the nozzle tip, the calculated value is larger than the experimental one.

This is because the slip factor is assumed to be peripherally constant, $k=0.3$, in the calculation, but it may actually become larger as the blade approaches the nozzle tip⁽³⁾.

Here, the fluid force on the blades F_t can be nondimensionalized using density ρ , angular velocity ω and the turbine dimensions as

$$F_t^* = F_t / (\rho \omega^2 B D_1^3). \quad (15)$$

Figure 6 shows the maximum tangential force $F_{t\max}$ and $\bar{F}_t$, the mean value of F_t over the nozzle region $\theta = 45^\circ \sim 135^\circ$. Both experimental and calculated forces are shown and are nondimensionalized using Eq. (15). The calculated force is represented by a single curve irrespective of the value of the available head H . The calculated value of $F_{t\max}^*$ is slightly larger than the experimental one. The calculated values of both $F_{t\max}^*$ and $\bar{F}_t^*$ well represent the effect of N_{11} . The ratios of the maximum to mean values, $F_{t\max}/\bar{F}_t$ of both the experimental and calculated forces are 2.2~2.3 near the point of maximum efficiency, $N_{11} = 40 \sim 42(\text{rpm, m})$.

4.2 Radial force F_r

Figure 7 shows the variation of the radial force F_r with the blade position θ at three values of N_{11} for the available head $H=1.25$ m. The radial force F_r shows the same tendency as the tangential force F_t , and the position of the maximum force is almost the

(a) $N_{11}=49.2(\text{rpm, m})$ (b) $H_{11}=41.7(\text{rpm, m})$ ($\eta_{\max}$ point) (c) $H_{11}=33.8(\text{rpm, m})$ Fig. 7 Variation of radial blade force F_r with blade position θ ($H=1.25$ m)

same, though the force is smaller than F_t . In addition, the maximum value at the second stage is close to that at the first stage. This is perhaps because a large negative pressure difference acts inwards on the blades at the nozzle area, while the pressure difference acts outwards at the second stage. As N_{11} decreases, F_r increases as well as F_t due to the increase of the flow rate and the momentum change between the

Fig. 8 Nondimensional radial blade forces

blade inlet and outlet. The calculated values almost coincide with the experimental ones at $N_{11}=33.8$ and $41.7(\text{rpm, m})$.

Figure 8 shows nondimensionalized experimental and calculated values of $F_{r\max}$ and $\overline{F_r}$. The calculated force is represented by a single curve irrespective of the value of the available head H . The calculated values of both $F_{r\max}^*$ and $\overline{F_r}^*$ well represent the effect of N_{11} . The ratio of the maximum and mean values, $F_{r\max}/\overline{F_r}$ is about 2.5 near the point of maximum efficiency. Comparing Fig. 6 with Fig. 8, we see that $F_{r\max}$ and $\overline{F_r}$ values are about 60% of corresponding $F_{t\max}$ and $\overline{F_t}$ values, respectively.

5. Conclusions

The unsteady fluid forces on the blades in a cross flow turbine are measured experimentally and analyzed numerically using the unsteady momentum theory. The following conclusions are obtained.

(1) Both the tangential and radial forces, F_t and F_r , take maximum values near the nozzle tip ($\theta=135^\circ$) where the blade escapes from the jet issuing from the nozzle outlet.

(2) As the rotational speed of the runner decreases, $F_{t\max}$, $F_{r\max}$, $\overline{F_t}$ and $\overline{F_r}$ increase.

(3) The values of $F_{r\max}$ and $\overline{F_r}$ are about 60% of corresponding values of $F_{t\max}$ and $\overline{F_t}$, respectively.

(4) The calculated values agree fairly well with the experimental results, so the unsteady fluid force and maximum force on the blades can be well estimated by the present analysis.

References

- (1) Kubota, T. and Kushimoto, S., Cross-Flow Tur-

- bine for Generation of Mini Hydro-Electric Power, J. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 86, No. 780 (1983), p. 52.
- (2) Sajiki, S., Present Condition of Mini Water Turbine, Turbomachinery, (in Japanese), Vol. 13, No. 5 (1985), p. 15.
- (3) Toyokura, T., Kanemoto, T., Suzuki, T. and Sato, T., A Study of Cross-Flow Turbine, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 461, B (1985), p. 143.
- (4) Yamabe, M., Ito, S., Okuzawa, T. and Takagi, Y., A Study on the Hydraulic Characteristics of Cross-Flow Turbine, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 472, B (1985), p. 3928.
- (5) Fukutomi, J., Nakase, Y. and Watanabe, T., A Numerical Method of Free Jet from a Cross-Flow Turbine Nozzle, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 50, No. 459 B (1984), p. 2790.
- (6) Johnson, W., Ely, R. and White, F., Design and Testing of an Inexpensive Crossflow Turbine, ASME Small Hydro Power Fluid Machinery, (1982), p. 129.
- (7) Van Dixhorn, L.R., Moses, H.L. and Moore, J., Experimental Determination of Blade Forces in a Cross-Flow Turbine, ASME Small Hydro Power Fluid Machinery, (1984), p. 67.
-