Numerical Simulation of Pseudoshock in Straight Channels*

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Pseudoshock in straight channel flows was numerically simulated by solving the two-dimensional time-averaged Navier-Stokes equations. The objectives of the present simulation were to investigate the influence of parameters on the structure of pseudoshocks and to validate the model of the pseudoshock mechanism proposed in the previous paper. In this simulation, Harten-Yee's second-order-accuracy TVD scheme was used for high resolution of the shock wave and numerical stability, and Baldwin-Lomax's algebraic model was used for turbulent flows. The computational results agreed well with the experimental ones and with the proposed model of λ - and X-type pseudoshocks.

Key Words: Fluid Dynamics, Compressible Flow, Shock Wave, Numerical Analysis, Pseudoshock

1. Introduction

When a supersonic flow is decelerated to subsonic in a confined channel, a pseudoshock appears due to the interaction between a boundary layer and a shock wave; this is not the case in the external flow around flying bodies, wings and aircraft. Air-breezing engines such as the turbo-ram jet engine and SCRAM jet engine are now under intensive investigation for applications to a hypersonic aircraft, and the pseudoshock is often encountered in such engines. This phenomenon is also a basic problem in supersonic diffusers, supersonic compressors and high-pressure power plants. Because the problem of the interaction between the boundary layer and the shock wave is very complex, theoretical analysis is very difficult, and the approach using the boundary layer theory has been attempted on the simplest problem of the interac-

tion between the incident oblique shock wave and the boundary layer. However, the problem of the interaction between multiple successive shock waves such as the pseudoshock and the boundary layer seemed to be almost impossible to treat analytically. On the other hand, numerical simulation is widely utilized in fluid dynamics due to the rapid development of high-speed, large-capacity computers together with the progress of techniques of numerical analysis. However, a numerical simulation of complex phenomena such as the pseudoshock remains to be developed, except for the simplest case of the low Mach number and the thin boundary layer, to confirm the possibility of the numerical simulation of a pseudoshock^{(1),(2)}.

Therefore, to date, a great number of studies have been carried out only experimentally⁽³⁾⁻⁽⁸⁾. Because the choice of parameters could not be done freely and independently due to the experimental conditions, they were limited to only some specific combinations. In the present study, a numerical simulation using the Harten-Yee's second-order-accuracy TVD scheme⁽⁹⁾ was carried out on the pseudoshock in two-dimensional supersonic straight flow, and the comparison with the experimental results and a model proposed in our previous reports⁽³⁾⁻⁽⁵⁾ was made in terms of the influence of the Mach number, the thickness of the boundary layer and the Reynolds number on the shape and the structure of the pseudoshock.

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2. Numerical Method

The basic equations are the time-averaged, two-dimensional, thin-layer-approximated Navier-Stokes equations, and are represented in the conservative form in the Cartesian coordinate as follows:

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{1}{Re} \frac{\partial S}{\partial y} \quad (1)$$

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, F = \begin{bmatrix} \rho v \\ \rho u^2 + p \\ \rho uv \\ (e + p)u \end{bmatrix}, G = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e + p)v \end{bmatrix} \quad (2.a)$$

$$S = \begin{bmatrix} 0 \\ (\mu + \mu_t) \frac{\partial u}{\partial y} \\ \frac{4}{3}(\mu + \mu_t) \frac{\partial v}{\partial y} \\ S_4 \end{bmatrix}$$

$$S_4 = \frac{1}{\gamma - 1} \left(\frac{\mu}{p_r} + \frac{\mu_t}{p_{rt}} \right) \frac{\partial}{\partial y} (c^2) + \frac{1}{2} (\mu + \mu_t) \frac{\partial}{\partial y} (u^2) + \frac{1}{6} (\mu + \mu_t) \frac{\partial}{\partial y} (v^2) \quad (2.b)$$

where ρ is density, u and v are velocity in the x - and y -directions, respectively, e is energy per unit volume, p is pressure, Re is Reynolds number, Pr is Prandtl number, Pr_t is turbulent Prandtl number, μ is viscosity, μ_t is eddy viscosity and c is sound velocity.

The pressure p is obtained by the following equation of state for a perfect gas:

$$p = (\gamma - 1) \left[e - \frac{1}{2} \rho (u^2 + v^2) \right] \quad (3)$$

Baldwin-Lomax's algebraic model⁽¹⁰⁾ was used for turbulent flows. The Harten-Yee's second-order-accuracy TVD scheme, known as the shock capturing method, was used to solve the equations.

As shown in Fig. 1, a two-dimensional parallel duct with the length of 8 times the height is used as the computational domain, and only the lower half of the symmetric duct was calculated to attain a sufficient number of effective mesh points. The numbers of mesh points were 301 in the x -direction with equal intervals, and 21 in the y -direction with unequal intervals so as to be close enough to the vicinity of the wall. The initial values were given by the inviscid normal shock relations, and the final values after the progression in time step, corresponding to the pseudoshock, were regarded as the steady state solutions.

Fig. 1 Computational domain

According to previous experimental results, the structure of the pseudoshock is influenced by the upstream Mach number of the main flow and the boundary layer thickness. Thus the effects of these parameters as well as the Reynolds number $Re (= hc_1/\nu_1$: subscript 1 represents the values of the upstream main flow) were investigated in the present simulation. The inlet condition was uniform mainflow with the turbulent boundary layer of 1/7 power law velocity profile. The outlet conditions were given by the first-order extrapolation, and the boundary conditions on the wall were nonslip and adiabatic. Pr was 0.72, Pr_t was 0.9, and the ratio of specific heat was 1.4.

3. Results and Discussion

The inlet Mach number M_{in} , the non-dimensional thickness of the boundary layer δ/h and the Reynolds number Re were the parameters to be varied. The ranges were $M_{in} = 1.5, 1.7, 2.0$, $\delta/h = 0, 0.2, 0.4, 0.6, 0.8$ and $Re = 10^5, 10^6, 10^7$. Typical results of these effects are shown in the following sections.

3.1 Effects of inlet Mach number on pseudoshock shape

For qualitative comparison with the optical observation, the contours of the iso-density obtained by the numerical simulations are shown in Fig. 2. The thickness of the boundary layer and the Reynolds number are fixed at 0.4 and 10^6 , respectively. With increasing Mach number, the shape of the first shock changes from λ -type to X-type at Mach number of around 1.8, and the number of shock waves increases; these are consistent with our previous experiments and model^{(3),(4)}.

3.2 Structure of λ -type pseudoshock

Figure 3 shows the static pressure distribution along the flow direction for $M_{in} = 1.7$, which is the

Fig. 2 Contour of density ($\delta/h = 0.4$, $Re = 10^6$)

typical case of the λ -type pseudoshock. x and y , the distances from the inlet of the computational domain and from the wall surface, respectively, are non-dimensionalized by the height of the passage $2h$. The static pressure at the central part of the duct rises oscillatorily due to the first shock and the subsequent shock waves, while the static pressure in the vicinity of the wall rises gradually and monotonically. The start of static pressure rise shifts upstream near the wall, and the pressure rise due to the first shock is larger than that due to any other successive shock waves. These are characteristic of the λ -type pseudoshock, and agree well with the experimental results⁽³⁾.

The Mach number distribution is shown in Fig. 4. The flow in the central part of the duct is decelerated once to subsonic and then accelerated again to supersonic, corresponding to the change of the static pressure, and the low-speed flow near the wall, which follows the boundary layer separation induced by the first shock, recovers gradually by mixing with the core flow downstream; these agree well with the experimental results⁽³⁾.

The distribution of the velocity vector is shown in Fig. 5 together with the density contour for correspondence with the position of the shock waves. The

Fig. 3 Distribution of static pressure ($M_{in}=1.7$, $\delta/h=0.4$, $Re=10^6$)

Fig. 4 Distribution of Mach number ($M_{in}=1.7$, $\delta/h=0.4$, $Re=10^6$)

deceleration of the flow in the central part of the duct is larger than that near the wall due to the normal part of the λ -type first shock, and the concave velocity profile is formed, which gradually recovers in the downstream direction. These agree well with the results of the previous experiments⁽³⁾ and the velocity profile measured by Sugiyama et al.⁽⁷⁾, and are characteristic of the λ -type pseudoshock. The region of adverse flow is not observed even at the foot of the strong first shock only with a small separation bubble.

For clarity, Fig. 6 shows a magnified lower half of the velocity vector of Fig. 5, ranging from immediately upstream of the first shock to the neighborhood of the fourth shock. The following mechanism has been proposed as the model of the structure of the pseudoshock in Ref. (4). (1) The separation shock occurs by the upstream boundary layer separation, and is reflected at the centerline (i.e., the axis of symmetry), forming the first shock. The separation shock is the front leg, and the reflected shock the rear leg. (2) The rear leg of the first shock is reflected on the surface of the low-speed layer, deforming the surface convex and leading to the formation of the expansion waves, which accelerates the core flow. (3) Then the outer boundary of the low-speed layer forms a concave surface, approaching the center of the duct, which causes the formation of compression wave or oblique shock wave. Hence, the second shock comprises the reflection of these two wave at the centerline. (4) The subsequent shock waves are formed in the same manner. Also, in the experimental results of Sugiyama et al.⁽⁷⁾, it is shown that the deceleration and acceleration of the main flow are caused by the change of the effective area of the core flow. In other words, the displacement thickness of the low-speed layer undulates, which corresponds to the alternating change of direction of the transverse velocity component toward the centerline and toward the wall. These are well reproduced in the velocity vector field

Fig. 5 Velocity vector field ($M_{in}=1.7$, $\delta/h=0.4$, $Re=10^6$)

Fig. 6 Magnified velocity vector field ($M_{in}=1.7$, $\delta/h=0.4$, $Re=10^6$)

of Fig. 6, validating the above-mentioned mechanism of the formation of the pseudoshock.

According to the observation⁽⁵⁾ by the oil film method, the boundary layer separates only slightly at the foot of the first shock, and immediately reattaches before the occurrence of the second shock, and the separation can hardly be observed at the foot of the second shock in the case of the λ -type pseudoshock. Thus the separation throughout the pseudoshock, as has been presumed, does not occur. These are also well reproduced in the velocity vector profile of Fig. 6.

3.3 Structure of X-type pseudoshock

Taking the case of $M_{in}=2.0$ as the typical X-type pseudoshock, the static pressure distribution is shown in Fig. 7. On the whole, a similar tendency to that of the λ -type is observed, but the change of the wall static pressure is steplike due to the large intervals between the successive shock waves. This is characteristic of the X-type as found in the experiment⁽⁴⁾.

Figure 8 shows the distribution of the Mach number. Almost the same tendency as that of the λ -type is observed, but in the X-type the region where the Mach number is above 1 is longer than that in the λ -type as reported in the experimental results⁽⁴⁾,

Fig. 7 Distribution of static pressure ($M_{in}=2.0$, $\delta/h=0.4$, $Re=10^6$)

Fig. 8 Distribution of Mach number ($M_{in}=2.0$, $\delta/h=0.4$, $Re=10^6$)

which is characteristic of the X-type.

Figure 9 shows the velocity vector. In contrast with the λ -type pseudoshock, the X-type pseudoshock exhibits no concave immediately downstream of the first shock, and the distribution becomes convex afterwards, which gradually recovers in the downstream direction. This was also observed in the experimental results. A large adverse flow region is observed at the foot of the first shock and the small adverse flow region at the foot of the second shock.

The magnified lower half of the velocity vector field of Fig. 9 is shown in Fig. 10. The boundary layer separates at the foot of the first and the second shocks as mentioned above, and reattaches afterwards. This agrees well with the visualization experiments by the oil film method⁽⁵⁾. It is confirmed that successive shock waves are formed due to the repeated deceleration and acceleration caused by the undulatory deformation of the low-speed layer, in the same manner as in the λ -type pseudoshock.

3.4 Effects of boundary layer thickness on pseudoshock shape

Figures 11 and 12 show, respectively, the contours of density and velocity vector fields and the distributions of the static pressure for various boundary layer thicknesses with $M_{in}=1.7$ and $Re=10^6$. Although the number of shock waves increases and the intervals between them become slightly wider with increasing boundary layer thickness, the shape of the first shock is hardly changed, suggesting that it is determined mainly by the Mach number. The thicker the boundary layer, the slower the pressure recovery and the lower the final pressure, which agrees with the previous experimental results and the model of the pseudoshock structure.

The velocity vector fields also show that the velocity distribution downstream of the first shock becomes concave, which is characteristic of the λ -type when the boundary layer is thin (Figs. 11 (a), (b)).

Fig. 9 Velocity vector ($M_{in}=2.0$, $\delta/h=0.4$, $Re=10^6$)

Fig. 10 Enlarged velocity vector field ($M_{in}=2.0$, $\delta/h=0.4$, $Re=10^6$)

Fig. 11 Contour of density and velocity vector ($M_{in}=1.7$, $Re=10^6$)

Fig. 12 Distribution of static pressure ($M_{in}=1.7$, $Re=10^6$)

However, when the boundary layer is thick (Fig. 11(c)), the concave part of the velocity distribution disappears, which is characteristic of the X-type. If the inlet boundary layer is eliminated as in the extreme case ($\delta/h=0$), the contour of the density is that shown in Fig. 13. The boundary layer is very thin, because it develops only within the distance from the inlet boundary of the computational domain to the position of the first shock, and the interaction with the first shock becomes weaker than in the case of a thick boundary layer. Accordingly, although the X-type is observed for the low Reynolds number of 10^5 (Fig. 13(a)), the λ -type is maintained at the Mach number as high as 2.0 for the high Reynolds number of 10^7 (Fig. 13(b)), while the X-type appears at Mach number 3.0 and Reynolds number of 10^7 (Fig. 13(c)). Although the results of the thin boundary layer at high Reynolds number are extremely different from those of the thick boundary layer, the above result is considered to be valid, because the boundary layer in the conventional duct is thick.

3.5 Effects of Reynolds number

Figures 14 and 15 show, respectively, the contours of the density and the static pressure distributions for various Reynolds numbers with $M_{in}=1.5$ and the boundary layer thickness $\delta/h=0.2$. The change of the Reynolds numbers with the fixed inlet boundary layer thickness means the change in the strength of the boundary layer against the adverse pressure gradient. The shape of the pseudoshock is almost the same irrespective of the Reynolds number. The difference, however, is that the rise of the static pressure at the first shock is not so steep and the first shock becomes relatively weaker than the second shock at Re of 10^5 , suggesting that the front leg of the first shock becomes weak and changes to the compression waves.

Fig. 13 Contour of density ($\delta/h=0$)

Fig. 14 Contour of density ($M_{in}=1.5$, $\delta/h=0.4$)

Fig. 15 Distribution of static pressure ($M_{in}=1.5$, $\delta/h=0.2$)

These results agree well with experimental ones ^{(3),(4)}.

4. Conclusions

A numerical simulation was performed and it is confirmed that the computational results agreed well with the previous experimental results and the previously proposed model.

(1) The inlet Mach number has a strong influence

on the entire structure of the λ -type and the X-type pseudoshocks.

(2) The thickness of the ordinary boundary layer exerts almost no influence on the shape of the pseudoshock, but exerts an influence on the number of shock waves.

(3) The λ -type pseudoshock is maintained even at a high Mach number when the Reynolds number is high, if the boundary layer is extremely thin.

(4) The influence of the Reynolds number is mainly on the structure of the front leg of the first shock.

(5) The boundary layer separation at the first shock is comparatively small. No separation at the second shock occurs for the λ -type pseudoshock; however, a small separation occurs for the X-type pseudoshock. In both cases, at the third shock and thereafter, the separation hardly occurs.

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