Combined Free and Forced Convection for Laminar Steady Flow in Horizontal Tubes*

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It is well known that free convection is caused by the buoyancy forces of a laminar steady flow in a heated pipe. However, this experimental investigation shows that free convection occurs when the temperature differences between air in a room and water in an unheated pipe are large. Under such experimental conditions, the cross-sectional distributions of axial velocities are obtained for water with $ReRa$ (Re : Reynolds number, Ra : Rayleigh number) as a parameter. Furthermore, a numerical analysis is also presented, and the calculated results are qualitatively compared with the above experimental data of axial velocities and those reported previously for air.

Key Words: Combined Free and Forced Convection, Laminar Flow, Numerical Analysis, Pipe Flow, Horizontal Tube

1. Introduction

Free convection in a heated horizontal tube is caused by buoyancy forces due to the temperature differences between the pipe wall and the fluid. According to the results of several experimental investigations⁽¹⁾⁻⁽⁶⁾ on combined free and forced convections for a laminar steady flow, the fluid near the pipe wall rises toward the top of the tube, and falls in the central portion; thus a secondary flow occurs. Furthermore, the maximum value of axial velocity in the cross section of the pipe exists below the pipe axis. This flow pattern for air has been photographed by Mori and Futagami⁽²⁾.

Theoretical studies on combined free and forced

convections for a laminar steady flow in a horizontal tube have been reported by a number of authors. The perturbation method reported by Morton⁽⁷⁾ is applicable only to relatively low values of the parameter $ReRa$. Therefore, several analytical and numerical analyses which are applicable to high $ReRa$ regime have been presented: the perturbation method by Faris and Viskanta⁽⁸⁾ and the finite difference methods by Siegwirth and Hanratty⁽³⁾, Hwang and Cheng⁽⁹⁾, and Ishigaki and Mochizuki^{(10),(11)}. The theoretical results, except those by Siegwirth and Hanratty⁽³⁾, are compared with the experimental data for air in this paper.

It was seen in the present work that free convection for laminar flow occurs when the differences between air in a room and water in an unheated pipe are large. There are very little data on this experiment. In this paper, under the experimental condition that the pipe wall was not heated, measurements of cross-sectional distributions of axial velocities for laminar flow were made for water with $ReRa$ as a parameter. Moreover, a numerical analysis was also made, and the calculated results were qualitatively compared with the experimental data for water and

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air^{(1),(2)}. The difference in flow characteristics between water and air was investigated, and the present calculated results were also compared with previously calculated results for air.

Nomenclature

- $c = \partial T / \partial z$
 g : acceleration due to gravity
 Pr : Prandtl number $= \nu / \alpha$
 P : pressure
 R : radius of pipe
 r : radial distance from pipe axis
 Ra : Rayleigh number $= \beta g c R^4 / (\nu \alpha)$
 Re : Reynolds number $= 2 R w_m / \nu$
 T : temperature of fluid
 T_c : temperature at the pipe center
 T_w : temperature at the pipe wall
 u : radial velocity
 v : angular velocity
 w : axial velocity
 z : axial distance
 α : thermal diffusivity
 β : coefficient of thermal expansion
 θ : dimensionless temperature difference
 $= (T_w - T) / (c R Pr Re)$
 ν : kinematic viscosity
 ξ : vorticity $= \nabla^2 \phi$
 ρ : fluid density
 ϕ : polar angle measured from top point of tube circumference
 ψ : dimensionless stream function
 Subscripts
 c : pipe centerline
 m : cross-sectional mean value
 w : pipe wall

2. Basic Equations

The problem considered in the present study is laminar steady flow with secondary flow produced by the buoyancy forces in a horizontal tube. The flow is expressed by cylindrical polar coordinates (r, ϕ, z) , with ϕ measured from top point of tube circumference; the velocity components are denoted by u, v and w . The following assumptions are made in the analysis.

- (1) Velocity and temperature fields in the fluid are fully developed.
- (2) Viscous dissipation is negligible.
- (3) Physical properties of the fluid are constant. The density is considered to be constant except for its variation in the buoyancy term.
- (4) The axial pressure gradient is constant.
- (5) The tube wall temperature is constant along the periphery, and heat flux along the tube is uniform.

The wall temperature gradient along the pipe axis is constant. Furthermore, the fluid temperature gradient along the pipe axis ($c = \partial T / \partial z$) is identical to the wall temperature gradient.

With the foregoing assumptions, the equations of continuity and motion, and the energy equation are written as follows:

$$\frac{\partial(ru)}{\partial r} + \frac{\partial v}{\partial \phi} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \phi} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \phi} \right) - \beta g (T_w - T) \cos \phi \quad (2)$$

$$u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \phi} + \frac{uv}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \phi} + \nu \left(\nabla^2 v + \frac{2}{r^2} \frac{\partial u}{\partial \phi} - \frac{v}{r^2} \right) + \beta g (T_w - T) \sin \phi \quad (3)$$

$$u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \phi} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w \quad (4)$$

$$u \frac{\partial T}{\partial r} + \frac{v}{r} \frac{\partial T}{\partial \phi} + w \frac{\partial T}{\partial z} = \alpha \nabla^2 T \quad (5)$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}.$$

The boundary conditions at the pipe wall are.

$$u = v = w = T_w - T = 0 \quad (6)$$

From assumptions (1) and (5) the following relation is obtained.

$$\frac{\partial T}{\partial z} = \frac{\partial T_w}{\partial z} = c \quad (\text{constant}) \quad (7)$$

From assumption (5) the following relation is obtained.

$$\frac{\partial T_w}{\partial \phi} = 0 \quad (8)$$

If the pressure term is eliminated from the Eqs. (2) and (3) after writing Eqs. (1) through (5) in dimensionless forms, the dimensionless equations describing the vorticity ξ , the axial velocity w' and the temperature θ are obtained. These equations are similar to those reported by Hwang and Cheng⁽⁹⁾. Then, the equation describing ξ is written as:

$$u' \frac{\partial \xi}{\partial r'} + \frac{v'}{r'} \frac{\partial \xi}{\partial \phi} = \nabla'^2 \xi - Re Ra \left(\frac{\partial \theta}{\partial r'} \sin \phi + \frac{1}{r'} \frac{\partial \theta}{\partial \phi} \cos \phi \right). \quad (9)$$

Here, the following dimensionless variables were introduced:

$$r' = r/R, \quad u' = Ru/\nu, \quad v' = Rv/\nu, \quad w' = w/w_c, \\ \partial T / \partial z = c, \quad \theta = (T_w - T) / (c R Pr Re), \quad Pr = \nu / \alpha, \\ Ra = \beta g c R^4 / (\nu \alpha), \quad Re = 2 R w_m / \nu, \\ u' = (1/r') \partial \psi / \partial \phi, \quad v' = -\partial \psi / \partial r'.$$

Such dimensionless equations are approximated in a finite-difference form, and the resulting equations are numerically calculated by the successive-over-relaxation method. As initial values, the velocity and

temperature profiles of the Poiseuille flow are given. The value of the relaxation factor of ξ is less than 1.0 because the solution diverges. The calculation is repeated until the difference between the angular velocities calculated from two successive iterations is less than some predetermined parameter. The number of divisions in the radial and circumferential directions is 20.

3. Experimental Apparatus and Procedures

The experimental setup is the same as that reported elsewhere⁽¹²⁾. The test section was made of a brass tube with a diameter of 79 mm (a wall 5 mm thick) and a length of 10.7 m. Water flowed through the test section which was connected to a head tank. A cock at the downstream end was employed to adjust the flow rate. Velocity measurements were made with a hot-film probe at distances of 3, 4, 5, 6, and 7 m from the pipe inlet. In measurements of axial velocities, a hot-film probe was used, traversing it across the tube. It is necessary to know the temperature gradient along the pipe wall in order to estimate the Rayleigh number Ra . Therefore, the temperature gradient was obtained by averaging the temperature of water between the inlet and the outlet of the tube.

4. Results and Discussion

4.1 The case of air

Experimental data for air reported by Mori et al.^{(1),(2)} were introduced here for comparison with our calculated results. The experimental conditions are shown in Table 1. Since the measured values have been reduced to nondimensional forms, only a qualitative comparison of those data with the calculated results is made.

4.1.1 Axial velocity and temperature distributions along vertical central axis Axial velocity and temperature distributions along the vertical central axis are plotted in Figs. 1 and 2, respectively, with $ReRa=0.89 \times 10^5$. For a fluid, $ReRa$ alone governs the flow and heat transfer characteristics. Both the velocity and temperature distributions exhibit a maximum in the lower half of the tube. Moreover, the present calculated result is compared with previously reported theoretical results. Although identical basic equations are used, there is some discrepancy among the four calculated results in the velocity and temperature distributions. In the velocity distribution, the four calculated results agree with the experimental data except in the lower half of the tube, where the present calculated result shows better agreement with the experimental data than the other calculated results. In the temperature distribution, the result calculated by Faris and Viskanta⁽⁸⁾ is markedly different from

Table 1 Experimental conditions for air ($Pr=0.72$)

Re	Ra	$ReRa$
2700	33.2	0.89×10^5
2200	—	0.89×10^5
2850	56.8	1.62×10^5
—	—	3.2×10^5

Fig. 1 Axial velocity distributions along vertical central axis

Fig. 2 Temperature distributions along vertical central axis

the other calculated results; the former shows better agreement with the experimental data near the top of the tube than the latter, but not near the bottom. The calculated results except by Faris and Viskanta⁽⁸⁾ show nearly the same distribution, and near the top of the tube, a marked discrepancy exists between the calculated and experimental results. Near the bottom of the tube, the present result shows better agreement with the experimental data than the other calculated results.

The velocity and temperature distributions are shown for $ReRa=0.89 \times 10^5$ and 3.2×10^6 in Figs. 3(a) and 3(b), respectively. The agreement between the present calculated result and experimental data is

Fig. 3 Axial velocity and temperature distributions along vertical central axis

Fig. 4 Axial velocity and temperature distributions along horizontal central axis

satisfactory for the velocity distribution, but for the temperature distribution substantial discrepancy exists between the two results near the top of the tube. It is seen from the calculated results that the maximum values of the velocity and temperature distributions shift toward the bottom of the tube as the value of $ReRa$ increases.

4.1.2 Axial velocity and temperature distributions along horizontal central axis The velocity and temperature distributions along the horizontal central axis are plotted in Figs. 4(a) and 4(b), respectively. The present calculated results agree well with the experimental data except near the wall. The maximum values of the velocity and temperature distributions appear near the wall, and these distributions become distinctly different from those of water, as shown later in Fig. 9.

From the above comparisons between the experimental and present calculated results for air, good agreement is seen for the velocity distributions, but for the temperature distributions the present calculated results agree well with the experimental data except near the top of the tube.

4.1.3 Streamlines and isotherms The streamlines corresponding to the secondary flow and isotherms in the cross section of the tube are given in Fig. 5 as a typical flow pattern, where $ReRa = 0.89 \times$

Table 2 Experimental conditions for water (Present work, $Pr = 8.0$)

Inlet temperature of water ($^{\circ}\text{C}$)	Outlet temperature of water ($^{\circ}\text{C}$)	Room temperature ($^{\circ}\text{C}$)	Re	Ra	$ReRa$
21.6	21.4	20.3	829	-310	-2.6×10^5
11.1	11.6	16.5	473	610	2.9×10^5
10.5	10.8	15.7	936	370	3.5×10^5
10.5	10.7	16.0	1573	250	3.9×10^5
10.5	11.1	17.0	562	760	4.3×10^5
10.3	11.5	19.4	338	1940	6.6×10^5
10.7	11.7	19.7	567	1610	9.1×10^5
11.6	12.5	22.5	782	1560	1.2×10^6

Fig. 5 Streamlines and isotherms

10^5 . In the streamline pattern, the center of circulation is almost at $r/R = 0.5$ and slightly below the central horizontal line. This type of flow pattern has been photographed by Mori and Futagami⁽²⁾. In the isotherms, the region of maximum temperature gradients and thus maximum heat transfer is found near the bottom of the tube where the isotherms are concentrated.

4.2 The case of water

4.2.1 Axial velocity distributions The present measured data for water are given below. A comparison between the present calculated and experimental results is made, and the difference in the flow patterns for air and water is shown. The experimental conditions for $ReRa = -2.6 \times 10^5 \sim 1.2 \times 10^6$ are given in Table 2.

As mentioned in the visual studies of Mori and Futagami⁽²⁾ for air, for heating, the wall temperature is higher than the fluid temperature; thus the fluid near the wall, being hotter than that in the core, rises toward the top. The two upward flows along both sides of the tube wall meet at the upper part of the

tube, change direction, and falls into the central position as the fluid flows through the tube. From this flow pattern, the maximum value of axial velocity along the vertical central axis exists the pipe axis. In contrast, in a cooled pipe the opposite phenomenon occurs.

Heating or cooling of the pipe wall was not performed in the present experiments. Therefore, it is inferred that the phenomenon discussed above, which corresponds to heating or cooling of the pipe, occurs due to the differences between room temperature and the temperature of water; the case in which room temperature is higher than the temperature of water corresponds to heating of a pipe, and vice versa.

The axial velocity distributions along the vertical central axis are plotted for $ReRa=2.9 \times 10^5$ and -2.6×10^5 in Fig. 6. For the case of $Ra < 0$, room temperature is lower than the temperature of water; the temperature gradient along the pipe axis takes a negative value. There are very little data in the case of $Ra < 0$. The maximum value of axial velocity along the vertical central axis in the case of $Ra > 0$ exists below the pipe axis, and that of $Ra < 0$ above it. Since the absolute values of the two $ReRa$ are approximately equal, the two distributions are also nearly symmetrical with respect to the pipe axis. However, the measured distribution at the lower part of the tube for $Ra < 0$ is slightly larger than the corresponding distribution at the upper part for $Ra > 0$. The solid and dashed lines in Fig. 6 are the present calculated results. The calculated results agree well with the experimental data for $Ra > 0$, but for $Ra < 0$ the former deviate from the latter at the lower part of the tube. The two distributions are nearly symmetrical with respect to the central axis. Therefore, it is inferred that for $Ra < 0$, the influence of the insertion

of a probe appears at the lower part of the tube. In comparison with the case of air, it is seen that the axial velocity distribution for water indicates a convex surface at the upper part of the tube.

The axial velocity distributions are shown for $ReRa=3.5 \times 10^5$ and 3.9×10^5 in Fig. 7. The two distributions in the case that the product of Re and Ra , $ReRa$, is approximately equal are similar; the same tendency is observed for air. The experimental and calculated results show good agreement. Figure 8 shows the axial velocity distributions for $ReRa=2.9 \times 10^5$ and 1.2×10^6 . It is seen that the effect of the buoyancy force is to shift the maximum value of axial velocity toward the lower part of the tube as the value of $ReRa$ increases; this is also observed in the case of air. For $ReRa=1.2 \times 10^6$, the calculated results agree with the experimental data except at $r/R > 0.5$. For $ReRa=2.9 \times 10^5$, the calculated and experimental

Fig. 6 Axial velocity distributions along vertical central axis

Fig. 7 Axial velocity distributions along vertical central axis

Fig. 8 Axial velocity distributions along vertical central axis

Fig. 9 Axial velocity distributions along horizontal central axis

Fig. 10 Temperature distributions along vertical central axis

Fig. 11 Streamlines and isotherms

results show good agreement.

The axial velocity distributions along the horizontal central axis are given for $ReRa = 4.3 \times 10^5$, 6.6×10^5 , and 9.1×10^5 in Fig. 9. The three distributions are similar and symmetrical with respect to the pipe axis. It is particularly interesting to observe that these distributions are parabolic and considerably different from the case of air in Fig. 4.

4.2.2 Temperature distribution The calculated result of the temperature distribution along the

vertical central axis is presented in Fig. 10 as a typical flow pattern, where $ReRa = 2.9 \times 10^5$. The maximum value of the temperature for water shows a larger shift toward the bottom of the tube than for air; from this result, it is inferred that large temperature gradients exist near the bottom of the tube.

4.2.3 Streamlines and isotherms Figure 11 shows the streamline patterns and isotherms. The center of the circulation for water moves downward and closer to the wall in comparison with that for air. The isotherms for water are more concentrated near the bottom of the tube than that for air.

5. Conclusions

Numerical analysis and experiments for laminar steady flow of water in a horizontal tube with free convection were presented and the following conclusions were obtained.

(1) Even with no heating or cooling of the pipe wall, it was seen that free convection occurs due to the difference between room temperature and temperature of water.

(2) On comparison of the experimental data^{(1),(2)} for air at $ReRa = 0.89 \times 10^5$ with four different calculated results⁽⁸⁾⁻⁽¹¹⁾, the present calculated result shows better agreement with the experimental data than the previously calculated results.

(3) The experimental data for air^{(1),(2)} except at $ReRa = 0.89 \times 10^5$ and for water show good agreement with the present calculated results.

(4) For water, the maximum value of the axial velocity exists below the pipe axis at $Ra > 0$ and above it at $Ra < 0$; the two distributions in the case where the absolute values of the two $ReRa$ are approximately equal are nearly symmetrical with respect to the pipe axis.

(5) The axial velocity distributions along the horizontal central axis for water at three $ReRa$ values are similar; the distributions for water are parabolic but the distribution for air is not.

(6) The position of the center of the circulation for water is below the central horizontal line and closer to the wall in comparison with the case for air.

(7) The present numerical analysis is applicable to both water and air flows.

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