Measurement of Transiently Changing Flow Rates in Oil Hydraulic Column Separation*

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Column separation and subsequent rejoining in an oil hydraulic line usually result in tensile waves and consequent ruptures in an oil column. In order to investigate these phenomena in more detail, it is essential to precisely measure transient flow rates in column separation and rejoining. The aim of the present work is to attain this objective: a method based on the theoretical relation of the flow rate change in a pipe with a kinetically induced pressure difference between the two separate sections. Instead of a conventional differential pressure transducer, two individual pressure gauges are used to measure pressures at the two sections and find the difference between them by subtraction. Theoretically predicted stepwise decreases of the flow rate until the column rejoining have been accurately measured, showing the effectiveness of the method. It is also pointed out that the successive calculus proposed by Zhao et al., which is expected to substantially economize the procedure to obtain flow rates from differential pressure data, may give unrealistic results.

Key Words: Unsteady Flow Rate, Measurement, Column Separation, Rejoining, Transient, Oil Hydraulic, Differential Pressure

1. Introduction

During column separation in an oil hydraulic line, cavities are not usually observed anywhere in the pipe except at the ends. It has recently been found that after column rejoining tensile waves are produced in the line and consequently cause ruptures in the oil column^{(1),(2)}. Observations of these tensile waves and subsequent ruptures inherent in oil hydraulic column separation would add valuable information to existing knowledge about cavitation study.

In the long history of column separation research, attention has been focused almost exclusively on pressure measurement and attempts to measure flow rates have been quite rare. In order to investigate the behavior of an oil column after rejoining, however, it

is essential to accurately measure transient flow rates because it cannot be determined from pressure data alone.

For this purpose the present study adopts the method of detecting 'kinetic' differential pressures which emerge from between two separate sections along a pipe when the fluid column is accelerated or decelerated. Although the idea is not new^{(3),(4)} and is based on a simple and exact theory, it has not always been effective in real applications. Obviously the success of the method depends mainly on the accuracy of measuring fluctuating differential pressures. Undoubtedly conventional differential pressure transducers cannot meet this demand for fidelity. Instead two semiconductor pressure transducers with similar characteristics are used in the present measurement; pressures are individually detected at two separate cross sections in a pipe and subtraction between them gives differential pressures. Actually the same technique has previously been applied by the present authors to measure pulsating flow rates⁽⁵⁾, proving suitable for measuring periodically changing differential pressures up to 800 Hz, although its applicability to transient ones has yet to be confirmed.

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Notation

- a : pipe inner radius
 c : velocity of sound in oil
 $I_n(z)$: n -th order modified Bessel function of the second kind
 l : distance between pressure holes
 L : pipe length
 p : pressure
 P : Laplace transform of p
 Δp : differential pressure
 ΔP : Laplace transform of Δp
 q : flow rate
 $\bar{q}$: average flow rate over axial distance l
 q_{st} : initial steady-state flow rate
 Q : Laplace transform of q
 $\bar{Q}$: Laplace transform of $\bar{q}$
 s : Laplace operator
 t : time
 Δt : sample time
 $w_q(t)$: weight function
 $W_q(s)$: weight function in Laplace domain
 x : distance along pipe axis
 η_i : i -th root of 0-th order Bessel function of the first kind, $J_0(x)$
 ν : kinematic viscosity of oil
 ρ : density of oil
 τ : nondimensional time, t/χ_0
 $\Delta\tau$: nondimensional sample time, $\Delta t/\chi_0$

2. Principle of Measurement

2.1 Basic theory

Solving the Laplace transformation of the linearized unsteady Navier-Stokes equation under an assumption of axisymmetry yields the following relation which holds for an unsteady viscous flow in a circular pipe :

$$\frac{\partial P}{\partial x} = -\frac{\rho s}{\pi a^2} \frac{I_0(z)}{I_2(z)} Q \quad (1)$$

where

$$z = \sqrt{\chi_0 s}, \quad \chi_0 = a^2/\nu \quad (2)$$

and the symbols are explained in Notation.

Integrating Eq. (1) between two cross sections A and B separated by length l results in

$$\bar{Q} = \frac{1}{l} \int_{x_A}^{x_B} Q dx = -\frac{\pi a^2}{\rho l} W_q(s) \Delta P \quad (3)$$

where

$$W_q(s) = \frac{1}{s} \frac{I_2(z)}{I_0(z)} \quad (4)$$

$$\Delta P = P_B - P_A$$

and $\bar{Q}$ is an average flow rate over the axial length l . $\bar{Q}$ can be approximately regarded as the flow rate at the midpoint of the distance AB when l is small.

Taking the inverse Laplace transform of Eq. (3)

gives the formula which relates the flow rate $\bar{q}(t)$ with the pressure difference $\Delta p(t)$:

$$\bar{q}(t) = -\frac{\pi a^2}{\rho l} \int_0^t \Delta p(t') w_q(t-t') dt' \quad (5)$$

where

$$w_q(t) = \mathcal{L}^{-1}[W_q(s)]. \quad (6)$$

Although it is difficult to find an exact analytical form of the weight function $w_q(t)$, the numerical inversion method by Bellman⁽⁶⁾ seems applicable to calculate $w_q(t)$ from $W_q(s)$ in the same way as in Ref. (7).

Moreover, approximate analytical expressions of $w_q(t)$ can be obtained in the following process: first, the inverse Laplace transform of $W_q(s)$ is derived using the residue theorem as

$$w_q(t) = \sum_{i=1}^{\infty} \left[\frac{\frac{1}{s} I_2(z)}{\frac{d}{ds} I_0(z)} \right]_{s=-\frac{\eta_i^2}{\chi_0}} e^{-\eta_i^2 t / \chi_0}. \quad (7)$$

Accordingly, approximating the infinite series in Eq. (7) by a finite term gives

$$w_{q1}(\tau) = \sum_{i=1}^m \frac{4}{\eta_i^2} e^{-\eta_i^2 \tau} \quad (\tau = t/\chi_0). \quad (8)$$

When $\tau \rightarrow 0$, however, the above approximation does not meet the mathematical requirement that the weight function $w_q(\tau)$ satisfies

$$w_q(0) = 1.$$

Hankel's asymptotic expansions of $I_0(z)$ and $I_2(z)$ are useful to find an alternative approximation for small τ :

$$w_{q2}(\tau) = \mathcal{L}^{-1} \left[\frac{1}{s} \left(1 - \frac{2}{z} + \frac{1}{z^2} + \frac{1}{4z^3} \right) \right]$$

$$= 1 - \frac{4}{\sqrt{\pi}} \tau^{\frac{1}{2}} + \tau + \frac{1}{3\sqrt{\pi}} \tau^{\frac{3}{2}}. \quad (9)$$

Zhao et al.⁽⁴⁾ have proposed a means of efficiently performing the convolutional integration in Eq. (5). Approximating $w_q(\tau)$ by exponential functions as

$$\bar{w}_q(\tau) = A_0 \delta(\tau) + m_1 e^{-n_1 \tau} + m_2 e^{-n_2 \tau}, \quad (10)$$

they have arrived at the following successive calculus for Eq. (5):

$$\bar{q}_{k+1} = \frac{\pi a^2}{\rho l} \left\{ \frac{\chi_0}{8} \Delta p_{k+1} - M_1 Y_{k+1}^{(1)} - M_2 Y_{k+1}^{(2)} \right\} \quad (11)$$

where

$$\left. \begin{aligned} Y_{k+1}^{(i)} &= e^{-n_i \Delta \tau} Y_k^{(i)} + \chi_0 e^{-n_i \Delta \tau / 2} (\Delta p_{k+1} - \Delta p_k) \\ Y_0^{(i)} &= 0 \\ \bar{q}_k &= \bar{q}(k \Delta t), \quad \Delta p_k = \Delta p(k \Delta t) \end{aligned} \right\}. \quad (12)$$

The coefficients in Eqs. (10) and (11) quoted from Ref. (4) are given in Table 1.

In Fig. 1 $w_q(\tau)$ calculated by the numerical Laplace inversion (solid line) is compared with the approximations, $w_{q1}(\tau)$ ($m=5$, broken line), $w_{q2}(\tau)$ (dotted line) and $\bar{w}_q(\tau)$ (dot-dashed line). Apparently $w_{q1}(\tau)$ and $w_{q2}(\tau)$ coincide with $w_q(\tau)$ for $\tau \geq 0.1$ and $\tau \leq 0.1$, respectively, and are smoothly connected to

Table 1 Coefficients in Zhao's weight function

$A_0 = \frac{1}{8} - M_1 - M_2 = 0.0003250$	
$m_1 = 0.6907211$	$m_2 = 0.1647906$
$n_1 = 5.7910644$	$n_2 = 30.508865$
$M_1 = 0.1192736$	$M_2 = 0.0054014$

Fig. 1 Comparison of weight functions

Fig. 2 Simple hypothetical differential pressure data

each other at around $\tau=0.1$. This result indicates that the numerical inversion method can give accurate values of the weight function $w_q(\tau)$. For $\tau < 0.01$ the approximate weight function $\bar{w}_q(\tau)$ of Zhao et al. differs from the exact one by about 10%.

2.2 Errors in Zhao's calculus

In an attempt to apply the successive calculus given by Zhao et al. to the present measurement, it has been discovered that their method has a defect.

We consider a very simplified example to clarify the point at issue. Assuming that sampled data of $\Delta p(\tau)$ take the values 1 at $\tau = \Delta\tau$ alone and 0 elsewhere as is illustrated in Fig. 2, and also that $\pi a^2 \chi_0 / \rho l = 1$, the first two discrete values of the flow rate are obtained from Eq. (5) by trapezoidal numerical integration as follows:

Fig. 3 Estimation of errors arising in Zhao's calculus via step wave propagation in a semi-infinite line

$$\bar{q}(\Delta\tau) \cong \frac{\Delta\tau}{2}, \quad \bar{q}(2\Delta\tau) \cong \Delta\tau$$

On the other hand Zhao's calculus, Eq. (11), gives

$$\bar{q}(\Delta\tau) \cong A_0 + \frac{m_1 + m_2}{2} \Delta\tau,$$

$$\bar{q}(2\Delta\tau) \cong (m_1 + m_2) \Delta\tau.$$

Comparison of these two differently obtained $\bar{q}(\Delta\tau)$ naturally leads to the suspicion that the latter can contain some error when $\Delta\tau$ is smaller than A_0 .

In order to examine what errors Eq. (11) will produce, a step flow-rate wave propagating in a semi-infinite oil hydraulic line is chosen as the next example. Along the line, consider two points which are l_2 apart and the upstream one of which is l_1 away from the inlet. Since the pressure difference Δp between the two points is analytically given by the viscous wave equation⁽⁷⁾, sampled data for the calculation are generated from the theoretical Δp .

In Fig. 3 profiles of flow rates obtained by convolutional integration of Eq. (5) (dot-dashed) and also by Zhao's successive calculus (broken) are compared with theoretical ones (solid) for three values of $\Delta\tau$. In the calculation $N = l_1 / (c\Delta t)$ and $M = l_2 / (c\Delta t)$ have been chosen as 100 and 6, respectively, and pressures and flow rates have been nondimensionalized by ρc^2 and $\pi a^2 c$, respectively.

Figure 3 clearly demonstrates that Zhao's calculus results in spike-shaped fluctuations on the wave front which grow in magnitude with decreasing $\Delta\tau$. These undesirable errors emerge when $\Delta\tau$ is less than A_0 , and did not appear in Ref. (4) because $\Delta\tau$ happened to be larger than A_0 for the measurement reported there. Thus we conclude that the use of Zhao's calculus should be avoided when the change of a flow rate is fast or the parameter χ_0 is large.

The origin of this defect seems obvious. According to Ref. (4), A_0 has the value that a step response of the transfer function $W_q(s)$ takes at $\tau=0$.

Mathematically that value is obtained by letting $s \rightarrow 0$ in $W_q(s)$, and thus is exactly equal to 0 and not to the value given in Table 1.

3. Experimental Apparatus

For the purpose of measuring differential pressures, it may seem inappropriate from a conventional standpoint to use two different pressure gauges and subtract one output from the other, because subtraction usually reduces the accuracy. However the present authors have previously ascertained that the method has its own merit of responsiveness when applied to the measurement of differential pressures in pulsating pipe flows⁽⁵⁾. Therefore the same device as reported previously is used again to measure kinetic differential pressures in transient pipe flows; the device consists of two semiconductor pressure transducers with similar characteristics (rated range 3 MPa) and a differential amplifier constructed on an IC board with operational amplifier IC's (Fig. 4), and outputs two individual pressures and the difference between the two.

An experimental apparatus used to measure flow rates during column separation in an oil hydraulic line is schematically illustrated in Fig. 5. A working oil (VG 46) discharged from a pump flows through an impact-driven quick-closing valve, a test line and a constant-pressure chamber with a gas-filled rubber bag, and returns to a reservoir⁽⁸⁾. A pipe joint with the same inner diameter as the test line ($2a=16.4$ mm)

Fig. 4 Amplifier for use in differential pressure measurement

Fig. 5 Schematic illustration of the experimental oil hydraulic line

and two axially separated pressure holes on its wall ($l=160$ mm apart, 5.3 mm in diameter) is inserted at the midpoint ($L_1=1.6$ m, $L_2=1.42$ m) or near the end ($L_1=3.19$ m, $L_2=0.14$ m) of the test line. The pressure transducers are mounted on the joint and connected with the amplifier.

After the closing valve is shut, transient waves travel up and down the test line and, if the initial flow rate is high enough, column separation occurs immediately downstream from the valve⁽⁸⁾. Throughout the process the differential pressure $\Delta p(t)$, the upstream port pressure $p_1(t)$ and the pressure behind the valve $p_s(t)$ are sampled ($\Delta t=30$ μ s) and recorded by AD converters. Steady-state fluxes are measured by an orifice flowmeter placed downstream of the constant-pressure chamber before the valve is closed; oil temperature is detected by a thermistor mounted at the closing valve.

4. Results

Results of the measurements for different initial fluxes are shown in Figs. 6 to 8, where (a) and (b) indicate the results at the midpoint and the end of the line, respectively. The measured pressures and flow rates are plotted on the left side, while their theoretical values calculated by the method of end histories⁽⁷⁾ with the initial flux and line pressure as the only data are shown on the right. Downward from the top the illustrations of differential pressure Δp , flow rate $\bar{q}$, upstream port pressure p_1 and pressure behind the closing valve p_s are plotted versus nondimensional time t/T_L where $T_L=L/c$ and $L=L_1+L_2+l$. ($c=1380$ m/s)

In the illustrations of theoretical flow rates, $\bar{q}$ computed using Zhao's successive calculus from measured differential pressure data are also plotted for reference as dotted lines. Among these figures Fig. 6 depicts ordinary transient wave transmission in a line without column separation. On the other hand Figs. 7 and 8 show typical transient waves with column separation. The behaviors of flow rates exhibited in Figs. 7 and 8 clearly reveal a process of column separation which is usually hard to deduce from conventional pressure data alone.

After the valve closure, a loaded flow-rate change with a negative magnitude travels as a wave between the cavity formed by column separation behind the valve and the constant-pressure end, repeating in-phase reflections at both ends and producing consecutive step decreases of the flow rate as shown in Figs. 7 and 8. As these decreases act in due course to reverse the motion of the oil column, the column starts to move upstream, collapses the cavity and finally collides against the closed valve accompanied

Fig. 6 Results ($q_{st}=30 \text{ cm}^3/\text{s}$, no separation) Fig. 7 Results ($q_{st}=70 \text{ cm}^3/\text{s}$, with separation and rejoining) Fig. 8 Results ($q_{st}=131 \text{ cm}^3/\text{s}$, with separation and rejoining)

by a high pressure surge. This process is called 'column rejoining'.

In Figs. 6 and 7 measured flow rates and pressures agree well with theoretical ones. In Fig. 8 the same is true until the rejoining. After the rejoining, however, theoretical calculation predicts that the pressure p_1 at the midpoint of the line will take a noticeable absolute negative value, whereas this is not appreciably observed in the measurement. This discrepancy shows that the rejoining has resulted in a tensile stress and consequent rupture in the oil column near the pressure hole of p_1 , the behavior of which is not incorporated in the theoretical model used here. Therefore measurement of flow rates in addition to pressures will be useful to obtain more information about these phenomena, such as the location of ruptures and the volume and lifetime of a resultant cavity.

Finally, it should be commented that flow rates obtained by Zhao's method in Figs. 6 to 8 exhibit many spike-shaped fluctuations which do not actually occur. This is because the nondimensional sample time $\Delta\tau = 4.4 \times 10^{-5}$ in the present measurement is less than one-seventh of A_0 .

5. Conclusions

(1) Transient flow rates in a pipe can be measured by detecting kinetically induced differential pressures. A method of measuring pressures at two separate points and obtaining the differences between them by subtraction has proved useful and accurate for this purpose.

(2) By this 'kinetic differential pressure' method, characteristic changes of flow rates during column separation and rejoining in an oil hydraulic line have been successfully measured for the first time. It is believed that the method will be useful in the study of rupture of an oil hydraulic column under tensile stress.

(3) In the actual measuring system, offset current from an operational amplifier can cause un-

expected serious errors in measured flow rates—the errors linearly increase with time. It can be concluded that those errors have been minimized in the present differential pressure amplifier.

(4) When change is fast or x_0 is large, the calculus proposed by Zhao et al. can give inaccurate results in the measurement of transient flow rates through kinetic differential pressure data.

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