Flow in an Annular-Conical Passage*

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An experimental study has been conducted on the flow behavior in an annular-conical passage. Experiments are carried out for three different cone apex angles ($\beta = 60^\circ, 90^\circ, 120^\circ$). For each apex angle two cases of different passage spacing are studied. The emphasis is placed on the transition from laminar to self-sustained oscillating flow regime in which the critical Reynolds number is investigated in relation to the apex angle and passage spacing. It is found that the critical Reynolds number depends on both the cone apex angle and passage spacing. For the self-sustained oscillating regime the Strouhal number is evaluated also with respect to both the passage spacing and cone apex angle and the dependence of the Strouhal number on the Reynolds number is determined.

Key Words: Flow Separation, Self-Sustained Oscillations, Annular-Conical Passage, Critical Reynolds Number, Strouhal Number

1. Introduction

The annular-conical passage, which belongs to the category of annular diffusers, has not received as much attention in the field of fundamental fluid dynamics research as it deserves. The annular conical passage is widely used in various technical applications such as a valve of an automobile engine, a poppet nozzle used for direct fuel injection into the cylinder of a gasoline engine, or an air outlet of an air-conditioning system for local cooling of large halls. The great efforts that are being made by the automobile industry to develop a clean and efficient engine further underline the importance of studying the flow behavior inside the annular-conical passage. This is because one of the crucial factors for improving fuel combustion is to advance the atomization process which might be affected by the behavior of flow passing through an annular conical passage in a valve.

There exist a number of types of annular diffusers⁽¹⁾ but we restrict our investigation to a diffuser with straight and parallel walls. This restriction brings us to a geometrically simple and fundamental flow passage. However, as was presented earlier in Refs. (2), (3) and (4), the flow behavior in the annular-conical passage is rather complicated due to self-sustained periodic flow separation caused by an adverse pressure gradient which is induced by virtue of the geometry of the flow passage. The flow visualization experiments show that the flow can roughly be divided into three regimes⁽⁴⁾:

1. laminar flow,
2. self-sustained oscillating flow, and
3. turbulent flow.

A similar observation was performed by Mochizuki and Yang⁽⁵⁾ when studying the flow in the passage between two parallel disks, which corresponds to an annular conical-passage with the apex angle of $\beta = 180^\circ$.

Due to many geometrical parameters that might influence the flow characteristics, extensive experimental study would be required to provide general description of the flow in an annular-conical passage.

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The present work covers three different apex angles ($\beta = 60^\circ, 90^\circ, 120^\circ$) and two different passage spacings. This study focuses on the first two flow regimes where the critical Reynolds number, which indicates the boundary between laminar flow and the onset of the self-sustained oscillating flow, is investigated in relation to the apex angle and a dimensionless passage spacing. A separation frequency measurement for the second flow regime is also performed and the relationship between the Strouhal and Reynolds number is presented.

2. Experimental Apparatus

The experimental setup to investigate the flow behavior inside the annular-conical passage is depicted in Fig. 1. Air is sucked by a blower from a large space into a test section, located on the top of an airtight chamber, through a bell mouth and a short inlet pipe. This suction arrangement is particularly important for obtaining a smooth and steady inlet flow. The flowrate is set by controlling the blower r.p.m. and measured with a laminar flowmeter.

Figure 2 shows a schematic of the test section. To form the annular-conical passage, two axially symmetric cones are used. The outer cone is made out of a 1-mm-thick transparent PVC plate to permit visual observation of the flow. The inner cone is constructed from a piece of wood with a highly polished matte black surface. The gap between the two

Fig. 1 Experimental apparatus

Fig. 2 Schematic of the test section

cone surfaces can be varied by moving the inner cone vertically. To change the cone apex angle, three different test sections with the apex angles of $60^\circ, 90^\circ, 120^\circ$ were made.

The flow visualization images were obtained using paraffin mist as a tracer and an argon laser light sheet as illumination. The flow visualization experiment is described in detail in Ref. (4).

3. Geometry and Dimensionless Parameters

The notation used to describe the geometry of the annular-conical passage is shown in Fig. 3. Full geometric specification of an annular conical passage with straight parallel walls requires a set of three independent dimensionless geometric parameters. A convenient set used in the present work consists of the apex angle β , the nondimensional passage spacing ψ , and the nondimensional length φ . The parameters are defined as follows:

$$\psi = \frac{s}{r_{01}}, \quad (1)$$

$$\varphi = \frac{l}{r_{01}}. \quad (2)$$

The Reynolds and Strouhal numbers used in the present study are defined as:

$$Re = \frac{U_i \cdot 2s}{\nu}, \quad (3)$$

$$S = \frac{f \cdot 2s}{U_i}, \quad (4)$$

where U_i denotes the average inlet-flow velocity, $2s$ is the hydraulic diameter with s representing the passage spacing, ν is the air kinematic viscosity, and f is the separation (vortex generation) frequency.

4. Frequency Measurement

The separation frequency (vortex generation frequency) or the Strouhal number is obtained by measuring local velocity fluctuations. A schematic of the measurement method is shown in Fig. 4. Fluctuations are measured with a hot-wire anemometer. The signal is linearized, filtered and amplified and finally processed by an FFT frequency analyzer which is

Fig. 3 Test section geometry

Fig. 4 Frequency measurement

Fig. 5 Power spectrum, $Re = 620$, $\beta = 90^\circ$, $\psi = 0.4$, $\varphi = 7.87$

installed on a computer.

After the flow visualization experiment and some preliminary frequency measurements, the streamwise location of the hot wire probe is selected so that the anemometer gives the strongest determined output signal. The distance of the hot wire from the inner wall is also so as to obtain the strongest output signal. Preliminary frequency measurements disclosed that the most powerful signal is obtained when the hot wire probe is set at about $1/4$ of the passage spacing from either the inner or outer wall.

Typical output from the FFT analyzer is present-

ed in Fig. 5. Each spectrum is the result of an average of two hundred data sets. Each data set consists of the signal taken through the Hann window for 20s and includes about 1000 data. The peak shown in the spectrum denotes the vortex generation frequency which is always verified by flow visualization. The frequency is obtained from the flow visualization by simply counting the number vortices generated over a certain time.

5. Critical Reynolds Number Determination

The critical Reynolds number, that is, the lower-bound Reynolds number below which the flow remains laminar and steady, is determined on the basis of flow visualization. The criterion for deciding whether the flow is steady laminar or in transition into the self-sustained oscillating regime is as follows: the flow is considered to be laminar and steady if at a constant Reynolds number, during the time period of five minutes no oscillations occur and the flow is smooth and steady. In addition, if a disturbance which causes separation is introduced into the inlet flow it must be damped soon. The transition from steady laminar flow to the self-sustained oscillating flow does not occur abruptly and there is always a certain transition region where intermittent flow occurs. This implies that another criterion is necessary to determine the Reynolds number at which the flow has completely transferred into the self-sustained oscillating regime. Flow visualization serves well for determining this Reynolds number too and the flow is considered to be in the self-sustained oscillating regime if for a fixed Reynolds number and time period of five minutes the flow undergoes steady oscillations and is free from occurrence of steady laminar flow.

6. Experimental Results and Discussion

Photographs (a) and (b) in Fig. 6 show the flow in steady laminar and self-sustained oscillating flow regime respectively. The vortex street shown in Fig. 6(b) is formed by axisymmetric toroidal vortices — schematically sketched in Fig. 4 — which are generated periodically and alternately on both side walls of the flow passage.

Typical results are presented in Fig. 7. Each graph is the plot of the Strouhal number against the Reynolds number for constant spacing and different apex angle. The critical Reynolds number, Re_{cr} , as well as the Reynolds number for the onset of the self-sustained oscillating regime are designated by vertical dotted lines.

As presented in Fig. 7, the flow is laminar when the Reynolds number is below Re_{cr} . When the Reynolds number increases intermittent flow separation

(a) $Re = 520$ (b) $Re = 730$ Fig. 6 Flow patterns for, $\beta=60^\circ$, $\psi=0.3$, $\varphi=10.69$

occurs in the flow passage and when the Reynolds number reaches a second critical value, the intermittence vanishes and the flow separation becomes regular. This denotes the beginning of the self-sustained oscillating regime in which the Strouhal number is evaluated. The Strouhal number decreases as the Reynolds number increases, and seems to reach a constant value before the transition into turbulence. The constant value of the Strouhal number falls within the range 0.2 - 0.3. With a further increase in the Reynolds number the turbulent flow region is reached. However, no boundaries are given in this paper since clarification of the transition into turbulence requires more detailed investigation.

Figure 7 also shows that for constant ψ the critical Reynolds number decreases when the apex angle β is increased. This tendency is replotted in Fig. 8 for two measured dimensionless passage spacings ψ . The decrease in the critical Reynolds number with an increase in β is attributed to an increase in adverse pressure gradient which is the driving force of the flow separation. The area-expansion-index parameter A_e which is a measure of the adverse pressure gradient induced by virtue of the geometry of the flow passage is defined as (refer to Fig. 3)

Fig. 7 Re - S relationship Fig. 8 Re_{cr} β relationship

$$A_e = \frac{1}{A} \cdot \frac{\partial A}{\partial \left(\frac{l_x}{r_{01}} \right)} = - \frac{\sin \alpha}{1 + \frac{l_x}{r_{01}} \cdot \sin \alpha - \frac{\psi}{2} \cdot \cos \alpha} \quad (5)$$

Figure 9 shows the variation of the area-expansion-index parameter throughout the flow passage. The parameter has its maximum value at the inlet of the flow passage and decreases towards the outlet. If the outlet is in infinity ($l \rightarrow \infty$) then irrespective of the apex angle, except for $\beta=0^\circ$, the situation at the flow passage outlet approaches the case of two infinite

Fig. 9 Variation of the area-expansion-index parameter throughout the flow passage

Fig. 10 Effect of the apex angle on the Strouhal number

Fig. 11 Effect of the passage spacing on the Strouhal number

parallel plates for which no adverse pressure gradient acts on the flow and $A_e \rightarrow 0$.

The smallest variations of the A_e parameter occur for the flow passages with the smallest apex angle ($\beta = 60^\circ$) and become larger as the apex angle is increased. The following discussion focuses on the relationship between geometry and pressure gradient (noting, however, that the pressure gradient does not depend only on a flow passage geometry). The larger the variations of the area-expansion-index produced by the flow passage shape are, the severer is the adverse pressure gradient induced in the flow. Looking at Fig. 8 it is easy to deduce that the flow will be exposed to the severest adverse pressure gradient in the limit case of $\beta = 180^\circ$ which corresponds to the flow passage formed by two parallel disks. In contrast the case of $\beta = 0^\circ$ corresponds to a flow passage for-

Fig. 12 Correlated Strouhal number-Reynolds number relationship for $\psi = 0.3, 0.4$ and $\beta = 60^\circ, 90^\circ, 120^\circ$

med by two concentric cylinders for which $A_e = 0$ and no adverse pressure gradient can be expected to affect the flow.

Summarizing the previous paragraph, an increase in the apex angle basically exposes the flow to a severer adverse pressure gradient which results in earlier separation when increasing the Reynolds number. Thus the critical Reynolds number decreases when the apex angle is increased, as shown in Fig. 8.

Figure 8 also reveals that the passage spacing ψ has a remarkable influence on the value of the critical Reynolds number. Comparing the critical Reynolds numbers measured for $\psi = 0.3$ and $\psi = 0.4$, the critical Reynolds number measured for $\psi = 0.3$ is for all measured apex angles larger than in the case of $\psi = 0.4$. This is again caused by the difference in the adverse pressure gradient induced in the flow by virtue of the geometry. As shown in Fig. 9, for a constant apex angle the area-expansion-index parameter is always smaller for smaller passage spacings which means that the severity of the adverse pressure gradient decreases with a decrease in passage spacing. This results in larger critical Reynolds numbers for smaller passage spacings.

The effects of apex angle β and passage spacing ψ on the Strouhal number are presented in Figs. 10 and 11 respectively. The apex angle β is found to have little effect on the Strouhal number which falls well onto one curve for all three measured apex angles. However, the Strouhal number differs for the two different passage spacings; it is always larger for smaller ψ . This indicates that the current Strouhal number definition makes the Strouhal number dependent not only on the Reynolds number but on the passage spacing as well. The Strouhal number becomes independent of the passage spacing when it is modified as:

$$S^* = \frac{f \cdot 2s}{U_1} \cdot \psi^{1.3} \quad (6)$$

Figure 12 shows data for all measured cases replotted according to Eq. (6). The new empirically determined Strouhal number modification results in the

Fig. 13 Situation at the inlet for two different cone spacings

data correlating well onto one curve, thus making the Strouhal number a function of the Reynolds number only. It is noted, however, that this correlation so far qualifies for only two measured passage spacings. To extend its reliability, further investigation of different passage spacings is needed.

It is necessary to mention that an effect from difference in inlet geometry can be expected. Despite all the precautions taken it is impossible to avoid changes to inlet geometry when changing either of the two parameters β and ψ . As shown in Fig. 13 the flow undergoes a different contraction ratio for two different cone spacings when entering the annular-conical passage. The contraction ratio differs by about 17% for the spacings $\psi=0.4$ and $\psi=0.3$. A larger contraction ratio always makes for a more uniform velocity profile which means that the local velocities near a wall become larger (when compared to a developed velocity profile) and thus the near-wall layer becomes more resistant to a separation. This could help to stabilize the flow and thus to a certain extent affect the value of the critical Reynolds number.

The present experimental apparatus also does not allow variation of the nondimensional length φ independently of the other geometrical parameters. Thus it is impossible to determine the pure effect of φ on the flow behavior. The effect is, however, considered to be very small, as compared to the effect of the

other two geometrical parameters.

7. Conclusions

The main contributions of this study are summarized as follows:

1. It is found that the critical Reynolds number is a function of both the apex angle β and the nondimensional spacing ψ . For constant ψ the critical Reynolds number decreases as the apex angle β is increased. When, on the other hand, the apex angle β is kept constant and ψ is varied, the critical Reynolds number is smaller for larger values of ψ .
2. The Strouhal number is found to decrease with increasing Reynolds number for all the measured cases.
3. The apex angle β is found to have almost no influence on the Strouhal number. The Strouhal number, however, varies with a change in the cone spacing ψ when the conventional Strouhal number definition is used.
4. New Strouhal number modification which produces correlated results for two passage spacings ψ and three apex angles β is found.

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