The Effect of Longitudinal Tension on Flow in Collapsible Tube*

Atsushi SAKURAI**, Kenkichi OHBA**, Yoshiyuki FUTAGAMI*** and Masami TSUJIMOTO**

Simulation experiments in vitro related to flow through a vein and an airway of the lung were performed. The effect of longitudinal tension applied to thin-walled silicone rubber tubes on the relationship between the cross-sectional area of the tube and the transmural pressure, and on static and dynamic relationships between pressure drop and flowrate through the tube was investigated using five different kinds of specially designed tubes of the same size with different applied longitudinal tensions. As a result, the following facts were clarified: Applying longitudinal tension to the tube made it more difficult for the tube to be collapsed. The movement of the tube wall along the tube axis during the self excited oscillation of flow was restricted and the most greatly collapsed portion shifted upstream. The amplitude of the oscillatory pressure drop was much larger than its time-averaged value. The time required for the tube to collapse gradually from a fully open state amounted to 80 to 85% of the period of the oscillation.

** Key Words**: Collapsible Tube, Self-Excited Oscillation, Effect of Longitudinal Tension on Oscillation, LDV Measurement, Bio Fluid Mechanics

1. Introduction

The flow through a collapsible tube, such as a venular vessel, an airway of the lung or a rubber tube, is very complicated, because the cross-section of the tube varies non-linearly with the transmural pressure (the difference between the internal and the external pressure of the tube). In experiments using thin-walled rubber tubes as in vitro models, which simulate blood vessels in vivo, it is known that a self-excited oscillation takes places in a flow through a collapsible tube when the tube is perfused at constant external pressure. Since these phenomena are physiologically important, many investigations have been done on this flow problem. In particular, much experimental and theoretical work associated with the self-excited oscillation has been performed. However the mechanism of the oscillation itself has not yet been elucidated.

In this paper, we describe the results of experiments regarding the effects of the longitudinal tension, such as that which acts on blood vessels in a living body, on the characteristics of flow through a collapsible tube. Five different kinds of specially designed thin-walled silicone rubber tubes of the same size with different applied longitudinal tension were used to investigate these effects. The effect of longitudinal tension on the relationship between the cross-sectional area of the tube and the transmural pressure, and on static and dynamic relationships between pressure drop and flowrate through the tube was investigated. Furthermore, local flow velocity at the tube axis during the oscillation was measured by using a fiber optic laser Doppler velocimeter (LDV) and the movement of the tube wall during the oscillation was recorded using a high-speed video camera. The interaction between the local flow velocity at the tube axis and the tube behavior during the oscillation was examined.
2. Apparatus and Experimental Procedure

As collapsible tubes we used thin-walled silicone rubber tubes of five different sizes. These tubes were designed so as to have the same size for each designated stretch ratio under the assumption that silicone rubber was incompressible and isotropic for a small stretch ratio. Table 1 shows tube dimensions of five different kinds of tubes for unstretched states (initial dimensions) and for stretched states. Measured values of an axial tension applied to the tube for each stretch ratio are also shown. Each tube becomes about 6.1 mm in outer diameter and about 0.2 mm in wall thickness for the corresponding designated stretch ratio ($\Delta l/l_0 = 0, 25, 50, 75$ and 100%). Therefore, the experiments were conducted under the same experimental conditions except for the applied axial tension (or stretch ratio).

A schematic diagram of the flow system is shown in Fig. 1. The collapsible tube is immersed in a liquid inside a transparent airtight box made of an acrylic acid resin. The tube is attached at both ends to rigid tubes in the box. These rigid tubes are set 160 mm apart and aligned along the tube axis, so the length of the collapsible tube L is constant (160 mm). The stretch ratio for each tube was achieved by using various initial lengths $l_0$. Five different stretch ratios $\Delta l/l_0 (\Delta l = L - l_0)$ within the elastic limit of silicone rubber, i.e., 0, 25, 50, 75 and 100%, were applied. An air compressor was used to apply a constant external pressure to the tube $p_0$, which was measured at the tube axis. A sucrose solution of concentration of 40.5 % was used as the working and surrounding fluids of the tube. Density and kinematic viscosity of the fluid were 1160 kg/m$^3$ and 3.66 x 10$^{-6}$ m$^2$/s, respectively. Fluid from the tank was accelerated by a pump and steady flow was supplied to the collapsible tube. After its time-averaged flowrate $Q_0$ was measured using a rotameter, it passed through a collapsible tube and flowed into the air at the same height of the tube and then returned back to the tank. The time-averaged pressure drop through the tube, $p_i - p_0$, and the flowrate at the downstream of the tube, $Q_0$, were measured using a semiconductor pressure transducer and an electromagnetic flowmeter, respectively. Since the time averaged value and the time variation of pressure and flowrate were measured using different instruments, the consistency between these measurements were estimated. Measured time variations of pressure and flowrate, $p_i - p_0$ and $Q_0$, were averaged over the measuring time (about 1 second) and these values were compared with $p_i$ and $Q_i$, respectively. The time-averaged value of $p_i - p_0$ and $p_0$ was agreed within the error of 5%. The error between the time-averaged value of $Q_i$ and $Q_0$ was also within 5%. The local flow velocity during a self-excited oscillation was measured using a fiber optic LDV, because the tube wall was almost transparent in the liquid used in this experiment. The motion of the tube wall was also recorded using a high speed video camera having 5 ms resolution. In order to clarify the interaction between the motion of the tube wall and the change in the local flow velocity, these data were examined and compared. Furthermore the effect of the longitudinal tension on the static relationship between tube cross sectional area and the transmural pressure (the difference between the internal and the external pressure of the
tube) was examined.

3. Results and Discussion

3.1 The effect of longitudinal tension on tube compliance

Figure 2 shows the effect of longitudinal tension on the relationship between the transmural pressure \( p_t \) and the mean cross sectional area of the tube \( A \). The quantity \( A \) was calculated by dividing the tube volume by the tube length \( L \). The quantity \( p_t \) and the compliance per unit length of the tube \( C/L \) are taken as the ordinate. The relationship between \( p_t \) and \( A \) was fitted by a cubic spline curve, because \( C/L \) was calculated as the reciprocal of the slope of the \( p_t - A \) characteristic curve, i.e., \( C/L = dA/dp_t \). Figure 2(a) shows the \( p_t - A \) characteristics for the whole measured \( A \), i.e., the region where the tube is entirely collapsed to the region where it is fully distended and the detail of the characteristics in the partially collapsed region is shown in Fig. 2(b). The origin of \( p_t \) for each stretch ratio in Fig. 2(b) is shifted to clarify the differences among characteristics. As is well known, \( C/L \) has a large peak in the region where the tube is partially collapsed and \( C/L \) is almost zero in the region where the tube is fully open and in the region where the tube is entirely collapsed. The region where \( C/L \) has a large value lies in almost the same range of \( A \) and is independent of the stretch ratio of the tube. It is also clear from Fig. 2(b) that the slope of the \( p_t - A \) curve tends to increase with increasing stretch ratio, whereas the maximum value of \( C/L \) tends to decrease. The characteristic for the stretch ratio \( \Delta U/U_0 = 25\% \) was different from that for the other stretch ratios despite repeated measurements. It seems that a tube to which a small longitudinal tension is applied may be easily influenced by experimental conditions such as the initial state of the tube, the tube mounting, etc. However, it was shown that the interaction between the flow through the tube and the tube deformation was reduced due to the decrease in tube compliance with the increase in the stretch ratio of the tube.

3.2 Relationships between time-averaged and dynamic pressure and flowrate characteristics

Figure 3 shows the effects of longitudinal tension and the external pressure \( p_e \) on time-averaged pressure drop through the tube, \( p_e \), and flowrate, \( Q_e \). The solid curve in Fig. 3 represents the \( p_e \) \( Q_e \) time-averaged characteristic of a rigid tube having the same inner diameter as that of the collapsible tube. It is known that the \( N \) shaped \( p_e \) \( Q_e \) characteristics which were measured in our experiments were caused by a comparatively high downstream resistance. Upward and downward vertical arrows near the \( p_e \) \( Q_e \) curves show the initiation and the termination of the self excited oscillation, respectively. Therefore, the self excited oscillation of flow exists in the range of flowrates between the arrow marks. Figure 4 shows the relationship between the dynamic characteristics of pressure, \( p_t - p_e \), and downstream flowrate, \( Q_e \) represented as a Lissajous's figure. The three figures in Fig. 4 correspond to the results for three different
stretch ratios, i.e., $\Delta l/l_0 = 0$, 50 and 100%. The external pressure $p_e$ is 3.92 kPa. The time-averaged flowrate $Q_n$ of the three Lissajous's figures in each figure correspond to flowrates of 0.5, 1.5 and 2.5 l/min. The operating point in each Lissajous's figure rotates counter-clockwise and dots on the figures represent a temporal position of operating point for every 1 ms. The dashed curve in each figure shows the $p_e-Q_n$ static characteristics with experimental conditions corresponding to those described for Fig. 3. The time-averaged $p_e-Q_n$ characteristics become almost flat in these figures due to the very large amplitudes of $p_1-p_2$ and $Q_n$, especially $p_1-p_2$. It is also shown that $p_e$ corresponds to the region of low $p_1-p_2$ which lies on the base of the triangle-like Lissajous's figure.

We can consider that Lissajous's figure is divided into two phases. One is the phase corresponding to the base of the triangle-like Lissajous's figure, in which $p_1-p_2$ shows a low value. In this phase dots on Lissajous's figure are clustered close together and about 80% to 85% of the period of the oscillation occurs in this phase. The tube is gradually collapsed from a fully open state, but not yet collapsed entirely in this phase. It is due to this process that $p_e$ has a small value compared with the amplitude of $p_1-p_2$ of the corresponding flowrate. The another is the phase in which $p_1-p_2$ has a large value. In this phase, which succeeds the above mentioned phase, the tube collapses rapidly and then opens rapidly. It takes an almost constant time independent of the stretch ratio and the flowrate. It took about 20 ms in the case of the stretch ratio $\Delta l/l_0 = 0\%$ and 50% and about 30 ms in the case of $\Delta l/l_0 = 100\%$, respectively. Therefore, it is shown that the period of the oscillation is dependent on the period in which the tube is gradually collapsed from the fully open state. In the case of $\Delta l/l_0 = 100\%$, there is a sudden decrease in $p_1-p_2$ in the base of the triangle-like Lissajous's figure. It is considered that the decrease of upstream pressure is caused by a propagation of the dent as is shown in Fig. 5, which corresponds to a local low pressure region in the tube. There is no such sudden decrease of $p_1-p_2$ for any other stretch ratio due to the large decay of the dent during a propagation.

3.3 The effect of the longitudinal tension on dynamic characteristics

The motion of the tube wall during the oscillation was observed by using a high speed video camera.

Fig. 3 Effects of longitudinal tension on the relationships between time averaged values of the pressure drop through the tube and flowrate

Fig. 4 Dependence of Lissajous's figures of pressure drop and downstream flowrate in one period of the oscillation on time averaged flowrate. Dashed curves show time averaged pressure drop and flowrate characteristics

JSME International Journal
The recorded tube motion for the three different stretch ratios, $\Delta l/l_0 = 0\%$, 50 and 100%, are shown in Fig. 5. The number shown at the right side of each photograph is the time elapsed until the tube is fully open and $l$ shows the distance measured from the downstream end of the tube. A rigid tube attached at the downstream end of the collapsible tube is also shown on the right side of each photograph. Fluid flows to the right in the photograph. In the case of $\Delta l/l_0 = 0\%$, the tube begins to collapse due to a pressure drop through the tube and the collapse becomes more marked with increasing elapsed time near the downstream end of the tube ($t = 0$ to 60 ms). The tube undergoes the greatest collapse at $t = 65$ ms and the dent is sucked into the downstream tube and moves toward the flow direction due to the inertia of fluid in a downstream tube ($t = 70$ ms). The tube opens abruptly in a short time ($t = 70$ to 80 ms) and the dent propagates upstream ($t = 80$ to 90 ms). It is considered that the dent is induced by the difference between the movement of the tube wall due to the elastic effect and the change of the pressure in the tube, i.e., the tube opens faster than the speed at which the pressure is increased in the tube.

For larger stretch ratios, the collapsed portion of the tube is shifted upstream and extends far upstream. Furthermore, the dent does not reach the downstream end. In the case of $\Delta l/l_0 = 100\%$, it is seen that the collapsed portion extends from the upstream to the downstream end during the collapsing phase of the oscillation. However, the tube remains open about 5 mm from the downstream end during the greatest collapse ($t = 100$ ms). The tube opens from the downstream end as it reverses the phase in which the tube collapses.

Figure 6 shows typical waveforms of the local velocity on the tube axis by using LDV, $u_{LDV}$, the pressure drop through the tube, $p_r - p_0$, and the downstream flowrate, $Q_0$, during the oscillation as measured in the experiments corresponding to the condition of Fig. 5. The time $t$ taken as an abscissa corresponds to the elapsed time in Fig. 5. The local velocity, $u_{LDV}$, near the downstream end of the tube increases in the phase that $Q_0$ gradually increases. However, $u_{LDV}$ begins to decrease just before the phase in which $Q_0$ reaches the maximum value because of the collapse of the tube. The phase of decreasing of $u_{LDV}$ becomes faster with the increase in $l$. In the case of $\Delta l/l_0 = 0\%$, there is a zero flow velocity phase because the tube is entirely collapsed near the downstream end. Furthermore, it is seen that the amplitude of $u_{LDV}$ decreases with the increase in $l$, especially when $l$ is larger than 10 mm. This is because the collapsed portion in the case of $\Delta l/l_0 = 0\%$ is localized in the vicinity of the downstream end of the tube as shown in Fig. 5. In the case of $\Delta l/l_0 = 50\%$, there is also a zero flow velocity phase in the waveform near the downstream end; however, a large amplitude of $u_{LDV}$ extends upstream in comparison with the case of $\Delta l/l_0 = 0\%$. In the case of $\Delta l/l_0 = 100\%$, a large amplitude oscillation of $u_{LDV}$ extends further upstream and $u_{LDV}$ at $l = 2$ to 6 mm shows a reverse flow just after the phase in which $p_r - p_0$ has the maximum value. It is thought that the fluid in the

![Fig. 5 Tube behavior in one period of oscillation for three different stretch ratios](image-url)
downstream of the collapsed portion is pulled to the upstream when the tube expands abruptly and the volume of the brought back fluid is depend on the change of the tube volume, i.e. the shape of the collapse. In the case of a large longitudinal tension a collapsed portion extends widely to upstream, so a reverse flow occurs further upstream.

It is also seen that a pulse of the flow velocity is generated just after the phase in which \( u_{LOS} \) decreases abruptly. This is thought to measure the velocity change which propagates with the dent of the tube, as shown in Fig. 5. This pulse propagates further upstream with the increasing stretch ratio.

From both the waveform of the flow velocity on the tube axis and the behavior of the tube wall during the oscillation, the decay of the velocity waveform depends on the longitudinal tension, because the shape of the collapse of the tube and the tube compliance change with the longitudinal tension.

An apparent propagation velocity of the pulse is calculated from the \( u_{LOS} \) data measured along the tube axis. As the decay of the pulse during propagation is large in the case of small stretch ratio, a mean propagation velocity at 30 mm from the downstream end was calculated. The propagation velocity, \( \bar{c} \), for the case of \( Q_o = 0.6 \text{ l/min} \) and 1.0 l/min were plotted against the stretch ratio. The results are shown in Fig. 7. The propagation velocity decreases with increasing stretch ratio and increases with the flowrate. This is considered to be caused by the difference in the shape of collapse for the different longitudinal tensions. In the case of a small longitudinal tension, the collapse of the tube locates in the vicinity of the downstream end the rest of the tube is almost fully opened. For a large longitudinal tension, the tube collapses over a great length along the tube.
axis. The propagation velocity of a fully opened tube is higher than that of a partially collapsed one, so that the propagation velocity for a small longitudinal tension is higher than that for a large tension.

Figure 8 shows the effect of longitudinal tension on the oscillation frequency of downstream flowrate \( f_{O} \). The frequency \( f_{O} \) is the greatest at the initiation of the oscillation and gradually decreases. In the midrange of \( Q_{o} \), \( f_{O} \) is almost constant regardless of \( Q_{o} \). For higher \( Q_{o} \), \( f_{O} \) gradually decreases again and the oscillation terminates. It is also seen that a smaller longitudinal tension yields a higher \( f_{O} \) for the same flowrate \( Q_{o} \). Because the fluid motion caused by the oscillation extends further upstream in the case of a large longitudinal tension, as is shown in Fig. 6, a larger mass of fluid oscillates for the larger longitudinal tension.

4. Conclusions

The effect of longitudinal tension applied to thin-walled silicone rubber tubes on the relationship between the cross-sectional area of the tube and the transmural pressure, and on static and dynamic characteristic relationships between pressure drop and flowrate through the tube was investigated using five different kinds of specially designed tubes of same size with different applied longitudinal tensions. As a result, the following facts were clarified.

(1) Applying longitudinal tension to the tube made it more difficult for the tube to be collapsed.

(2) The movement of the tube wall along the tube axis during the oscillation was restricted and the most largely collapsed portion shifted upstream.

(3) The period of the oscillation can be divided into two phases by the relationship between pressure drop through the tube and downstream flowrate represented as a Lissajous's figure. One is the phase in which the tube gradually collapses from a fully open state. The time required for this phase amounted to 80 to 85% of the period of the oscillation. The pressure drop through the tube in this phase remained a low value. The other phase is that in which the tube is rapidly collapsed and rapidly opened with a large variation in the pressure drop. The period of the oscillation depends on that of the former phase.

(4) The amplitude of the oscillatory pressure drop was ten to twenty times larger than its time-averaged value. The oscillatory pressure drop in the phase in which the tube collapses gradually from a fully open state corresponded to time-averaged value of the pressure drop.

(5) The apparent propagation velocity of the dent generated by the rapid opening of the tube became greater as the longitudinal tension was reduced.

A part of this research was supported by the Grant in Aid for Specially Remoted Research "Biomechanics" from the Ministry of Education, Science and Culture. The authors would like to express their thanks to Mr. Masayuki Toda, Mr. Yoshinao Matsui and Mr. Kohji Iizuka for their valuable contribution in performing the present experiments.

References


