Asymptotic Analysis on the Extinction of Diffusion Flames in Supersonic Stagnation-Point Flow*

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An asymptotic analysis is applied for the extinction of diffusion flames in supersonic stagnation-point flow. (It can be considered as the Tsuji-burner in a supersonic flow.) If airflow velocity is subsonic, the flame temperature decreases as airflow velocity increases because the flame stretch rate increases, and finally flame extinction takes place at a certain airflow velocity. However, in the case of supersonic airflow, the region in which a diffusion flame can be established between a detached shock wave and a porous body reappears, and flame extinction occurs when the airflow velocity decreases. This indicates shock heating has a stronger effect than high stretch-rate flow and dominant parameter for the phenomena is a temperature behind a detached shock wave. Moreover, it is clarified that the flame temperature rises sharply with Mach number of airflow.

Key Words: Diffusion Combustion, Extinction, Supersonic Combustion, Shock Wave, Scramjet Engine

1. Introduction

The effect of the strain rate on counterflow diffusion flames has been extensively explored both experimentally and theoretically as the most fundamental problem in combustion science. In some theoretical studies^{(1),(2)}, such effect was explained by the well-known S-curve theory, namely, a flame temperature of the counterflow diffusion flame becomes lower as the flow stretch rate increases (the Damköhler number decreases), and the flame finally disappears at a certain strain rate. Of course, this theory is limited to the case in which the flow velocity is subsonic.

However, diffusion flame in supersonic airflow has become of major interest recently, because it concerns the development of the SCRAM-jet engine. As for counterflow diffusion flame, the case of a high

temperature oxidizer has attracted the interest of many combustion researchers. For instance, Darabiha and Candel⁽³⁾ showed that flame extinction never occurs at any stretch rate when the temperature of the oxidizer exceeds the critical temperature, though it was already noted by Chung et al.⁽⁴⁾ and Saitoh⁽⁵⁾.

In this paper, diffusion flame of the Tsuji-burner⁽⁶⁾ in supersonic airflow was analyzed. There has been no research on this problem except a few analyses^{(4),(7)}. The purpose of the present study is to clarify the interaction between the effect of a large stretch rate and that of shock heating on flame extinction in supersonic airflow. Moreover, a basic outline for the more detailed numerical simulation is given.

Nomenclature

- a : velocity gradient in the radial distance (stretch rate) [1/s]
- B : frequency factor [$\text{m}^3/(\text{s kg})$]
- c : specific heat at constant pressure [$\text{J}/(\text{kg K})$]
- d : distance between a shock wave and a porous body [m]
- E : activation energy [kcal/mole]
- D : diffusion coefficient [m^2/s]

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- f_i : function given by Eq.(29)
 M : Mach number of airflow
 p : pressure [atm]
 Q : heat of reaction [kJ/g]
 R : universal gas constant [J/(mole K)]
 R_0 : radius of a porous sphere or cylinder [m]
 T : temperature [K]
 T_{ref} : reference temperature equal to $Q/(cY_{Fw})$ [K]
 u : velocity in the direction of x [m/s]
 v : velocity in the direction of y [m/s]
 x : distance from stagnation point along stagnation plane [m]
 y : distance normal to stagnation point [m]
 Y_i : mass fraction of species i
 z : nondimensional space variable given by Eq.(28)

Greek symbols

- β : parameter given by Eq.(27)
 δ : Damköhler number given by Eq.(31)
 θ : nondimensional temperature equal to T/T_{ref}
 ω_i : rate of reaction of species i [kg/(m³s)]
 λ : thermal conductivity [W/(m K)]
 γ : parameter given by Eq.(30)
 ρ : density [kg/m³]
 ν : mass stoichiometric ratio
 κ : specific heat ratio
 A : Damköhler number given by Eq.(13)
 η : nondimensional space variable
 $= \left(\frac{2a\rho_s c_s}{\lambda_s} \right)^{1/2} \int_0^y \frac{\rho}{\rho_s} dy$
 ξ : nondimensional space variable
 $= \frac{1}{2} \operatorname{erfc} \left(\frac{\eta}{\sqrt{2}} \right)$

Subscripts

- i : O or F
 F : fuel
 O : oxidant
 s : behind the shock wave
 w : the surface of a porous body
 a : the value of the flame sheet model
 1 : upstream of the shock wave
 2 : downstream of the shock wave

2. Formulation

The governing equations

The flow model and the coordinate system are schematically shown in Fig. 1. A fuel stream issued uniformly from a porous body and a supersonic airflow form a counterflow field with a detached shock wave, in which the location of the stagnation plane is in the shock layer. To simplify the governing equations, the following assumptions are introduced.

- (1) inviscid flow
- (2) changes of concentrations and temperature in

the radial direction (x) are negligible.

(3) The velocity of a fuel ejected from the surface of a porous body is small in comparison with the velocity of airflow behind the shock wave.

(4) The overall one-step reaction is considered.

The governing equations can be written as follows

$$\frac{1}{x} \frac{\partial}{\partial x}(\rho u x) + \frac{\partial}{\partial y}(\rho v) = 0, \quad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}, \quad (2)$$

$$\rho c v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + Q \dot{\omega}_F, \quad (3)$$

$$\rho v \frac{\partial Y_i}{\partial y} = \frac{\partial}{\partial y} \left(\rho D \frac{\partial Y_i}{\partial y} \right) - \dot{\omega}_i, \quad (i = O, F). \quad (4)$$

The reaction rate is defined as

$$\dot{\omega}_i = B_i \rho^2 Y_F Y_O \exp \left(-\frac{E}{RT} \right). \quad (5)$$

The radial velocity (u) is regarded to be proportional to the radial distance (x), $u = ax$. The axial velocity (v) is given by substituting this relation into Eq.(1).

$$v = -\frac{2a}{\rho} \int_0^y \rho dy. \quad (6)$$

Since the momentum Eq.(2) becomes independent of the others, only equations of energy and mass conservation of species remain to be solved. Equations (3) and (4) are transformed into their nondimensional forms by using a space variable and symbols shown in the nomenclature. The following well-known Eq.(8) are obtained.

$$\frac{d^2 \theta}{d\eta^2} + \eta \frac{d\theta}{d\eta} = -\frac{Y_{Fw} \dot{\omega}_F}{2a\rho}, \quad (7)$$

$$\frac{d^2 Y_i}{d\eta^2} + \eta \frac{dY_i}{d\eta} = \frac{\dot{\omega}_i}{2a\rho}, \quad (i = O, F). \quad (8)$$

The Le number $Le = 1$, $\rho\lambda = \text{const.}$ and $c = \text{const.}$

Fig. 1 Schematic of the problem

are assumed in the above equations. As shown in Fig. 1, the flow has a discontinuity at a shock wave. Therefore, paying attention only to the shock layer, boundary conditions of the air side are substituted from the conditions behind the shock wave. Moreover, since we consider only the stagnation point region, the normal shock conditions, the so-called 'Rankine-Hugoniot relations' are applied. Boundary conditions can be expressed as follows

$$\theta(\eta_s) = \theta_s, \quad \theta(\eta_w) = \theta_w, \quad (9)$$

$$Y_o(\eta_s) = Y_{os}, \quad Y_o(\eta_w) = 0, \quad (10)$$

$$Y_F(\eta_s) = 0, \quad Y_F(\eta_w) = Y_{Fw}. \quad (11)$$

The specific heat ratio is assumed to be constant in those relations. Therefore, the boundary values at the air side can be determined only by conditions upstream of the shock wave. Flame characteristics can be obtained explicitly as a function of Mach number of airflow with this simplicity.

A new transformation with yields and the following equations are obtained.

$$-\frac{1}{Y_{Fw}} \frac{d^2\theta}{d\xi^2} = \frac{d^2(\nu_i Y_i)}{d\xi^2} = \Lambda, \quad (12)$$

where,

$$\Lambda \equiv \frac{\pi \dot{\omega}_F \exp\{\sqrt{2} \operatorname{erfc}^{-1}(2\xi)\}^2}{a\rho}. \quad (13)$$

Actually η_s is comparatively large and so ξ_s is approximately regarded as 0. Since a stagnation plane is formed near the surface of a porous body from assumption (3), η_w can be estimated as 0 and so ξ_w becomes 1/2. Although Chung et al.⁽⁴⁾ treated η_w as $-\infty$, 0 is more close to a problem in actually.

Accordingly, boundary conditions are simplified as follows,

$$\theta(0) = \theta_s, \quad \theta(1/2) = \theta_w, \quad (14)$$

$$Y_o(0) = Y_{os}, \quad Y_o(1/2) = 0, \quad (15)$$

$$Y_F(0) = 0, \quad Y_F(1/2) = Y_{Fw}. \quad (16)$$

3. Asymptotic Analysis

3.1 Outer solution

The following equations are derived by using the similarity contained in Eq.(12).

$$\frac{d^2}{d\xi^2} \left(\frac{\theta}{Y_{Fw}} + \nu Y_o \right) = 0, \quad (17)$$

$$\frac{d^2}{d\xi^2} \left(\frac{\theta}{Y_{Fw}} + Y_F \right) = 0. \quad (18)$$

The relations between the mass fraction of species and the temperature are obtained by integrating Eqs. (17), (18) and boundary conditions.

$$Y_o = 2 \left(\frac{\theta_w - \theta_s}{\nu Y_{Fw}} - Y_{os} \right) \xi + \frac{\theta_s - \theta}{\nu Y_{Fw}} + Y_{os}, \quad (19)$$

$$Y_F = 2 \left(\frac{\theta_w - \theta_s}{Y_{Fw}} + Y_{Fw} \right) \xi + \frac{\theta_s - \theta}{Y_{Fw}}. \quad (20)$$

If the flame sheet model is employed to determine the flame location and the adiabatic flame temperature, temperature distribution at the fuel side becomes

$$\theta = 2(\theta_w - \theta_s - \nu Y_{Fw} Y_{os}) \xi + \theta_s + \nu Y_{os} Y_{Fw}, \quad (21)$$

and that at the air side becomes

$$\theta = 2(\theta_w - \theta_s + Y_{Fw}^2) \xi + \theta_s. \quad (22)$$

The flame location $\xi = \xi_a$ and the adiabatic flame temperature $\theta = \theta_a$ are calculated easily from Eqs. (21) and (22).

$$\xi_a = \frac{\nu Y_{os}}{2(Y_{Fw} + \nu Y_{os})}, \quad (23)$$

$$\theta_a = 2(\theta_w - \theta_s + Y_{Fw}^2) \xi_a + \theta_s. \quad (24)$$

In the outer region where a reaction does not occur, Eqs.(21) and (22) give the first approximation, and the following expansions of θ are derived as outer solutions.

$$\theta = 2(\theta_w - \theta_s - \nu Y_{Fw} Y_{os}) \xi + \theta_s + \nu Y_{os} Y_{Fw} + \frac{A_1 \xi}{\beta} + O\left(\frac{1}{\beta^2}\right), \quad (25)$$

$$\theta = 2(\theta_w - \theta_s + Y_{Fw}^2) \xi + \theta_s + \frac{B_1 \xi}{\beta} + O\left(\frac{1}{\beta^2}\right). \quad (26)$$

where β is the large parameter for expansion, defined as

$$\beta = \frac{E}{RT_{ref}} \left(\frac{T_{ref}}{T_a} \right)^2 = \frac{E}{RT_{ref} \theta_a^2}. \quad (27)$$

3.2 Inner solution

The same procedure as Liñán⁽²⁾ was utilized for the inner solution. The new stretched variable z is introduced.

$$z \equiv Y_{Fw} \delta_0^{1/3} \beta (Y_{Fw} + \nu Y_{os})(\xi - \xi_a). \quad (28)$$

The temperature distribution in the reaction zone, in which the adiabatic flame temperature is taken as the first approximation, is obtained as follows:

$$\theta = \theta_a - \frac{1}{\delta_0^{1/3}} \left[\frac{f_1}{\beta} + \frac{\gamma z}{\beta} + O\left(\frac{1}{\beta^2}\right) \right], \quad (29)$$

where,

$$\gamma = \frac{\nu Y_{os} - Y_{Fw}}{\nu Y_{os} + Y_{Fw}} + \frac{2(\theta_s - \theta_w)}{Y_{Fw}(Y_{Fw} + \nu Y_{os})}. \quad (30)$$

δ_0 is the new reduced Damköhler number, expressed by

$$\delta_0 = \frac{\pi B \rho}{\nu a} \frac{1}{(\beta Y_{Fw})^3} \times \exp \left\{ \frac{\sqrt{2} \operatorname{erfc}^{-1}(2\xi_a)^2}{(\nu Y_{os} + Y_{Fw})^2} \right\} \exp \left(-\frac{E}{RT_a} \right). \quad (31)$$

The exponential term in the reaction rate is approximated as

$$\exp \left(-\frac{E}{RT} \right) \cong \exp \left(-\frac{E}{RT_a} \right) \exp \{ -\delta_0^{-1/3} (f_1 + \gamma z) \}. \quad (32)$$

and Eqs.(19), (20), (29) and (32) are substituted into Eq.(12). Finally, the energy equation is transformed into a simple ordinary differential equation, which has the same form as that of Liñán⁽²⁾, namely,

$$\frac{d^2 f_1}{dz^2} = (f_1 + z)(f_1 - z) \exp \{ -\delta_0^{-1/3} (f_1 + \gamma z) \}. \quad (33)$$

The boundary conditions and coefficients in Eqs.(25) and (26) can be obtained for matching the inner and outer solutions as follows,

$$z = -\infty : \frac{df_1}{dz} = -1, \quad (34)$$

$$z = \infty : \frac{df_1}{dz} = 1, \quad (35)$$

$$A_1 = -\frac{1}{\delta_0^{1/3} \xi_a} \lim_{z \rightarrow -\infty} (f_1 + z),$$

$$B_1 = -\frac{1}{\delta_0^{1/3} \xi_a} \lim_{z \rightarrow \infty} (f_1 - z). \quad (36)$$

In particular, A_1 and B_1 can be calculated from a relation between f_1 and z .

4. Result and Discussion

4.1 The case of a sphere body

Equations (33)–(35) have been solved numerically. The following values are used for calculation, hydrogen being chosen as the fuel :

$$Y_{os}=0.233, Y_{fw}=1.0, R_0=0.025 \text{ [m]}, \nu=0.125$$

$$E=30 \text{ [kcal/mole]}, Q=121 \text{ [kJ/g]}, T_w=500 \text{ K.}$$

In particular, B is calculated from Nicholls's result⁽⁹⁾.

$$B=1.3 \times 10^8 \text{ [m}^3/\text{s kg]}$$

Other parameters such as ρ (a function of T) or c are suitably adopted from JANAF Tables⁽¹⁰⁾.

Figure 2 shows the Damköhler number in terms of the Mach number of airflow, calculated from Eq. (31). The stretch rate ($2a$) is needed for the calculation. In this calculation, it is obtained by dividing the flow velocity (v_2) behind the shock wave by the distance (d) between the shock wave and the porous body. The distance between the shock wave and a porous body is quoted from the experimental data⁽¹¹⁾. Then,

$$2a = \frac{v_2}{d}, \quad (37)$$

$$v_2 = M_2 \sqrt{\kappa R T_2}, \quad (38)$$

$$d = 0.143 \exp\left(\frac{3.24}{M_1^2}\right) R_0. \quad (39)$$

To confirm this empirical formula (39), numeri-

Fig. 2 The relation between Damköhler number and the Mach number of airflow

cal simulation was carried out. Euler equations which includes the conservation of 9 species were calculated. Figure 3 shows the result of the relation between the non-dimensional shock stand-off distance and the Mach number of airflow past a sphere. Data from CFD agrees well with Eq. (39).

The Damköhler number increases sharply when M_1 becomes large as in Fig. 2. Figure 4 shows the relation between $\theta_{\max}$ (nondimensional maximum temperature in the reaction zone which is regarded as the flame temperature) and the Mach number of airflow. The left ends of the curves show the limit of the stable solution of Eqs.(33)–(35) and express extinction points. The decrease of the Mach number of airflow (i.e., the decrease of airflow velocity) leads to extinction in the case of supersonic airflow. The extinction

Fig. 3 The comparison of experimental data and CFD data

Fig. 4 Dependence of the Mach number of airflow on the flame temperature

phenomena are demonstrated well by taking δ_0 as a parameter, as seen in Fig. 5. It is evident that the flame temperature decreases rapidly near extinction δ_0 . Slight differences for each T_1 are due to changes of γ in Eq.(30).

Large changes of the flame temperature are obtained in the case of supersonic airflow as in Fig. 4, compared with the upper branch of normal 'S-curve' for the case of subsonic airflow. The flame temperature of subsonic airflow, which is ordinary plotted on the log scale, approaches the constant value (adiabatic flame temperature). Such changes in the case of supersonic airflow are due to the increase of adiabatic flame temperature, as the first approximation, with the increase of boundary temperature (T_2) resulting from the shock heating. If detailed reaction model is considered in the analysis, the increase of flame temperature may not become so sharp.

4.2 The case of a cylinder body

The same analysis is conducted for a cylinder body in cases of both subsonic and supersonic airflow after rearrangement of the continuity equation. Flame stretch rates defined by the following formulas are used.

Supersonic airflow⁽¹¹⁾,

$$a = \frac{v_2}{d}, \quad (40)$$

$$d = 0.386 \exp\left(\frac{4.67}{M_1^2}\right) R_0. \quad (41)$$

When an airflow is subsonic, a shock wave does not exist and a flame stretch rates are quoted from the values of Jain and Mukunda⁽¹²⁾.

Subsonic airflow,

$$a = \frac{3v_1}{2R_0}, \quad (\text{sphere}) \quad (42)$$

Fig. 5 Dependence of the Damköhler number on the flame temperature

$$a = \frac{2v_1}{R_0}, \quad (\text{cylinder}). \quad (43)$$

Figure 6 shows $\theta_{\max}$ as a function of M_1 when the upstream air temperature is 700 K. As compared with the case of a sphere body which has the same radius, extinction points shift to the low Mach number side in cases of both subsonic and supersonic airflow. If airflow velocity is subsonic, the increase of the stretch rate expected by comparing Eqs.(42) and (43) results in extinction at a lower flow velocity than in the case of a sphere body. Accordingly, the region in which a flame can be established is reduced.

On the other hand, the distance between the shock wave and the cylinder body is much longer than in the case of the sphere body, especially at a low Mach number. Therefore, the stretch rate decreases and

Fig. 6 The comparison of a cylinder body and a sphere body

Fig. 7 Distribution of the pressure along the stagnation streamline for $M=3.0$ flow past a sphere

extinction takes place at a lower Mach number. As a result, the region in which a flame can be established is extended. This suggests that the shock wave affects the stretch rate as well as the adiabatic flame temperature.

Although an outline of the phenomena was obtained in this analysis, it was not possible to include consideration of more complicated problems, such as the effects of the curvature of a flame or a shock wave and pressure gradient. One of such example is shown in Fig. 7 by numerical simulation. This shows that the pressure along the stagnation streamline for $M=3.0$ airflow increases gradually in the shock layer, in which flame exists. More detailed numerical simulation is needed to overcome these problems.

5. Conclusion

Extinction analysis of a diffusion flame in a supersonic airflow was calculated. The effects of an increase of stretch rate and shock heating were considered simultaneously in this analysis. Two regions in which a flame can be established when airflow velocity changes from subsonic to high Mach number were demonstrated. In addition, it was shown that the flame temperature changes very sharply in the case of supersonic airflow.

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