Numerical Simulation of a Taylor Bubble in a Stagnant Liquid inside a Vertical Pipe*

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The feasibility of a detailed numerical simulation of a Taylor bubble in a stagnant liquid filling a vertical pipe was examined in the present study. The simulation was carried out using the volume of fluid method. Since there have been few quantitative experiments on Taylor bubble shape, physical experiments under a wide range of Eötvös and Morton numbers were also conducted using sucrose solution and air at room temperature and atmospheric pressure. It was confirmed by experiments that the bluntness of the nose of the bubble, the flatness of the tail and the liquid film thickness around the bubble are strongly affected by the two dimensionless numbers. Calculated terminal rising velocities and bubble shapes agreed fairly well with all the measured ones, which indicates that the effects of drag force, buoyancy and surface tension force on the bubble were well predicted by the simulation.

Key Words: Multiphase Flow, Computational Fluid Dynamics, Taylor Bubble, Bubble Shape, Terminal Rising Velocity, Fluid Properties

1. Introduction

Most of the numerical simulations of gas-liquid two-phase flow have been based on averaged field equations such as the drift-flux model and two-fluid model^{(1),(2)}. Recent rapid progress in computer performance and numerical methods, however, enables us to predict gas-liquid two-phase flow in more detail. Particle tracking methods^{(3),(4)} may be one of the promising approaches to detailed simulation. The other promising method is an interface tracking method^{(5),(6)}. In the latter method, time evolution of the gas-liquid interface is solved using local-instantaneous field equations and jump conditions at the interface⁽¹⁾. We⁽⁷⁾⁻⁽¹⁰⁾ have been examining the feasibility of detailed simulation using one of the latter methods, the volume of fluid (VOF) method^{(5),(11)}. It

was confirmed in the previous examinations that we can obtain good predictions for (1) terminal rising velocity and shape of a single bubble in an infinite stagnant liquid, (2) fluctuating motion of a single bubble in a stagnant liquid, (3) lateral migration of a single bubble in a linear shear field and (4) indirect interaction of two in-line bubbles. As a result, we could demonstrate a high feasibility of detailed numerical simulations of bubbly flow.

On the other hand, the feasibility of detailed simulation of two-phase slug flow has not yet been confirmed since the previous examinations were conducted only for small bubbles. Gas-liquid two-phase slug flow in a vertical pipe is characterized by a large bullet-shaped bubble, the so-called Taylor bubble⁽¹²⁾. Since the terminal rising velocity and shape of the Taylor bubble are affected by the existence of the pipe wall and fluid properties, the analysis of the Taylor bubble can be regarded as a good benchmark test for a detailed numerical method of two-phase flow. Therefore, detailed simulation of the Taylor bubble in a vertical pipe was conducted in the present study using the VOF method. Since the amount of available experimental data on precise bubble shapes is small,

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physical experiments were also conducted in a wide range of flow condition.

2. Survey of Previous Works on Taylor Bubbles

Characteristics of the Taylor bubble such as its terminal rising velocity and shape have been studied both experimentally and theoretically since the 1910's⁽¹³⁾⁻⁽²⁶⁾. Early theoretical works⁽¹⁵⁾⁻⁽¹⁷⁾ were based on a potential flow theory so that the effects of surface tension and viscosity were not taken into account. On the other hand, experimental data obtained by extensively varying the fluid properties and pipe diameter indicated that the terminal rising velocity and shape are significantly affected by viscosity, surface tension, buoyancy and inertia. By taking into account these effects, White and Beardmore⁽²¹⁾ conducted a dimensional analysis using a large amount of available experimental data, and proposed a reliable graphical correlation of the terminal rising velocity shown in Fig. 1. The three dimensionless numbers, Eo , M and Fr , in Fig. 1 are the Eötvös, Morton and Froude numbers defined by

$$Eo = \frac{g(\rho_L - \rho_G)D^2}{\sigma}, \quad (1)$$

$$M = \frac{g\mu_L^4(\rho_L - \rho_G)}{\rho_L^2\sigma^3}, \quad (2)$$

$$Fr = \frac{V_B}{\sqrt{gD(\rho_L - \rho_G)}}, \quad (3)$$

where g is the gravity constant, ρ the density, D the pipe diameter, σ the surface tension, μ the viscosity, V_B the terminal rising velocity, and the subscripts G and L denote the gas and liquid phases, respectively. The Eötvös number represents the effects of surface tension and buoyancy. The Morton number, which is

also referred to as the property group, is mainly governed by liquid viscosity. This figure indicates that the dimensionless terminal rising velocity Fr is determined by Eo and M .

As for the bubble shape, Bretherton⁽¹⁸⁾ conducted an analysis concerning the effect of σ on the nose shape. His results are schematically drawn in Figs. 2(a) and (b). He pointed out that the nose of the bubble takes the configuration shown in Fig. 2(a) when $Eo \geq 3.73$ and in Fig. 2(b) when $Eo \leq 3.73$. In his analysis, the value 3.73 corresponds to the minimum Eötvös number necessary for the bubble to rise. Goldsmith and Mason⁽²⁰⁾ showed that in the case of high viscosity, (1) both ends of the bubble are spheroid, and (2) the top end is prolate and the bottom is oblate, as shown in Fig. 2(c). They also indicated that the degrees of prolateness of the nose and oblateness of the tail increase with σ , as schematically shown in Fig. 2(c). In addition, they reported that a wave disturbance shown in Fig. 2(d) will appear at the tail of the bubble when μ_L is high. Brown⁽²²⁾, Özgü et al.⁽²³⁾ and Fukano et al.⁽²⁴⁾ also investigated the effect of μ_L on the bubble shape. They observed that the thickness of the liquid film around the bubble increases with μ_L . Recently, Nickens and Yannitell⁽²⁵⁾ conducted a theoretical analysis on the effect of σ . They showed that the bluntness of the nose increases with σ , which is consistent with the result of Goldsmith and Mason.

As summarized above, although much effort has been made to clarify the effects of surface tension and viscosity on the bubble shape, no systematic investigations have been conducted to examine the effects of Eo and M on the Taylor bubble shape. In addition, there are no available experimental data applicable for comparison with calculated bubble shapes. Hence,

Fig. 1 White and Beardmore's graphical correlation of terminal rising velocity⁽²¹⁾; symbols indicate test conditions of the present experiments

Fig. 2 Bubble shapes reported by Bretherton⁽¹⁸⁾ (a), (b) and Goldsmith and Mason⁽²⁰⁾ (c), (d)

the first half of this paper is devoted to the presentation of a database concerning the effects of Eo and M on the shape of the Taylor bubble.

3. Experiments

3.1 Experimental techniques

Working fluids of the experiments were sucrose solution and air at room temperature and atmospheric pressure. Distilled tap water was used to make the sucrose solution. The solution was filled in a vertical acrylic pipe of 2 m length. The Eötvös number was varied by changing the pipe diameter so that three different pipes ($D=5.4, 10.2$ and 15.1 mm) were used. The Morton number was varied by adjusting the sucrose concentration. Air was injected from the bottom of the pipe. The volume of the injected air was adjusted so as to maintain the bubble length at about $2D$. This is because much CPU time would be required if the length of the bubble is long. The terminal rising velocity V_B was measured at a location 1.3 m above the bottom of the pipe using a VTR system. The measurement was conducted more than 20 times under constant M and Eo . Then, the measured V_B was determined by averaging all the obtained data. It was confirmed that no source of fixed error was detectable and uncertainty estimated at the 95% confidence level was 1.3% of the measured values. The test section for photographing the bubble is shown in Fig. 3. The pipe was surrounded by an acrylic block in order to eliminate optical distortion produced by the refraction at the pipe wall. Error evaluation based on geometric optics clarified that the acrylic block reduced the maximum error of measured bubble shapes to less than $0.02D$. The density of the sucrose solution ρ_L was measured by dividing the mass of the fluid by its volume. The liquid viscosity μ_L and the surface tension σ were measured using a rotational viscometer (Viscotester; VT-03, Rion Ltd.) and a capillary tube tensiometer, respectively. The uncertainties estimated at the 95% confidence level for ρ_L , μ_L and σ were 2.7%, 2.0% and 3.3%, respectively. The temperature of the air was also measured to obtain the physical properties of the air from a property table.

Test conditions are summarized in Table 1. They are also plotted in Fig. 1. In order to examine the effects of M and Eo , the experiments were conducted under the conditions of constant M or constant Eo . The experiments covered conditions in which the viscous force as well as the surface tension was dominant or negligible. Consequently, the entire range of possible value of Fr , i.e., $0 \leq Fr \leq 0.35$, was covered in the experiments, as shown in Fig. 1. It should be noted, however, that the maximum bubble

Reynolds number of the test conditions defined by

$$Re = \frac{\rho_L V_B D}{\mu_L} \quad (4)$$

was 87.8 so that all the flows examined were laminar. In the case of turbulent flow, bubble shape will not be axisymmetric or stable so that transient 3-D simulation is required. In addition, the numerical method adopted in this study does not take into account the turbulence. For these reasons, the present experiments were limited to laminar flow conditions.

3.2 Results and discussion

In order to verify the validity of the present experiments, measured terminal rising velocities were compared with White's graphical correlation in terms of the Froude number. The results are shown in Fig. 4. They agreed within $\pm 4.1\%$ error so that the validity of the present experiments was confirmed. Figure 5 shows photographs of the Taylor bubbles for all the test conditions. The size of each photograph in this figure was adjusted so as to make every pipe diameter look the same. It was confirmed by observation using the VTR system that the bubble shapes were so stable that they did not change with time. All

Table 1 Test conditions

case	ρ_L [kg/m ³]	ρ_G [kg/m ³]	$\mu_L \times 10^{-3}$ [kg/ms]	$\mu_G \times 10^{-5}$ [kg/ms]	σ [N/m]	D [mm]	Eo	$\log M$ [·]
1	1249	1.22	25.0	1.81	0.0798	5.4	4.47	-5.22
2	1249	1.22	25.7	1.81	0.0796	10.2	15.97	-5.17
3	1249	1.22	25.7	1.80	0.0793	15.1	35.14	-5.16
4	1268	1.22	55.0	1.81	0.0778	5.4	4.65	-3.82
5	1268	1.22	55.0	1.81	0.0778	10.2	16.60	-3.82
6	1268	1.22	55.0	1.81	0.0778	15.1	36.38	-3.82
7	1311	1.21	100.0	1.81	0.0760	5.4	4.92	-2.77
8	1311	1.22	97.0	1.81	0.0760	10.2	17.57	-2.82
9	1311	1.21	93.0	1.81	0.0760	15.1	38.50	-2.90
10	1313	1.23	153.0	1.80	0.0791	5.4	4.74	-2.08
11	1313	1.23	162.0	1.80	0.0765	10.2	17.48	-1.94
12	1313	1.23	169.0	1.80	0.0738	15.1	39.71	-1.82

Fig. 3 Schematic of test section for taking photographs

Fig. 4 Comparison between White and Beardmore's Froude numbers and the measured ones; symbols correspond to those in Fig. 1

Fig. 5 Photographs of Taylor bubbles

the measured bubble shapes traced from enlarged photographs are shown in Fig. 6. The following tendencies, which are consistent with those described in previous works, can be observed from this figure:

- (1) the tail of the bubble is flattened by increasing Eo or by decreasing M ,
- (2) the liquid film thickness increases with M ,
- (3) the nose of the bubble takes the configuration shown in Fig. 2(b) when $Eo=4.7$, that is, when the bubble does not rise at all,
- (4) when $M \geq -2.83$, that is, when μ_L is high, the oblateness of the tail increases with decreasing Eo

Fig. 6 The effects of Eo and M on the bubble shape

Fig. 7 The effects of Eo and M on the nose shape

from 16.9 to 4.7.

(5) the wave disturbance shown in Fig. 2(d) appears when Eo is low.

The effects of M and Eo on the nose shapes are summarized in Figs. 7(a) and (b), which indicates (6) the bluntness of the nose of the bubble increases with decreasing M under a constant Eötvös number, which results in an increment of the liquid

film thickness around the bubble (tendency (2)),

(7) the bluntness of the nose increases with decreasing Eu .

Tendencies (6) and (7) are also consistent with those in previous works^{(20),(25)}. The difference between the present and previous works is that the present work was carried out in terms of Eu and M .

Since various kinds of Taylor bubble shapes were obtained in a wide range of Eu and M , the present data will be useful for the verification of a detailed numerical method based on an interface tracking technique.

4. Numerical Analysis

4.1 Numerical method

An axisymmetric cylindrical coordinate system, the (r, z) system, was used in the analysis. In this system, local-instantaneous mass and momentum equations of incompressible two-phase flow are given by

$$\frac{\partial u_k}{\partial r} + \frac{\partial v_k}{\partial z} + \frac{u_k}{r} = 0, \quad (5)$$

$$\frac{\partial u_k}{\partial t} + u_k \frac{\partial u_k}{\partial r} + v_k \frac{\partial u_k}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu_k \left[\frac{\partial^2 u_k}{\partial r^2} + \frac{\partial^2 u_k}{\partial z^2} + \frac{1}{r} \frac{\partial u_k}{\partial r} - \frac{u_k}{r^2} \right], \quad (6)$$

$$\frac{\partial v_k}{\partial t} + u_k \frac{\partial v_k}{\partial r} + v_k \frac{\partial v_k}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu_k \left[\frac{\partial^2 v_k}{\partial r^2} + \frac{\partial^2 v_k}{\partial z^2} + \frac{1}{r} \frac{\partial v_k}{\partial r} \right] - g, \quad (7)$$

where the subscript k denotes the phase ($k=G$ or L), u the velocity in the radial direction, v the velocity in the axial direction, p the pressure, ν the kinematic viscosity and t the time. The density ρ in Eqs. (6) and (7) is defined by

$$\rho = (1-F)\rho_G + F\rho_L, \quad (8)$$

where F is the volumetric fraction of the liquid phase in a computational cell ($0 \leq F \leq 1$). The gas-liquid interface exists in a computational cell whose F value lies between 0 and 1. The location and curvature κ of the interface are obtained using the F values in the eight cells surrounding the interface cell. An additional pressure, p_s , due to surface tension is calculated by

$$p_s = \kappa \sigma. \quad (9)$$

The force resulting from p_s is added to the right-hand side of Eqs. (6) and (7) to include the effect of σ on the velocity and pressure fields. Since F moves with the liquid, it satisfies the following volume conservation of the liquid phase.

$$\frac{\partial F}{\partial t} + \frac{1}{r} \frac{\partial r F u_L}{\partial r} + \frac{\partial F v_L}{\partial z} = 0 \quad (10)$$

The above equation is solved to determine the time-advanced interface location.

The differences between the original VOF

method⁽⁵⁾ and the present method are as follows.

(1) The original VOF method assumes that $\mu_L = \mu_G$. On the other hand, different viscosities can be used for μ_L and μ_G in the present method.

(2) Calculations are conducted using double-precision variables in the present method to avoid numerical instability caused by round-off errors.

(3) The modified SOLA method⁽²⁷⁾ is adopted as an iterative solution scheme of the present method in order to reduce the CPU time.

The boundary conditions and initial bubble shape are shown in Fig. 8. Since the velocity gradient is large in the liquid film region, fine cells are used in this region. The top and bottom of the initial bubble shape were hemispheres of radius 0.8D. The two hemispheres were connected to a cylinder of the same radius. The number of cells assigned to the cylinder was adjusted to make the volume of the calculated bubble almost equal to that of the measured one. The liquid was fed from the top boundary in the downward direction at velocity V_{in} . The left wall, i.e., the pipe wall, was also moved in the downward direction at V_{in} . Symmetric and continuous outflow conditions were adopted for the right and bottom boundaries, respectively. The value of V_{in} was adjusted to be equal to the terminal velocity V_B of the calculated bubble in order to realize the simulation in the frame of reference fixed at the top of the bubble. In this frame of reference, the axial location of the nose of the calculated Taylor bubble must be fixed. Hence, the calculation was conducted several times by changing V_{in} until the nose of the bubble ceased to move in the axial direction. Each computation required about 8 hours of CPU time using a workstation, HP-730 (72 Mips, 22 MFLOPS).

Fig. 8 Initial bubble shape and boundary conditions

Fig. 9 Time evolution of calculated bubble shape ; $Eo=39.71$, $\log M=-1.82$

Fig. 10 An example of the time history of bubble location and velocity ; $Eo=39.71$, $\log M=-1.82$

Fig. 11 Comparison of the measured and calculated terminal rising velocities

4.2 Results and discussion

Figure 9 shows an example of calculated bubble shape evolution. In this figure, t^* is the dimensionless time defined by

$$t^* = \frac{V_{int}}{D} = \frac{V_B t}{D} \quad (11)$$

Since V_{in} was adjusted to be equal to V_B , the nose of the bubble soon ceased to move. In addition, the bubble shape became stable after $t^*=6$. The resulting stable shape was used in the comparison with the measured shape. Figure 10 shows an example of the time history of the axial position and velocity of the

Fig. 12 Comparison of the measured and calculated shapes

bubble nose in the frame of reference fixed to the pipe wall. As shown in this figure, the bubble was moving at a constant velocity soon after traveling a distance comparable to the pipe diameter, i.e., $t^* \geq 1$. It is well known that the velocity of the Taylor bubble reaches the terminal condition after traveling only a short distance, D or $2D^{(21)}$. Hence, the result shown in Fig. 10 is consistent with actual bubble behavior.

Comparisons between the measured and calculated terminal rising velocities are shown in Fig. 11. They agreed within $\pm 3.1\%$ error. It follows that drag, buoyancy and surface tension forces acting on the Taylor bubble were accurately predicted in the simulation. Figure 12 shows a comparison of the

Fig. 13 Dye traces below the bubble (from Ref. (26))

(a) $\log M = -1.82$ (b) $\log M = -2.90$ (c) $\log M = -5.16$

Fig. 14 Calculated velocity distributions; $Eo = 37.3$

measured and calculated Taylor bubble shapes. The velocity vectors in this figure were drawn in the frame of reference fixed at the nose of the bubble. As shown in this figure, the effects of Eo and M on the bubble shape were predicted well.

As for the velocity distribution inside the Taylor bubble, Goldsmith and Mason⁽²⁰⁾ observed that a single toroidal vortex exists inside the bubble. On the other hand, Kawaji et al.⁽²⁶⁾ recently measured velocity profile below the bubble using a dye generated by a photochromic method. Figure 13 shows the velocity profile taken from Ref. (26). Their experiments were conducted under the condition of $\log M = -8.26$. As shown in this figure, the liquid supplied from the film is falling like a jet and the liquid below the bubble is almost uniformly following the bubble. The calculated velocity distributions are shown in Fig. 14. Only

one-fourth of the velocity vectors were drawn in this figure so as to make the distribution clear. As can be seen from this figure, the toroidal vortex also appeared in the calculated velocity distributions inside the bubbles. As for the profile below the bubble, we can see that the velocity profile becomes similar to that in Fig. 13 as the Morton number approaches to those of Kawaji et al.'s experiments. Hence, quantitative agreement was obtained for the velocity distribution.

5. Conclusions

The feasibility of detailed numerical simulation of Taylor bubbles in a vertical pipe was examined using the volume of fluid method. Physical experiments were also conducted in order to obtain bubble shapes in a wide range of flow conditions. As a result, the following conclusions were obtained.

(1) Experimental data on the effects of Eötvös and Morton numbers on Taylor bubble shapes were presented, which will be of use in the verification of a detailed numerical method based on an interface tracking technique.

(2) The terminal rising velocity and shape of the Taylor bubble for a wide range of Eo and M were predicted fairly well with the VOF method.

(3) The calculated velocity distribution inside and below the bubble agreed with available experimental data.

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