Acceleration of Abrasive Particles in Premixed Abrasive Water Jet Nozzle*

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It has been shown that the premixed abrasive water jet system has a greater capacity for drilling and cutting than the conventional abrasive water jet system. In the present investigation, the motion of spherical particles in a premixed abrasive water jet nozzle is investigated theoretically. The flow in the nozzle is treated as one-dimensional flow. Convergent nozzles with and without a cylindrical focusing section are studied. The numerical results show that the particle velocity at the outlet of the nozzle has a tendency to increase in proportion to the square root of the injection pressure. The outlet particle velocity decreases with increasing diameter and specific gravity of the particles. The optimum length of the focusing section increases with increasing diameter and specific gravity of the particles.

Key Words: Multiphase Flow, Numerical Analysis, Jet Cutting, Premixed Abrasive Water Jet, Nozzle, Particle Velocity

1. Introduction

Since the rate of material removal by an abrasive water jet is considered to be a function of the abrasive velocity, the cutting ability of the jet depends on the abrasive velocity at the outlet of the nozzle. Acceleration of abrasive particles in a conventional abrasive water jet system, in which an abrasive water jet is formed by entraining abrasive particles into a high-velocity water jet stream in a tubular nozzle, was studied by Isobe et al.⁽¹⁾ Hamada et al.⁽²⁾ studied the acceleration of a steel ball in an annular jet. It has been shown⁽³⁾ that a premixed abrasive water jet system, in which an abrasive suspension jet is formed by propelling premixed abrasive suspension through a

nozzle, has a greater capacity for drilling and cutting than the conventional abrasive water jet system. The erosive properties of the premixed abrasive water jet in air^{(4),(5)} and under a submerged condition^{(6),(7)} have been studied by many investigators. Laurinat et al.⁽⁸⁾ studied the optimum nozzle configuration of the premixed abrasive water jet. They tested five types of nozzles, which have a tapered inlet section followed by a straight passage (focusing section) with different lengths, and pointed out the existence of the optimum length of the focusing section. Guo et al.⁽⁹⁾ studied the accelerating process of particles in the premixed abrasive water jet system theoretically. Nevertheless, there are still unclear points about the acceleration of abrasive particles and the optimum nozzle configuration.

In order to develop an efficient abrasive water jet system, the motion of abrasive particles in the premixed abrasive water jet nozzle is investigated theoretically. The optimum nozzle configuration and the effects of size and specific gravity of abrasive particles on the acceleration of the particles in the nozzle are clarified through numerical analysis. The numerical result on the optimum length of the focusing

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Fig. 1 Configuration of nozzles

section is compared with existing experimental results⁽⁸⁾.

2. Mathematical Formulation

2.1 Nozzle configuration and flow in the nozzle

Consider a convergent nozzle followed by a constant-diameter straight passage (focusing section) of diameter D_F ($=D_2=D_3$) as shown in Fig. 1. The lengths of the convergent section and the focusing section are L_C and L_F , respectively. The nozzle with a linearly convergent section is designated Type 1. The Type 2 nozzle has a curved convergent section with radius of curvature R_C .

In the following, we assume that the concentration of the abrasive suspension is low and the flow is not affected by the existence of the particles. The compressibility of water is neglected. When the flow in the nozzle is one-dimensional and the velocity and pressure of water do not change in the radial direction, the equation of continuity is

$$D^2 V_w = D_1^2 V_{w1} = D_2^2 V_{w2} = D_3^2 V_{w3} \quad (1)$$

where D is the diameter of the passage and V_w is the velocity of water. Subscripts 1, 2 and 3 designate the inlet of the nozzle, the inlet of the focusing section, and the outlet of the nozzle, respectively. The pressure in the nozzle is calculated by Bernoulli's equation.

$$\begin{aligned} p_1 + \frac{\rho V_{w1}^2}{2} &= p_2 + \frac{\rho V_{w2}^2}{2} + \zeta \frac{\rho V_{w2}^2}{2} \\ &= p_3 + \frac{\rho V_{w3}^2}{2} + \zeta \frac{\rho V_{w3}^2}{2} + \lambda \frac{L_F}{D_F} \frac{\rho V_{w3}^2}{2} \end{aligned} \quad (2)$$

Here p , ρ , ζ , and λ are the pressure, the density of water, the loss coefficient of contraction, and the coefficient of pipe friction, respectively. In the following numerical analysis, ζ is assumed to be zero unless otherwise specified. In conventional premixed abrasive water jet system, the diameter of the nozzle focusing section is 1 to 2 mm. Since the maximum jetting pressure of the premixed abrasive water jet system to date is 200 MPa⁽⁵⁾, the jetting pressure in the numerical analysis is taken to be 10 to 200 MPa.

Table 1 Drag coefficient equations

Re	C_D
$Re \leq 0.1$	$24.0/Re$
$0.1 < Re \leq 1.0$	$22.73/Re + 0.0903/Re^2 + 3.69$
$0.1 < Re \leq 10.0$	$29.1667/Re - 3.8889/Re^2 + 1.222$
$10.0 < Re \leq 100.0$	$46.5/Re - 116.67/Re^2 + 0.6167$
$100 < Re \leq 1000.0$	$98.33/Re - 2778.0/Re^2 + 0.3644$
$1000.0 < Re \leq 5000.0$	$148.62/Re - 4.75 \times 10^4/Re^2 + 0.357$
$5000.0 < Re \leq 10000.0$	$-490.546/Re + 57.87 \times 10^4/Re^2 + 0.46$
$10000.0 < Re \leq 50000.0$	$-1662.5/Re + 5.4167 \times 10^6/Re^2 + 0.5191$
$Re > 50000.0$	0.44

Accordingly, the Reynolds number in the focusing section is on the order of 10^5 and the coefficient of pipe friction can be calculated by Nikuradse's formula.

2.2 Equation of particle motion

Actual abrasive particles have irregular shapes. However, abrasive particles are assumed to be spherical for simplicity. Interactions between the particles are not considered. The equation of particle motion is given by^{(10),(11)}

$$M_A(dV_A/dt) = F_P + F_D + F_V + F_B \quad (3)$$

where M_A and V_A are the mass and the velocity of the particle, respectively. The first term on the right-hand side is the force due to the pressure gradient in the nozzle. The second term is the drag in steady motion at an instantaneous velocity. The third term is the virtual mass contribution which arises because acceleration of the particle requires acceleration of the fluid. The final term is the Basset history integral. The pressure gradient term F_P is given by

$$F_P = -\frac{4}{3}\pi r^3 \frac{dp}{dx} \quad (4)$$

where r is the radius of the particle. The drag term F_D is given by

$$F_D = \frac{1}{2}\pi r^2 \rho C_D (V_w - V_A) |V_w - V_A| \quad (5)$$

where C_D is the drag coefficient which is a function of Reynolds number based on the particle diameter and the velocity difference $V_w - V_A$. The drag coefficient is calculated using the formulas shown in Table 1⁽¹²⁾. The virtual mass term and the Basset history integral term are given by⁽¹¹⁾

$$F_V = \frac{2}{3}\pi r^3 \rho C_V \left(\frac{dV_w}{dt} - \frac{dV_A}{dt} \right) \quad (6)$$

$$F_B = 6C_B r^2 \sqrt{\pi \rho \mu} \int_0^t \frac{d(V_w - V_A)/d\tau}{\sqrt{t - \tau}} d\tau \quad (7)$$

$$C_V = 2.1 - 0.132 C_A^2 / (1 + 0.12 C_A^2) \quad (8)$$

$$C_B = 0.48 + 0.52 C_A^3 / (1 + C_A^3) \quad (9)$$

where μ is the viscosity of water and C_A is the acceleration modulus defined by Eq.(10)⁽¹¹⁾.

$$C_A = \frac{2r}{(V_w - V_A)^2} \frac{d(V_w - V_A)}{dt} \quad (10)$$

3. Numerical Results and Discussion

Equations (3) to (7) are solved numerically by use of a Runge-Kutta scheme. The following numerical calculations are carried out under the conditions mentioned below unless otherwise specified. The particle velocity at the nozzle inlet is the same as the velocity of water. The specific gravity of the abrasive particle is assumed to be 4.0. The jet is injected into the air at atmospheric pressure. The inlet diameter of the nozzle D_1 , the diameter of the focus section $D_F (= D_2 = D_3)$, and the length of the convergent section L_C are 7, 1, and 10 mm, respectively. The inlet radius of curvature of Type 2 nozzle R_C is 18.2 mm.

3.1 Nozzle without focusing section

Figure 2 shows the effects of the injection pressure on the water velocity V_w^* and the particle velocity V_{A2} at the outlet of the nozzle without focusing section. The water velocity V_w^* increases in proportion to the square root of the injection pressure. The particle velocity V_{A2} also increases in the same manner as V_w^* . Since the loss in the nozzle is assumed to be zero, V_w^* is the attainable maximum water velocity. In the following results, the velocities of water and the particle are normalized by V_w^* .

Figures 3 and 4 show the effects of the particle diameter $2r$ and the specific gravity of the particle ρ_A/ρ on the particle outlet velocity V_{A2} , respectively. The results shown in Figs. 3 and 4 are calculated for the injection pressure of $p_1 = 20$ MPa. The particle outlet velocity V_{A2} is hardly affected by the injection pressure in the range of $p_1 = 10$ to 200 MPa. Smaller and lighter particles can attain larger velocity at the nozzle outlet. The Type 2 nozzle can accelerate the particle more effectively than the Type 1 nozzle.

3.2 Nozzle with focusing section

In the constant-diameter straight passage downstream of the convergent section, the particle velocity increases because of the velocity difference between the particle and water. With increasing length of the focusing section L_F , the particle velocity approaches to the water velocity. The velocity of water in the focusing section, however, decreases with increasing L_F due to the pipe friction. Accordingly, an optimum length of the focusing section exists. Figure 5 shows the velocity distributions of water and the particle in the nozzle. The injection pressure is 20 MPa and the length of the focusing section is 10 mm. The diameter of the particle is taken to be $100 \mu\text{m}$. At the inlet of the focusing section, the particle velocity in the Type 1 nozzle is smaller than that in the Type 2 nozzle. However, the particle velocities at the outlet of the

Fig. 2 Effect of injection pressure on particle velocity and water velocity at outlet of nozzle (Nozzle without focusing section)

Fig. 3 Variations of particle outlet velocity with particle diameter (Nozzle without focusing section)

Fig. 4 Variations of particle outlet velocity with specific gravity of particle (Nozzle without focusing section)

Fig. 5 Distributions of water and particle velocity in nozzle (Nozzle with focusing section)

focusing section of Type 1 and Type 2 nozzles are almost the same. In the focusing section of Type 1 nozzle, a larger drag force acts on the particle and the particle velocity approaches the water velocity more rapidly than in the focusing section of Type 2 nozzle.

Figure 6 shows the effects of the nozzle inlet shape on the particle velocity at the outlet of the focusing section. The injection pressure is 20 MPa and the diameter of the particle is 100 μm . The particle velocity at the nozzle outlet V_{A3} increases steeply with L_F/D_F , takes a maximum value, and then

Fig. 6 Effect of nozzle inlet shape on particle outlet velocity (Nozzle with focusing section)

Fig. 7 Effect of particle diameter on particle outlet velocity (Nozzle with focusing section)

Fig. 8 Relationship between optimum focusing length and injection pressure

decreases gradually. When L_F/D_F is larger than 5, the particle velocity at the nozzle outlet is the same irrespective of the nozzle inlet shape.

The effect of particle diameter on the particle velocity at the outlet of the Type 1 nozzle is shown in Fig. 7. The injection pressure is 20 MPa and the length of the focusing section is 10 mm. A smaller particle attains a larger velocity at the nozzle outlet. The optimum length of the focusing section which gives the maximum particle velocity at the nozzle outlet is shown by the dotted line in Fig. 7. The optimum length tends to increase with increasing particle diameter. The ratio of the particle outlet velocity V_{A3} of the nozzle with the optimum focusing section to that of the nozzle without the focusing section is approximately 1.23 for the particle of 50 μm diameter and 1.16 for the particle of 400 μm diameter. In the case of smaller abrasive particle, the focusing section plays a more important role in the acceleration of the particle.

Figure 8 shows the relationship between the optimum focusing length $L_{F(opt)}$ and the injection pressure p_1 . The particle velocity $(V_{A3}/V_w^*)_{max}$ at the outlet of the nozzle with the optimum focusing section versus the injection pressure is shown in Fig. 9 for the Type 1 nozzle with the convergent section of $L_c = 10$ mm. The optimum focusing length tends to increase slightly with increasing injection pressure. The value of $(V_{A3}/V_w^*)_{max}$ is hardly affected by the injection pressure, i.e. the maximum particle velocity at the nozzle outlet increases in proportion to the square root of the injection pressure.

Figure 10 shows the effects of the specific gravity of the particle on the particle velocity at the nozzle outlet for the Type 1 nozzle of $L_c = 10$ mm. The injection pressure is 20 MPa and the particle diameter is 100 μm . A lighter particle attains a larger outlet

Fig. 9 Relationship between particle velocity at outlet of nozzle with optimum focusing section and injection pressure

Fig. 10 Effect of specific gravity of particle on particle outlet velocity (Nozzle with focusing section)

velocity. The optimum focusing length shown by the dotted line tends to increase with increasing specific gravity of the particle. The ratio of the particle outlet velocity V_{A3} of the nozzle with the optimum focusing section to that of the nozzle without focusing section is approximately 1.06 for the particle of $\rho_A/\rho=2$ and 1.45 for the particle of $\rho_A/\rho=8$. In the case of heavier abrasive particles, the focusing section plays a more important role in the acceleration of the particle.

The rate of material removal by the abrasive water jet is considered to be a linear function of the cumulative kinematic energy of impinging particles⁽¹³⁾. With increasing diameter and specific gravity of the particle, the particle velocity at the nozzle outlet decreases. On the other hand, the kinematic energy of the particle increases with increasing mass of the particle. Accordingly it is considered that an optimum diameter and specific gravity of the abrasive particle exist.

The numerical results presented above are obtained by assuming the loss coefficient of the contraction ζ to be zero. When ζ is 0.1 and 0.2, the particle velocity and the water velocity at the outlet of the nozzle are shown in Fig. 11 for the Type 1 nozzle of $L_c=10$ mm. The loss in the convergent section reduces the velocities of the particle and water at the nozzle outlet. However, the variations of V_{w3}/V_w^* and V_{A3}/V_w^* with L_F/D_F are similar to those at $\zeta=0$. The optimum length of the focusing section shown by the dotted line is hardly affected by ζ .

3.3 Comparison with experimental results

Laurinat et al.⁽⁸⁾ determined the optimum focusing length of the premixed abrasive water jet nozzle experimentally. They used five nozzles with different lengths. All nozzles had the same nozzle inlet geometry, i.e., a 3.8 mm long, 60 degree tapered section and a nozzle diameter of 1.6 mm. The abrasive was HP 50 Barton Garnet (specific gravity of 3.9 to 4.1 and grain

Fig. 11 Effect of loss coefficient of contraction on particle outlet velocity (Nozzle with focusing section)

Fig. 12 Effect of nozzle length on depth of cut and particle outlet velocity

size of 355 to 710 μm) and the abrasive flow rate was 1.3 kg/min. Aluminum specimens were cut at an injection pressure of 175 bar and a traverse speed of 50 mm/min. Figure 12 shows the relationships between the depth of cut and the nozzle length $L (= L_c + L_F)$ at the standoff distances of 5 and 10 mm. The particle velocity and the normalized kinematic energy of the particle $(V_{A3}/V_{A3max})^2$ at the nozzle outlet obtained by the present method are also shown in Fig. 12. The particle velocity is calculated for $\rho_A/\rho=4$, $2r=500 \mu m$, and $\zeta=0$. The optimum length of the focusing section obtained numerically agrees well with the experimental results.

4. Conclusions

The motion of spherical particles in a premixed abrasive water jet nozzle is investigated theoretically. The flow in the nozzle is treated as one-dimensional flow. The numerical result on the optimum focusing length is compared with the existing experimental results. The main conclusions are as follows.

(1) The particle velocity at the outlet of the premixed abrasive water jet nozzle increases in proportion to the square root of the injection pressure.

(2) The convergent nozzle followed by a constant-diameter straight passage (focusing section) can accelerate abrasive particles effectively.

(3) The existing experimental relationship between the depth of cut and the nozzle length can be explained well by the present results on the optimum focusing length.

(4) Smaller and lighter particles can attain larger velocity at the nozzle outlet.

(5) The optimum length of the focusing section tends to increase with increasing particle diameter and particle density.

In order to clarify the relationship between the cutting ability of the premixed abrasive water jet and the nozzle shape, the motion of abrasive particles not only in the nozzle but also in the jet stream must be investigated. The effects of the shape of the nozzle convergent section on the structure of the jet are not considered in the present investigation. The structure of the premixed abrasive water jet is also affected greatly by the grain size of the abrasive⁽¹⁴⁾. Further investigation is necessary.

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