Research on Fluid Elastic Vibration of Cylinder Arrays by Computational Fluid Dynamics* (Analysis of Two Cylinders and a Cylinder Row)

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Fluid elastic vibration of tube bundles is one of the most important factors that must be considered in the design of heat exchangers. Since this phenomenon is caused by the interaction between flow and structure, and is hence very complex and sensitive in nature, the design criteria established through many experiments, based on envelopes of the scattered data, have been the only reliable tools for the designers. The authors have developed an accurate CFD technique to solve this moving boundary problem. The fluid elastic vibration problems of two cylinders and a cylinder row are simulated using the CFD technique. The characteristic phenomena of vortex shedding, coupled with two-cylinder motion and fluid elastic instability of a cylinder row are successfully simulated.

Key Words: Computational Fluid Dynamics, Flow-Induced Vibration, Moving Boundary, Cylinders, Critical Velocity

1. Introduction

Cylinder array structures are used as heat transfer surfaces in boilers, heat exchangers, and the like in various power generation plants and chemical plants. In recent years, this kind of equipment has been designed with higher flow velocity; therefore, fluid elastic vibration should be discussed in greater detail. A number of studies⁽¹⁾ have been made on this phenomenon by experimental approaches mainly consisting of the analysis of design criteria⁽²⁾ which were obtained by summarizing the results of model tests. Although these evaluation techniques are now applied to the design of actual equipment, they are still not accurate

enough to predict all the problems caused fluid elastic vibration. However, the previous results varied considerably among experimenters because the phenomena are very sensitive to conditions which are difficult to control precisely. Therefore, it is very difficult to develop an experimental analysis of the mechanism of flow-induced vibration.

On the other hand, as a technique in computational fluid dynamics, the moving mesh method⁽³⁾ was developed for simulating flow-induced vibration. Recently, this method has been used by many researchers to study the unsteady flow around a single cylinder. Oogane et al.⁽⁴⁾ and Kuroda and Taniguchi⁽⁵⁾ have presented reports on the lock-in phenomenon in the case where a single cylinder is vibrated forcibly, and Tamura et al.⁽⁶⁾ and Oogane and Sakuta⁽⁷⁾ have also presented ones on the case where a cylinder is elastically supported. While there are few examples on the case of plural cylinders and those on a cylinder array are also scarce, though they are very important from a practical viewpoint.

The aim of this study is to clarify the fluid elastic vibration of cylinder arrays using the highly accurate moving mesh method which was developed by us⁽³⁾. In

* Received 18th August, 1995. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 61, No. 582, B (1995), pp. 503-509 (Received 17th June, 1994)

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the first step, two parallel cylinders, which are supported elastically and rigidly, are analyzed. This numerical technique is extended to a cylinder row which is arrayed infinitely in the direction perpendicular to the flow by applying the cyclic boundary condition. This analytical method is evaluated by comparing the results with the previous experimental data.

2. Nomenclature

- c : Viscous damping coefficient
 C_D : Drag coefficient
 C_L : Lift coefficient
 d : Diameter of a dimensionless cylinder (typical length)
 $F_{L,D}$: Dimensionless external force (lift direction and drag direction)
 f_n : Natural frequency
 K : Limiting coefficient for occurrence of fluid elastic vibration
 k : Spring constant
 m : Mass
 P : Dimensionless pressure
 $Re(Re_i, Re_t)$: Reynolds number
 (Suffix i indicates flow velocity at infinity and t indicates flow velocity between cylinders.)
 s : Pitch between cylinders
 S_t : Strouhal number
 t : Dimensionless time in physical space
 V : Dimensionless typical flow velocity
 V_{cr} : Critical flow velocity for occurrence of vibration
 x, y : Physical coordinates (dimensionless)
 γ : Mass ratio of fluid to cylinder
 δ : Logarithmic damping factor
 ζ : Damping ratio
 ξ, η : Computational coordinates
 ρ : Fluid density
 τ : Dimensionless time in computational space
 ω_n : Natural frequency in dimensionless angle
 Suffix
 x, y, z : Partial differential by suffix in physical space
 ξ, η, τ : Partial differential by suffix in computational space

3. Method of Analysis

In this analysis, calculation was performed by applying the moving mesh method⁽³⁾ to the flow analysis technique based on the MAC method, and also by introducing the equation of motion for the structure system to analyze interaction between of the fluid and the structure system. Here, the outline of the method of analysis and the introduction of the equation of motion for the structure system will be described.

3.1 Fundamental equations and discretization

The Navier-Stokes equation (N-S equation), which is a governing equation for two-dimensional incompressible viscous fluid, with the term of external force being omitted, is expressed as follows.

$$V_t + V \cdot \text{grad } V = -\text{grad } P + 1/Re \Delta V \quad (1)$$

Also, the continuous equation and the Poisson equation are expressed as

$$D = \text{div } V = 0 \quad (2)$$

$$\Delta P = -\nabla \cdot \text{grad } V - (D_t - 1/Re \Delta D) \quad (3)$$

These equations were discretized by finite differential method, and the simultaneous equations were converged by the iteration method (pseudo-line-SOR method). At this time, the moving mesh method, described later, was introduced to handle moving objects. To discretize the governing equations, the Euler backward difference was used for time, the upwind difference with the third-order accuracy (K-K scheme⁽⁸⁾) was used for the convection term, and the center difference with the second-order accuracy was applied to other terms.

The MAC method was applied to the calculation of the Poisson equation for the pressure field, and the pseudoimplicit method was applied to the N-S equation.

3.2 Moving mesh method⁽³⁾

To solve the above-mentioned governing equations by finite differential method, the generalized coordinate system is adopted, and the two-dimensional physical coordinates (x, y, t) are converted into the computational system (ξ, η, τ) taking into consideration the time t taken to handle moving objects. Specifically, by performing coordinate transformation on the governing equations, the flow around objects moving with time can be calculated:

$$\partial_x = \frac{1}{J} (y_\eta \partial_\xi - y_\xi \partial_\eta) \quad (4)$$

$$\partial_y = \frac{1}{J} (-x_\eta \partial_\xi + x_\xi \partial_\eta) \quad (5)$$

$$\partial_t = \partial_\tau + \frac{1}{J} (x_\eta y_\tau - y_\eta x_\tau) \partial_\xi + \frac{1}{J} (x_\xi y_\tau - y_\xi x_\tau) \partial_\eta \quad (6)$$

where $J = x_\xi y_\eta - x_\eta y_\xi$ (Jacobian).

3.3 Equation of motion for structure system

To analyze the fluid system and the interaction between of the fluid and the structure system, a vibration system model on the two-dimensional plane, as shown in Fig. 1, is introduced. Since this model is elastically supported independently in the X and Y directions, vibration in each direction is allowed. The equations of motion for this vibration system are described as

$$m \ddot{y} + c \dot{y} + ky = F_L \quad (7)$$

$$m \ddot{x} + c \dot{x} + kx = F_D \quad (8)$$

By nondimensionalizing these equations, the following equations can be obtained:

$$\ddot{y} + 4\pi\zeta f_n \dot{y} + (2\pi f_n)^2 y = \frac{1}{2} C_L \gamma \quad (9)$$

$$\ddot{x} + 4\pi\zeta f_n \dot{x} + (2\pi f_n)^2 x = \frac{1}{2} C_D \gamma \quad (10)$$

where

$$\left. \begin{aligned} \zeta &= \frac{c}{2\sqrt{mk}}, \quad \omega_n = \sqrt{\frac{k}{m}} = 2\pi f_n \\ C_{L,D} &= \frac{F_{L,D}}{(1/2)\rho V^2 d}, \quad \gamma = \frac{\rho d^2}{m} \end{aligned} \right\} \quad (11)$$

For discretization of Eqs. (9) and (10), the center difference with second-order accuracy is used.

As described above, the pressure field is determined by solving the Poisson equation and the velocity field is determined by solving the N-S equation by which the fluid force is determined. Then, the displacement of the object can be determined by solving the equation of motion for the structure system. All equations (Eqs. (1), (3), (9) and (10)) are solved for each time step to simulate the flow field including the moving boundary with the elastically supported cylinder.

Fig. 1 Vibration system model

Fig. 2 Flowchart of analytical method

ders.

The flowchart for this analytical program is shown in Fig. 2.

4. Analytical Results

4.1 Two cylinders arranged normal to flow

The flow around two cylinders arranged normal to the flow was analyzed. This analysis was also performed to verify the accuracy of our analytical method. For mesh generation in this analysis, a box mesh⁽⁹⁾ was adopted to increase the number of cylinders easily. Figure 3 shows the mesh used in this analysis. This mesh provides a method in which the calculation region is divided into a number of blocks and calculation is performed for each region. With this method, mesh generation is relatively easy, even in the flow field around a complicated shape, but a round object has a fictitious corner. At this portion, the boundary conditions of pressure are set by using the forward and backward difference scheme of second-order accuracy. Two sets of analytical conditions for the two parallel cylinders were selected: the two cylinders are rigidly supported and the two cylinders are elastically supported. The analysis conducted made under the conditions shown in Table 1.

4.1.1 Two fixed cylinders First, from among the analytical results for two fixed cylinders, the flow field at typical two pitch ratios between two cylinders, $s/d=2.0$ and 1.5 , with different flow phenomena is shown.

For two cylinders arranged parallel to the flow, the downstream flow is the same as that in the case of a single cylinder when the pitch ratio between cylinders is large. However, when the pitch ratio between cylinders is small ($s/d < 2.0$), vortices discharged in

Table 1 Analytical conditions for two cylinders

	Re_l	s/d	f_n	ζ	$\frac{\rho d^2}{m}$	Δt	Mesh
Rigid Supported	1000	1.2 ~3.0	—	—	—	0.01	95×113
Elastic Supported	1000	1.5	0.20	0.001	0.03	0.01	95×113

Fig. 3 Analytical mesh for two cylinders

the downstream flow interfere with each other. As a result, it is known that asymmetry occurs in the jetstream formed between two cylinders.^{(10),(11)} Figure 4 shows the reproduction of this phenomenon in this analysis. It is seen that in Fig. 4(b), in which the pitch ratio between cylinders is large, that the jet flow is almost symmetric, but the jet flow seen in Fig. 4(a), in which the pitch ratio between cylinders is small, is shifted greatly to one side of the cylinder.

Figure 5 shows the time history of lift fluctuation

Fig. 4 Velocity vector around two rigidly supported cylinders

obtained occurring in this analysis for pitch ratios of 1.5 and 2.0 between cylinders. It is apparent that for $s/d=2.0$, the lift fluctuation of two cylinders shows a waveform close to sinusoidal, which is typically observed with Karman vortices, and vortices are discharged with a phase difference of 180° , i.e., in reverse phase. At a dimensionless time t of about 100, the phase difference changes slightly. This indicates that the vortices discharged from two cylinders interfere slightly with each other. It is also found that for $s/d=1.5$, the waveform of the lift fluctuation is greatly disturbed, and the vortices discharged from the cylinders is no longer generated as a regular Karman vortex street because of the effect of the jetstream and the interference between vortices.

Figure 6 shows the frequency analysis result of the lift fluctuation. It is seen that for $s/d=2.0$, the discharge period of the Karman vortex street is predominant approximately at a Strouhal number of 0.2,

Fig. 5 Time history of lift fluctuation

Fig. 6 Frequency analysis result for lift fluctuation

while for $s/d=1.5$, the peak frequencies are observed in a certain frequency region. Figure 7 shows the comparison of typical peak frequencies obtained by this analysis with the existing experimental results⁽¹²⁾. In this figure, the above-mentioned phenomenon is expressed in relation to the Strouhal number and the pitch ratio, s/d , between two cylinders. As shown in the figure, for $s/d < 2$, the measured peak Strouhal numbers are distributed in a certain region, and the analytical result of the peak distribution region coincides approximately with the experimental value.

4.1.2 Two elastically supported cylinders

Figure 8 shows the flow state of two elastically supported parallel cylinders ($s/d=1.5$) for each time step. This figure shows that when the cylinders are supported elastically, they vibrate in a reverse phase mode in which the cylinders approach and separate from each other in the direction perpendicular to the flow. This is because the vortices discharged from two cylinders are in out of phase, as described above. Figure 9 shows the frequency analysis result for the lift fluctuation of two cylinders. It is found that as

with the fixed cylinders, plural peaks are observed, but the number of peaks decreases as compared with the case of fixed cylinders (Fig. 6). This is probably because, like the lock-in phenomenon of a single cylinder, all of the lift fluctuation components close to the natural frequency of the cylinder are drawn into it. Also, in addition to the peak at $St=0.2$, a second peak is observed at $St=0.26$ for cylinder A and at $St=0.12$ for cylinder B. This is thought to be the vortex discharge frequency excited by the jetstream between the two cylinders. That is to say, the mechanism is assumed to be that the jetstream is drifts to the cylinder A side due to the Coanda effect, the vortex formed on the jetstream side of cylinder A decreases and the vortex discharge frequency becomes increases ($St=0.26$); inversely, the vortex formed on the jetstream side of cylinder B increases and the vortex discharge frequency becomes lower ($St=0.12$). Such a jetstream drift also has a large effect on the vibration amplitudes of two cylinders. Figure 10 shows the displacement of the two cylinders in the Y direction. An interesting phenomenon is reproduced in which

Fig. 7 Strouhal number for two cylinders normal to flow

Fig. 9 Frequency analysis result for lift fluctuation

Fig. 8 Analysis of two elastically supported cylinders

Fig. 10 Displacement of two cylinders

because the jetstream drifts greatly toward cylinder A, the vibration amplitude of cylinder A becomes larger than that of cylinder B. Such asymmetry was reported in the previous experiment⁽¹³⁾.

4.2 Row of cylinders

The analytical conditions are shown in Table 2. The analytical region was a flow passage through five cylinders, as shown in Fig. 11, and a flow field in which cylinders make an infinite row in the direction perpendicular to the flow was analyzed by introducing the cyclic boundary condition.

In this analysis, the fluid elastic vibration in the case of a high-density fluid (water) with a relatively low Reynolds number was simulated for ease of calculation. Therefore, the mass damping parameter was set at a relatively low value. In setting the analytical conditions, the test results⁽¹⁴⁾ obtained by Chen and Jendrzejczyk, which describe the experimental conditions precisely, were consulted.

4.2.1 Fixed row of cylinders In the previous experimental study⁽¹⁵⁾, it is shown that the formation of vortices downstream of the cylinder array is very complicated, unlike in the case of a single cylinder, and when the cylinder pitch is small ($s/d < 2$), there exists a peak Strouhal number within a certain range,

Table 2 Analytical conditions for a cylinder row

	Re_t	s/d	f_n	ζ	$\frac{\rho d^2}{m}$	Δt	Mesh
Rigid Supported	4667 ~10800	2.5, 1.35	—	—	—	0.01	95×194
Elastic Supported	5786 ~10414	1.35	0.386 0.445	0.029 0.111	0.792 0.750	0.01	95×194

Fig. 11 Cyclic boundary condition

like in the case of two parallel cylinders. Therefore, the analytical results for small cylinder pitch ($s/d = 1.35$) and large cylinder pitch ($s/d = 2.5$) in the case where a row of cylinders is fixed are shown in Fig. 12. For large cylinder pitch, vortices are discharged regularly at a synchronized period on all cylinders, while for small cylinder pitch, irregular vortices are discharged with interfere with each other, so that the jet flow between the cylinders vibrates unsteadily. Figure 13 shows the lift fluctuation and its frequency analysis result for the typical cylinder, C (refer to Fig. 11), among the four cylinders. Figure 13 indicates that when the cylinder pitch is small, the power spectrum of the lift fluctuation has plural peaks corresponding to the flow state of complicated discharged vortices, and, like the case of two cylinders, a spectrum peak is present within a certain range of the Strouhal number. When the cylinder pitch is large, like in the case of a single cylinder, vortices are discharged at a single

(a) Pitch ratio $s/d = 2.5$

(b) Pitch ratio $s/d = 1.35$

Fig. 12 Velocity vector

Fig. 13 Frequency analysis result for lift fluctuation of cylinder (C)

frequency. This result is the same as that obtained from the previous experiment, and it is found that it agrees well qualitatively with the experimental result. Figure 14 compares the Strouhal number at this time with the existing experimental result⁽¹⁵⁾. They show good quantitative agreement, demonstrating that this analytical technique is valid for fixed cylinder arrays.

4.2.2 Elastically supported row of cylinders

Next, the analytical result will be described for the case where the cylinder array is fixed in the flow direction and elastically supported in the direction perpendicular to the flow. The previous study⁽¹⁶⁾ revealed that the occurrence of unstable vibration of a

Fig. 14 Strouhal number for a cylinder row

cylinder array is mainly caused by vibration in the direction perpendicular to the flow, which occurs even if the movement of the cylinder array in the flow direction is restrained. Therefore, in this analysis as well, the movement of tubes in the flow direction was restrained to facilitate the analysis of tube vibration, and analysis was made for the system vibrating only in the direction perpendicular to the flow.

Figure 15 shows the time history of the displacement for a row of cylinders at each Reynolds number. It is seen from Fig. 15 that while the vibration amplitude of cylinders at $Re \leq 10030$ is very small, the vibration amplitude increases gradually at $Re = 10400$, and eventually, the cylinders collide with each other. These results are plotted on the graphs in Fig. 16 where dimensionless flow velocity is on the horizontal axis and RMS value of the vibration amplitude (Y direction) of the cylinder is on the vertical axis. From this figure, the critical flow velocity in which the vibration amplitude increases rapidly can be determined. Figure 17 compares the divergence limit of fluid elastic vibration obtained in this analysis with the existing experimental result⁽¹⁷⁾. Figure 17 shows the experimental results obtained by other researchers. The results of this analysis agree well with the experimental results⁽¹⁸⁾ of Halle and Lawrence in terms of general trends, but the critical flow velocity tends to shift slightly downward compared

Fig. 15 Time history waveform of cylinder displacement

Fig. 16 Vibration amplitude of each cylinder

Fig. 17 Critical flow velocity of fluid elastic vibration for a cylinder row

with the results⁽¹⁴⁾ of Chen and Jendrzejczyk. In this respect, the combination of element parameters, including the mass damping parameter, and the experimental error as much as the error in this analysis are thought to be its causes. This is one of the important problems which require further study. If the evaluation equation of critical flow velocity is defined as Eq. (12) in accordance with the conventional evaluation method, the critical coefficient K and the power exponent α in the range of this analysis can be obtained as follows from the analytical result.

$$\frac{V_{cr}}{f_n d} = K \left(\frac{m \delta}{\rho d^2} \right)^\alpha \quad \delta = 2\pi\zeta \quad (12)$$

$$K = 1.71, \alpha = 0.09$$

5. Conclusions

The fluid elastic vibration phenomena on two parallel cylinders and a cylinder row are simulated by introducing the equation of motion of the structure system to the flow analysis technique using the mov-

ing mesh method. As a result, the following points were clarified.

(1) For two fixed parallel cylinders, it is known that the peaks in the lift power spectrum are in a certain range of Strouhal numbers at a pitch ratio of $s/d < 2$, and the same phenomenon was reproduced in this analysis. The analytical results agreed well with the previous experimental results.

(2) For two elastically supported parallel cylinders ($s/d = 1.5$), the vibration of cylinders out of phase with each other was reproduced, and an interesting phenomenon in which the vibration amplitude of one cylinder is larger than that of the other cylinder because of the drift of the jetstream between cylinders due to the Coanda effect was observed.

(3) For a fixed cylinder row, as in the case of two fixed parallel cylinders, the lift spectrum peak was reproduced in a certain range of Strouhal numbers at $s/d < 2.5$, this result agreed well with the previous experimental result.

(4) For an elastically supported cylinder row, the fluid elastic vibration phenomenon was reproduced. It was demonstrated that compared with the design criteria used so far. This analysis almost reproduced the experimental result not only qualitatively but also quantitatively.

The present study was carried out in cooperation with five electric power companies operating PWR plants in Japan. This cooperation is gratefully acknowledged.

References

- (1) Belevins, R. D., Flow-Induced Vibration, (1990). p. 88, Van Nostrand Reinhold.
- (2) Chen, S. S., Flow-Induced Vibration of Circular Cylindrical Structures, (1987). p. 424, Hemisphere Pub. Co.
- (3) Izumi, H., Taniguchi, N., Kawata, Y., Adachi, T. and Kobayashi, T., Numerical Simulation of Flow Fields with Moving Boundaries, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 60, No. 572, B (1994), p. 1218.
- (4) Oogane, K., Sakuta, H., Takahashi, T. and Shirakashi, M., Numerical Analysis for Lock-In Phenomenon of Vibrating Cylinder in Uniform Flow, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 57, No. 544, B (1991), p. 4058.
- (5) Kuroda, S. and Taniguchi, K., Flows Around an Oscillating Cylinder in a Uniform Flow, Proc. Jpn. Soc. Mech. Eng., Fluids Eng. Conf., (in Japanese), No. 920-68 (1992), p. 65.
- (6) Tamura, T., Tsuboi, K. and Kuwahara, K., Numerical Simulation of Unsteady Flow Patterns around a Vibrating Cylinder, AIAA Paper, 88-0128 (1988).
- (7) Oogane, K. and Sakuta, H., Numerical Analysis

- for Fluid Flow around an Elasticity-Supported Cylinder, Proc. Jpn. Soc. Mech. Eng., Spring Annu. Mtg., (in Japanese), No. 920-17 (Vol. A), (1992), p. 39.
- (8) Kawamura, T. and Kuwahara, K., Computation of High Reynolds Number Flow around a Circular Cylinder with Surface Roughness, AIAA Paper, 84-0340 (1984).
 - (9) Nakahashi, K., Grid Generation for Computational Fluid Dynamics, J. Information Processing Soc. Jpn., (in Japanese), Vol. 30, No. 7 (1989), p. 767.
 - (10) Spivack H. M., Vortex Frequency and Flow Pattern in the Wake of Two Parallel Cylinders at Varied Spacing Normal to an Air Stream, J. Aeron. Sci., (1946), p. 289.
 - (11) Bearman, P. J. and Wadcock, A. J., The Interaction Between a Pair of Circular Cylinders Normal to a Stream, J. Fluid Mech., Vol. 61, No. 3 (1973), p. 499.
 - (12) Kobayashi, T., Characteristics of Fluid-Dynamic Forces Acting on Circular or Square Cylinders in Close Proximity, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 42, No. 357 (1976), p. 1452.
 - (13) Jendrzejczyk, J. A., Chen, S. S. and Wambsganss, M. W., Dynamic Responses of a Pair of Circular Tubes Subjected to Liquid Cross Flow, J. Sound Vib., Vol. 67, No.2 (1979), p. 263.
 - (14) Chen, S. S. and Jendrzejczyk, J. A., Experiment and Analysis of Instability of Tube Rows Subject to Liquid Crossflow, J. Appl. Mech., Vol. 49 (1982), p. 704.
 - (15) Ishigai, S. and Nishikawa, E., Experimental Study of Structure of Gas Flow in Tube Banks with Tube Axes Normal to Flow, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 40, No. 337 (1974), p. 2599.
 - (16) Funakawa, M., Ishimatsu, T., Kumon, K. and Edakuni, K., Excitation Mechanism of Fluid-Elastic Vibration in Tube Array, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 471, C (1985), p. 2997.
 - (17) Chen, S. S., Guidelines for the Instability Flow Velocity of Tube Arrays in Crossflow, J. Sound Vib., Vol. 93, No. 3 (1984), p. 439.
 - (18) Halle, H. and Lawrence, W. P., Crossflow-Induced Vibration of a Row of Circular Cylinders in Water, ASME-IEEE Joint Power Generation Conf., ASME Paper, No. 77-JPGC-NE-4 (1977).