Simulation of Laminar Flow over a Backward-Facing Step Using the Lattice BGK Method*

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The lattice BGK method is applied to the simulation of a two-dimensional laminar flow over a backward-facing step. The method and its incompressible version, as well as the new algorithm for boundary conditions, are described. Quantitative comparisons with experimental and other numerical studies are made for three cases.

Key Words: Computational Fluid Dynamics, Numerical Method, Lattice Boltzmann Scheme, Laminar Flow, Backward-Facing Step, Benchmark Study

1. Introduction

A considerable number of studies⁽¹⁾⁻⁽⁴⁾ have been done on lattice-gas hydrodynamics over the past few years. These studies have provided a sound basis for the application of lattice gas methods to computational fluid dynamics (CFD). In this regard, the lattice Bhatnagar-Gross-Krook (BGK) method^{(5),(6)} can be considered as the state-of-the-art technique. Compared to its predecessor schemes, such as lattice gas automata (LGA) and lattice Boltzmann equation (LBE) methods, the lattice BGK method is adapted for the simulation of fluid flow. The deviation of macroscopic equations from the Navier-Stokes (N-S) equation is shifted to higher orders, and the transport coefficient can be changed by varying a relaxational parameter. Yet it is an evolutionary algorithm, like its predecessor, which can be numerically solved without any iteration schemes or matrix opera-

tions. The locality of particle interactions, which is also inherited from its predecessors, ensures a linear speed-up in the massively parallel implementation^{(8),(9)} of the procedure.

Recently, lattice BGK solutions have been carefully compared with those obtained using conventional methods. Martinez et al.⁽¹⁰⁾ performed and compared the lattice BGK and spectral simulations of two-dimensional turbulence. Hou et al.⁽¹¹⁾, on the other hand, compared the lattice BGK solution of a cavity flow with a finite difference solution. The conclusions obtained in both these studies are optimistic, and indicate the possibility of applying the lattice BGK method in more realistic situations. In this paper, we will present another comparative study concerning the lattice BGK solution of laminar flow over a backward-facing step.

2. The Method

2.1 General lattice BGK model

The lattice BGK method is very compact in all dimensions. It consists of a regular lattice, whose nodes can be used to discretize the continuous space, and a governing equation in finite difference form for

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the distribution function of particles. A two-dimensional D2Q9 model⁽⁶⁾ will be used in the following. The square lattice is composed of two "sublattices". For each sublattice, coordinates of the lattice link vectors are permutations of $(\pm 1, 0)$ and $(\pm 1, \pm 1)$. There are two types of moving particles on the lattice. As particles would hop from one lattice node to the other, their velocity vectors coincide with the lattice link vectors. Therefore, particles of type I move along the axis with speed defined as $\vec{c}_{1i} = [\cos(\pi(i-1)/2), \sin(\pi(i-1)/2)]$, and particles of type II move along the diagonal directions with speed defined as $\vec{c}_{2i} = \sqrt{2} [\cos(\pi(i-\frac{1}{2})/2), \sin(\pi(i-\frac{1}{2})/2)]$, where i can be any of 1, 2, 3, 4, indicating the direction of velocity. Particles at rest are also allowed at each node. Distribution functions of the three types of particles are denoted as $N_{pi}(\vec{x}, t)$, where p is the index for the particle type (0: rest, 1: type I, 2: type II) and i is only relevant to the moving particles. The particle distribution function satisfies the following lattice Boltzmann equation with a collision operator of BGK type,

$$N_{pi}(\vec{x} + \vec{c}_{pi}, t+1) - N_{pi}(\vec{x}, t) = -\frac{1}{\tau}(N_{pi} - N_{pi}^{(0)}), \quad (1)$$

where $N_{pi}^{(0)}(\vec{x}, t)$ is the equilibrium distribution at $\vec{x}, t$ and τ is the single relaxation time. Note that both lattice spacing and time increment are of unit value. The density per node, ρ , and the macroscopic velocity, $\vec{u}$, are defined in terms of particle distribution by

$$\rho = \sum_{pi} N_{pi}, \quad \rho \vec{u} = \sum_{pi} N_{pi} \vec{c}_{pi}. \quad (2)$$

The equilibrium distribution can be chosen from the following forms for particles of each type (particular for D2Q9 model).

$$N_0^{(0)} = 4\rho/9 \left[1 - \frac{3}{2} u^2 \right] \quad (3)$$

$$N_1^{(0)} = \rho/9 \left[1 + 3(c_{pia}u_a) + \frac{9}{2}(c_{pia}u_a)^2 - \frac{3}{2}u^2 \right]$$

$$N_{2i}^{(0)} = \rho/36 \left[1 + 3(c_{pia}u_a) + \frac{9}{2}(c_{pia}u_a)^2 - \frac{3}{2}u^2 \right]$$

With the use of a standard multiscale analysis⁽²⁾, the macroscopic equations derived from the lattice BGK model are

$$\partial_t \rho + \partial_a(\rho u_a) = 0, \quad (4)$$

$$\partial_t(\rho u_a) + \partial_\beta(\rho u_a u_\beta) = -\partial_a(c_s^2 \rho)$$

$$+ c_s^2 \left(\tau - \frac{1}{2} \right) \partial_\beta [\rho(\partial_a u_\beta + \partial_\beta u_a)] + O(u^3), \quad (5)$$

where c_s is the speed of sound and $c_s^2 = 1/3$ for the model. Equation (4) can be easily recognized as the continuity equation. Equation (5) is similar to the N-S momentum equation, with the kinetic viscosity defined by $\nu = c_s^2 \left(\tau - \frac{1}{2} \right)$. The higher-order term, $O(u^3)$, can be explicitly derived⁽⁷⁾:

$$\left(\frac{1}{2} - \tau \right) \partial_\beta \partial_\gamma (\rho u_a u_\beta u_\gamma). \quad (6)$$

In the small velocity limit, the effect of this term is negligible. Note that Eqs. (4) and (5) are accurate to the second order of the unit lattice spacing and time increment. Therefore, we prove here that solving the lattice BGK equation [Eq. (1)] is equivalent to solving the N-S equation by a second-order (in both space and time) finite difference method.

2.2 Steady incompressible model

It is clear that Eqs. (4) and (5) are for compressible flows. If the flow velocity $\vec{u}$ is small enough or in the limit of small Mach number, the lattice BGK solution can be a good approximation of the incompressible N-S solution. However, this is inconvenient, because flows of practical interest are often at high Reynolds number but nominally incompressible regimes. In this sense, the lattice BGK solution includes an additional compressibility error in addition to that due to the finite size of the lattice. Zou et al.⁽¹²⁾ proved that this difficulty can also be overcome in the case of steady flow.

Here we describe the improved incompressible model in a slightly different manner. Since the density distribution is irrelevant in incompressible flows, the definition of this quantity [in Eq. (2)] becomes unnecessary. Instead, the definitions of pressure and velocity can be introduced.

$$p = c_s^2 \sum_{pi} N_{pi}, \quad \vec{v} = \sum_{pi} N_{pi} \vec{c}_{pi} \quad (7)$$

The equilibrium distributions are transformed into

$$N_0^{(0)} = 4/3 \left[p - \frac{1}{2} v^2 \right] \quad (8)$$

$$N_1^{(0)} = 1/3 \left[p + (c_{pia}v_a) + \frac{3}{2}(c_{pia}v_a)^2 - \frac{1}{2}v^2 \right]$$

$$N_{2i}^{(0)} = 1/12 \left[p + (c_{pia}v_a) + \frac{3}{2}(c_{pia}v_a)^2 - \frac{1}{2}v^2 \right].$$

The following new set of macroscopic equations could be brought forward.

$$\partial_t p + c_s^2 \partial_a v_a = 0 \quad (9)$$

$$\partial_t v_a + \partial_\beta (v_a v_\beta) = -\partial_a p + \nu \partial_\beta (\partial_a v_\beta + \partial_\beta v_a) + O(v^3) \quad (10)$$

Here the error term, $O(v^3)$, must be rewritten as

$$\left(\frac{1}{2} - \tau \right) \partial_\beta [\partial_\gamma (v_a v_\beta v_\gamma) + (v_a \partial_\beta p + v_\beta \partial_a p)]. \quad (11)$$

Note that the first part of this error term is similar to the one in the compressible regime [Eq. (6)]. The second part is $O(v^3)$, i.e., $\partial_a p \sim \partial_\beta (v_a v_\beta)$ at the Euler level for the lattice BGK model. Both the terms will vanish completely in the steady state, since they are generated by $\partial_t \Pi_{a\beta}^{(0)}$, where $\Pi_{a\beta}^{(0)}$ is the equilibrium momentum flux tensor. In summary, for the steady case, the macroscopic equations for the lattice BGK model are

$$\partial_a v_a = 0, \quad (12)$$

$$v_\beta \partial_\beta v_\alpha = -\partial_\alpha p + \nu \partial_\beta \partial_\beta v_\alpha. \quad (13)$$

Apart from the second-order error due to the finite lattice size, they are exactly equal to the steady incompressible N-S equations.

The difficulty in solving the incompressible N-S equations by the general lattice BGK method is that one must determine the pressure, through an equation of state (EOS), using a sound speed on the order of lattice spacing and the density distribution. Hence the continuity equation in a divergence-free form [Eq. (12)] can never be accurately satisfied due to the temporal and spatial variations of density. In the incompressible lattice BGK formulation, Eq. (9) provides a time evolution scheme for the pressure, which will automatically converge to the incompressible continuity equation in the steady case. In fact, this is identical to the pseudocompressible scheme for the conventional N-S solver.

2.3 Boundary conditions

The algorithm to specify the boundary conditions in the lattice BGK simulation was ambiguous. The bounce-back method used on the solid wall produces first-order errors^{(11),(14)}. To solve this problem, Zou and He⁽¹³⁾ proposed a new method, using the idea of bounce-back of the nonequilibrium distribution. Through numerical examples, they were able to show that second-order accuracy could be achieved by specifying various boundary conditions using the new algorithm.

To describe their method, we take the case of a node on the bottom wall shown in Fig. 1, and assume that the wall is moving with a velocity $\vec{v}^w$. This assumption is solely for the generality of description. In practice, since the wall is stationary, $\vec{v}^w$ is substituted with zero. According to the coordinate system in the figure, it is obvious that N_{21} , N_{12} and N_{22} , which point into the fluid, must be determined each time a particle propagation is completed. If the incompressible model is considered, pressure p would be another unknown variable on this node. Using Eq. (7), we have

$$\begin{aligned} N_{12} + N_{21} + N_{22} \\ = \frac{1}{c_s^2} p - (N_0 + N_{11} + N_{13} + N_{14} + N_{23} + N_{24}), \end{aligned}$$

$$N_{21} - N_{22} = v_x^w - (N_{11} - N_{13} - N_{23} + N_{24}),$$

$$N_{12} + N_{21} + N_{22} = v_y^w + (N_{14} + N_{23} + N_{24}).$$

Hence there are three equations for a total of four unknowns. At this stage, bounce-back of the nonequilibrium part of particle distribution normal to the boundary is introduced,

$$N_{12} - N_{12}^{(0)} = N_{14} - N_{14}^{(0)}. \quad (14)$$

The physical meaning of this assumption is as follows. The interpretation of the BGK approximation might be that only some of particles at a lattice node attain

equilibrium after collision, while others keep moving along their old directions. When this is applied at the wall, however, the nonequilibrium particles can no longer move on, because of the obstruction of wall. Therefore, they must be bounced back. With Eqs. (14) and (8), the four unknowns can be readily solved:

$$p = c_s^2 [v_y^w + N_0 + N_{11} + N_{13} + 2(N_{14} + N_{23} + N_{24})],$$

$$N_{12} = N_{14} + \frac{2}{3} v_y^w,$$

$$N_{21} = N_{23} - \frac{1}{2} (N_{11} - N_{13}) + \frac{1}{2} v_x^w + \frac{1}{6} v_y^w,$$

$$N_{22} = N_{24} + \frac{1}{2} (N_{11} - N_{13}) - \frac{1}{2} v_x^w + \frac{1}{6} v_y^w.$$

For other kinds of boundary nodes, including the nodes at the inlet and outlet of the flow region, either the velocity condition or the pressure condition can be properly specified with the same principle. However, it is found in practice that the new method often causes instability. In particular, the creation of negative particle distribution on boundary nodes is observed in the collision process. Nevertheless, this does not happen when the Reynolds number is small enough.

3. Description of Flow Geometry

The reason why we chose flow over a backward-facing step as the problem for analysis is two-folds. First, there have been many studies⁽¹⁵⁾ devoted to this moderately complicated flow, so that both experimental and numerical data are available. Next, the rectangular geometry can be easily described by nodes of the square lattice. For other geometries, for example, circular cylinders, they can only be approximately created on the regular lattice. The edges of the approximation are often jagged, producing discernible effects in the flow near the object.

The geometry of flow is sketched in Fig. 1, where the open circles represent the fluid nodes and the solid squares are for boundary nodes. A no-slip boundary is used on all the solid walls. The inflow condition is a parabolic profile of velocity with a maximum speed $V_{\max}$, while a constant pressure p_0 is applied to the outflow boundary. All these conditions can be

Fig. 1 Schematic 2D lattice geometry for the flow over a backward-facing step

specified using the method described in the previous section, except for the corner nodes. For the bottom corner A, which is also the origin of the coordinates in the figure, N_{11} and N_{24} are also to be specified. They could be determined, by assuming that the pressure of node A is equal to that of neighboring nodes. The problem for node B is lack of freedom, that is, only N_{21} can be specified. In the simulation, we chose $N_{21}=N_{23}$, hence only avoiding the slip velocity in the diagonal direction. The total length of the flow region is $L=23.0$, consisting of $n_L=829$ lattice nodes. There are $n_H=109$ or $n_H=73$ nodes spanning the flow region, so that the height can be $H=3.0$ or $H=2.0$, respectively, for different cases. The length of the entry region is chosen as $l=3.0$. Three test cases obtained from the combinations of two geometries and two Reynolds numbers were computed,

- case (i) $h=2.0$, $H=3.0$, $Re=50$;
- case (ii) $h=2.0$, $H=3.0$, $Re=150$;
- case (iii) $h=1.0$, $H=2.0$, $Re=150$;

where the Reynolds number is defined by

$$Re = V_{\max}(H-h)/\nu\delta, \quad (15)$$

$\delta=H/n_H$ being the resolution of the lattice. Since the experimental data⁽¹⁶⁾ of these cases are published in a well-organized way, systematic comparisons are made in the following section. For case (i), an additional computation [case (i')] was carried out, with the use of a coarser lattice ($n_L=415$, $n_H=55$). Meanwhile, another computation of case (ii), i.e., case (ii') was also done. In this case, L and l are set to 36.0 and 6.0 respectively, to study the effect of the lengths of entry and downstream regions.

4. Numerical Results

4.1 Accuracy and rate of convergence

Flow is considered to be steady if both of the following inequalities are satisfied:

Fig. 2 Evolution of $\sum |\text{div } \vec{v}| / \sum |\vec{v}|$ in the lattice BGK simulation of flow over a backward-facing step

$$\epsilon^t = \frac{\sum_{ij} |\text{div } \vec{v}|_{ij}}{\sum_{ij} (|v_x|_{ij} + |v_y|_{ij})} < \text{TOLA}, \quad (16)$$

and

$$\frac{|\epsilon^{t+200} - \epsilon^t|}{\sum_{ij} (|v_x|_{ij} + |v_y|_{ij})} < \text{TOLB}, \quad (17)$$

where i, j are indices of lattice nodes. The first criterion [Eq.(16)] is to check the satisfaction of the incompressible continuity equation which holds if the steady stage is reached. The second one [Eq.(17)] rules out the transitional stage in which $|\text{div } \vec{v}|$ happens to be small. In the simulation, $\text{TOLA}=1.0 \times 10^{-6}$ and $\text{TOLB}=1.0 \times 10^{-8}$ were selected. For case (i') with a lower lattice resolution, these values are relaxed to $\text{TOLA}=1.0 \times 10^{-5}$ and $\text{TOLB}=1.0 \times 10^{-7}$. Codes are usually terminated after 150,000~300,000 time steps, which costs about 2~3 days of CPU time on a HP 700 machine. The evolution of ϵ^t for each case is shown in Fig. 2. Here for the same lattice resolution, the value of ϵ^{steady} becomes slightly larger when the Reynolds number increases [see curves for cases (i) and (ii)]. Clearly, more time steps are required for case (ii) to converge to the steady state. On the other hand, for the same Reynolds number, the finer the lattice, the higher the accuracy achieved. For example, while the lattice resolution of case (i) is 2 times higher than that of case (i'), the corresponding value of ϵ^{steady} is only about 1/8 that of case (i'). As a trade-off, the calculation time for the case with the finer lattice becomes longer, though the convergence rate does not change. The decrease of accuracy for case (iii) is caused by lowering of the lattice resolution in the flow region, which may be improved if $H=3.0$ and $h=1.5$ are used.

4.2 Results and discussion

Global features of the flow are well described in Figs. 3, 4 and 5, where streamlines, the normalized pressure $[(p-p_B)/V_{\max}^2]$ contour lines and curves for the evolution of the maximum velocity $[V_{\max}(x)/V_{\max}]$ are drawn over the whole domain. The streamlines are plotted according to the normalized stream functions $[\psi \rightarrow \phi/(H-h)V_{\max}]$. It is found that $\phi_{\max}=1.334$ and 0.666, as opposed to the theoretically obtained $\phi_{\max}=1.3$ and 0.6, for $H/h=1.5$ and 2.0, respectively. Note that stream functions in the circulation zone are negative due to the reversed flow. Characteristics of the pressure contours are strongly affected by the Reynolds number. It is interesting to compare the graph of case (i) with that of case (ii), since their flow geometries are exactly the same. First, the pressure difference between the inlet and outlet of the flow domain decreases when the Reynolds number increases. Next, the minimum pressure appears in the circulation zone, while the maximum is found right after it. It is important that the reversed pressure

Fig. 3 Streamlines in flow over a backward-facing step

Fig. 4 Pressure contours in flow over a backward-facing step

gradient, for case (ii), is also very strong on the wall opposite to the step. This presents a greater possibility of inducing a secondary vortex there. Although it is well known that the pressure contour is one of the most important features of flow, we do not give detailed discussion on the comparison between the results of the lattice BGK simulation and the N-S solutions. In particular, there is no experimental

Fig. 5 The streamwise evolution of the maximum velocity. Solid lines are results of the lattice BGK simulation, symbols are those of the experiment⁽¹⁶⁾

information available due to the difficulty in measurement, therefore, no valid conclusion can be drawn from such a comparison. We indicate, however, that the pressure contours obtained in the lattice BGK simulation are qualitatively similar to the N-S solutions. Experimental data⁽¹⁶⁾ for the evolution of the maximum velocity are plotted in Fig. 5. They are in good agreement with the lattice BGK solution, except in the transitional area for case (i), where the circulation zone is formed.

For a close look at the transitional region, streamlines of the circulating flow are plotted in Fig. 6, and specific characteristic values are generalized in Table 1. From these, it is clear that the lattice BGK method did give reasonable solutions. However, there are several issues that must be explained. First the reattachment length is obtained by finding the zero wall shear stress position (see Fig. 8), which is strongly recommended in the experimental study. However, the method of extrapolating the $\psi=0$ line is often adopted in other numerical calculations. We therefore give, in parenthesis, in Table 1, the value obtained by employing this method. Nevertheless, the difference between the two is trivial. Second, the deviation from the experimental results may not be caused completely by the errors in the lattice BGK method. It is pointed out that the expansion ratio H/h of the experimental apparatus is not exactly 1.5 or 2.0, which might bring about errors in the measurement⁽¹⁶⁾. Correspondingly, the minimum of the stream function and the vortex center, as well as the reattachment length of the lattice BGK solution, are closer to the average value of other numerical solutions. Third, the results obtained by employing a coarser lattice or a longer entry region, namely, results for case (i') and case (ii') are also plotted in Figs. 6(a) and 6(b). It is noteworthy that the

(a) Solid lines and the star are for case (i). Dashed lines and the diamond are for case (i'). The contours of stream function, from inside to outside, are -0.015 , -0.013 , -0.010 , -0.0080 , -0.0053 , -0.0026 and 0.0 .

(b) Solid lines and the star are for case (ii). Thick disconnected lines and the diamond are for case (ii'). The contours of stream function, from inside to outside, are -0.027 , -0.021 , -0.018 , -0.014 , -0.011 , -0.0071 , -0.0036 and 0.0 .

(c) Case (iii). The contours of stream function, from inside to outside, are -0.038 , -0.033 , -0.027 , -0.020 , -0.013 , -0.0067 and 0.0 .

Fig. 6 Streamlines in the circulation zone. Stars and diamonds are used to indicate the locations of the vortex centers

decrease of lattice resolution seems to slightly shift the center of the vortex and the reattachment point to the left. The increase of the length of the entry region, on the other hand, has no effect on these parameters.

Velocity profiles at different locations are given in Fig. 7, with the results of experiment. They agree well with each other. There is a small difference between the velocity profiles at $x=8.0$ and $x=16.0$, for the lattice BGK solution shown in Fig. 7(a) (solid lines). For the results of experiment (squares), however, no such difference is found. This indicates that the flow in experiment becomes fully developed through a shorter length in the downstream region, which may be also caused by the above-mentioned error in the experimental apparatus.

Finally, the distribution of the dimensionless wall shear stress $[\sigma(x) \rightarrow \sigma/V_{\max}^2]$ is compared with the experimental result in Fig. 8. Many numerical methods failed in this test, because it involves the

Table 1 Values of characteristic parameters. The results calculated using the conventional N-S solvers are obtained by taking averages. Ranges of variation for the N-S solutions are also indicated

(a) Case (i), $Re=50$, $H/h=1.5$

	Recirculation	Vortex Center		Reatt. Length
	$\psi_{\min}/\psi_{\max}$	$x_c/(H-h)$	$y_c/(H-h)$	$x_r/(H-h)$
experiment	-0.012	0.8	0.72	3.0
lattice BGK	-0.016	0.91	0.56	$2.85(2.82)$
N-S Solvers	-0.017	0.94	0.61	2.83
	$-0.0088 \sim -0.021$	$0.83 \sim 1.0$	$0.55 \sim 0.68$	$2.5 \sim 3.4$

(b) Case (ii), $Re=150$, $H/h=1.5$

	Recirculation	Vortex Center		Reatt. Length
	$\psi_{\min}/\psi_{\max}$	$x_c/(H-h)$	$y_c/(H-h)$	$x_r/(H-h)$
experiment	-0.016	2.2	0.64	6.0
lattice BGK	-0.022	1.83	0.61	$6.43(6.41)$
N-S Solvers	-0.021	1.89	0.63	6.06
	$-0.013 \sim -0.024$	$1.68 \sim 2.0$	$0.60 \sim 0.64$	$5.05 \sim 7.25$

(c) Case (iii), $Re=150$, $H/h=2.0$

	Recirculation	Vortex Center		Reatt. Length
	$\psi_{\min}/\psi_{\max}$	$x_c/(H-h)$	$y_c/(H-h)$	$x_r/(H-h)$
experiment	-0.053	1.88	0.58	4.5
lattice BGK	-0.061	1.86	0.56	$4.94(4.96)$
N-S Solvers	-0.060	1.77	0.63	4.73
	$-0.051 \sim -0.066$	$1.5 \sim 2.0$	$0.56 \sim 0.66$	$3.7 \sim 5.8$

calculation of derivatives which may be greatly affected by the differencing scheme. In the lattice BGK simulation, the stress can be simply calculated by

$$\sigma_{\alpha\beta} = \left(1 - \frac{1}{2\tau}\right) \sum_i N_{pi}^{\text{neq}} c_{pia} c_{pi\beta}, \quad (18)$$

where $N_{pi}^{\text{neq}} = (N_{pi} - N_{pi}^{(0)})$ is the nonequilibrium part of the particle distribution function. The lattice BGK method gives fairly good results, for the wall shear stress. In particular, the constant values both upstream and downstream are in good agreement with theoretical predictions for all three cases. Formulas for these predictions are written in the following,

$$\sigma_{\text{upstr.}} = \frac{4}{Re} \frac{(H-h)}{h}, \quad \sigma_{\text{downstr.}} = \frac{4}{Re} \frac{h(H-h)}{H^2}. \quad (19)$$

It is clear that even though the velocity variation is continuous throughout the whole domain, because of the existence of the backward-facing step, the shear stress on the bottom wall is discontinuous at the end of the entry channel. A sharp rise of the shear stress, together with a strong oscillation, is observed

Fig. 7 Velocity profiles. Solid lines are lattice BGK solutions, while symbols are experimental results

immediately before the discontinuity in the lattice BGK solution. On the other hand, in the experiment, the probes are not able to detect this drastic change, so that only a mild increase of the stress is measured. We argue that such overshoot and oscillation can be taken as evidence of a second-order numerical mode. Typically, describing the discontinuity with a second-order differencing scheme, such as the Lax-Wendroff scheme, often causes overshoots and Gibbs oscillations because the short-wavelength components are strong. For first-order schemes, such as the famous upwind scheme, the discontinuity can be represented as a monotonic transition due to the introduction of artificial viscosity. Figures 8(a) and 8(b) show that, as expected, the oscillating interval enlarges with the reduction of lattice resolution, while the overshoot becomes larger with the increase of Reynolds number. On the wall opposite the step, both the lattice BGK solution and the experimental measurement show a local minimum near the reattachment point. This will induce flow separation on the wall for a sufficiently large Reynolds number or a sufficiently high step,

Fig. 8 Wall shear stress distribution. Lines are the lattice BGK solutions, and symbols are for the experimental results. The distribution of shear stress is discontinuous on the wall with the step

which will also form the secondary vortex. Finally, the physical prediction of wall stress suggests that the new algorithm for boundary conditions in the lattice BGK simulation is hydrodynamically consistent.

5. Conclusion

The lattice BGK method can be applied in the calculation of nontrivial flows. Accuracy of the results is comparable to that obtained by using conventional N-S solvers. However, there exists a gap which must be bridged when considering engineering applications, such as the description of curved geometries, the use of flexible lattice resolution, and the incorporation of turbulent models into the lattice BGK method.

References

- (1) Wolfram, S., Cellular Automaton Fluids 1: Basic Theory, J. Stat. Phys., Vol. 45, No. 3/4(1986),

- p. 471.
- (2) Frisch, U., d'Humières, D., Hasslacher, B., Lallemand, P., Pomeau, Y. and Rivert, J.P., Lattice Gas Hydrodynamics in Two and Three Dimensions, *Complex Systems*, Vol. 1(1987), p. 649.
 - (3) McNamara, G. and Zanetti, G., Use of The Boltzmann Equation to Simulate Lattice-Gas Automata, *Phys. Rev. Lett.*, Vol. 61, No. 20(1988), p. 2332.
 - (4) Doolen, G.D., Frisch, U., Hasslacher, B., Orszag, S. and Wolfram, S., *Lattice Gas Methods for Partial Differential Equations*, (1989), Addison-Wesley, Redwood City, CA, USA.
 - (5) Chen, H., Chen, S. and Matthaeus, W.H., Recovery of Navier-Stokes Equations Using a Lattice-Gas Boltzmann Method, *Phys. Rev. A*, Vol. 45(1992), p. 5771.
 - (6) Qian, Y.H., d'Humières, D. and Lallemand, P., Lattice BGK Models for Navier-Stokes Equation, *Europhys. Lett.*, Vol. 17, No. 6(1992), p. 479.
 - (7) Qian, Y.H. and Orszag, S., Lattice BGK Models for the Navier-Stokes Equation: Nonlinear Deviation in Compressible Regimes, *Europhys. Lett.*, Vol. 21, No. 3(1993), p. 255.
 - (8) Punzo, G. and Succi, S., High-Resolution Lattice-Boltzmann Computing on the IBM SP 1 Scalable Parallel Computer, *Computers in Physics*, Vol. 8, No. 6(1994), p. 705.
 - (9) Hirono, S., Chen, Y., Ohashi, H. and Akiyama, M., Parallel Computing of Flow Simulation Using a Lattice BGK Method, *Proc. JAS Spring Meeting (in Japanese)*, (1996).
 - (10) Martinez, D.O., Matthaeus, W.H., Chen, S. and Montgomery, D.C., Comparison of Spectral Method and Lattice Boltzmann Simulation of 2-Dimensional Hydrodynamics, *Phys. of Fluids*, Vol. 6, No. 3(1994), p. 1285.
 - (11) Hou, S.L., Zou, Q., Chen, S., Doolen, G.D. and Cogley, A.C., Simulation of Cavity Flow by the Lattice Boltzmann Method, *J. Comp. Phys.*, Vol. 118, No. 2(1995), p. 329.
 - (12) Zou, Q., Hou, S.L., Chen, S. and Doolen, G.D., An Improved Incompressible Lattice Boltzmann Model for Time-Independent Flows, *J. Stat. Phys.*, Vol. 81, No. 1/2(1995), p. 35.
 - (13) Zou, Q. and He, X., On Pressure and Velocity Flow Boundary Conditions for the Lattice Boltzmann BGK Model, preprint (1995).
 - (14) Cornubert, D., d'Humières, D. and Levermore, D., A Knudsen Layer Theory for Lattice Gas, *Physica D*, Vol. 47, No. 6(1991), p. 241.
 - (15) Morgan, K., Periaux, J. and Thomasset, F., Analysis of Laminar Flow over a Backward Facing Step, (1984), Friedr. Vieweg & Sohn, Braunschweig, Wiesbaden.
 - (16) Kuney, J.L. and Binder, G., Viscous Flow over Backward Facing Steps: An Experimental Investigation, Analysis of Laminar Flow over a Backward Facing Step, (1984), p. 32.