MHD Mixed Convection from a Horizontal Cylinder in a Porous Medium*

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Mixed convection from a horizontal cylinder imbedded in saturated electrically conducting fluid is investigated using nonsimilarity boundary layer transformation. Variable wall temperature is assumed at the cylinder surface as the thermal boundary condition. The problem is solved numerically using the finite difference technique and double-checked using a local nonsimilarity solution. The effects of the magnetic field on the velocity and temperature profiles, wall shear stress, and Nusselt number are presented. The non-Daric flow model is adopted. The effects of the inertial and boundary parameters on the solution in the presence of a magnetic field are also presented. The problem is solved covering the entire regime of mixed convection.

Key Words: MHD, Mixed Convection, Horizontal Cylinder, Porous Medium

1. Introduction

Geothermal gases are electrically conductive, as is the fluid used in cooling systems for nuclear reactors; such flows undergo the influence of magnetic fields. Magnetic fields are found to affect the local shear stress as well as the heat transfer coefficient. Designing systems which incorporate electrically conducting fluids for applications in space vehicle propulsion, power generating systems, pumps and heat exchangers requires a better understanding of magnetic field effects.

The magnetic field effect on fluid flow and heat transfer has been investigated by many authors. In Refs. (1)–(6), MHD free convection from a vertical plate in a nonporous medium with steady and unsteady flow with different temperature boundary conditions is considered. MHD convection from a vertical flat plate imbedded in a porous medium is reported

in Refs. (7)–(10). Fewer works on MHD convection from geometries other than that of a flat plate are available⁽¹¹⁾.

In the present work, mixed convection from a horizontal circular cylinder embedded in electrically conducting fluid and exposed to a transverse magnetic field is considered. A nonsimilarity transformation to the boundary layer flow about the cylinder is applied such that the entire mixed convection regime is confined within a finite range of a mixed convection parameter. Numerical calculation is performed and the effect of the magnetic field on the local shear stress and local Nusselt number are presented. A local nonsimilarity solution for the problem is also carried out as a double check.

Nomenclature

- B : applied magnetic flux density, Wb/m²
- Da_s : local Darcy number, K/x^2
- f : dimensionless stream function
- g : acceleration of the gravity, m/s²
- Ha : Hartmann number, $B_s \sqrt{\frac{\sigma}{\mu/\epsilon}}$
- $h(x)$: local heat transfer coefficient, W/m²·K

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- k : thermal conductivity, $\text{W/m}\cdot\text{K}$
 K : permeability coefficient, m^2
 K' : coefficient of Forchheimer's equation
 Nu_x : local Nusselt number, hx/k
 Pe_s : local Peclet number, $2U_\infty s/\alpha$
 s : $R/(1 + \cos \frac{x}{R})$
 R^* : inertial effect parameter
 Ra_s : local Rayleigh number, $\frac{kg\beta(T_w - T_\infty)s}{\nu\alpha}$
 Re : Reynolds number, $U_\infty s/\nu$
 T : temperature, K
 T_∞ : free stream temperature, K
 T_w : wall temperature, K
 U : velocity component in the s -direction, m/s
 V : velocity component in the y (or r)-direction, m/s
 x : the streamwise direction along the cylinder surface
 y : normal to the cylinder surface

Greek Symbols

- α : effective thermal diffusivity of saturated porous medium, m^2/s
 β : volumetric coefficient of thermal expansion, K^{-1}
 δ : boundary layer thickness
 η : pseudo-similarity variable
 θ : dimensionless temperature
 μ : dynamic viscosity, $\text{N}\cdot\text{s}/\text{m}^2$
 ν : kinematic viscosity, m^2/s
 ξ : nonsimilarity parameter
 ρ : density of the fluid, kg/m^3
 τ_w : local wall shear stress, $\mu(\partial u/\partial y)_{y=0}$, N/m^2
 ϵ : porosity
 ϕ : stream function

2. Problem Formulation

Consider a horizontal circular cylinder embedded in a viscous incompressible and conducting fluid-saturated porous medium. The cylinder is subjected to a transversely applied magnetic field of strength B acting normal to the cylinder surface, as shown in Fig. 1. A curvilinear coordinate system is used, x is the streamwise direction along the cylinder surface measured from the lower stagnation point, and y is the coordinate normal to the cylinder surface. Consider the boundary layer flow and, adopting the Brinkman-Forchheimer model of saturated-fluid porous medium flows, assume a small magnetic Reynolds number such that the induced magnetic field is negligible and in the absence of the electric field; by applying the Boussinesque approximation, the governing equations for the above problem become (12) and (13),

Fig. 1 The physical problem

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial y} + \frac{2F\sqrt{K}}{\nu}u \frac{\partial u}{\partial y} = \frac{K}{\mu} \left[g \sin \frac{x}{R} \frac{\partial \rho}{\partial y} - \sigma B^2 \frac{\partial u}{\partial y} + \frac{\mu}{\epsilon} \frac{\partial^3 u}{\partial y^3} \right] \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

In the above equations, the buoyancy force normal to the boundary layer is neglected compared with that in the streamwise direction. Also the boundary layer thickness adjacent to the cylinder surface is considered to be thin compared with the cylinder radius, i.e., $R \gg \delta$. u and v are the Darcian volume averaged velocity components in the x - and y -directions, respectively. T is the temperature, B is the transverse magnetic field, ρ , μ , and α are the density of the fluid, and the equivalent viscosity and thermal diffusivity of the saturated porous medium respectively. In this analysis, viscous dissipation is neglected.

In terms of a stream function that satisfies the continuity equation identically, the governing equations become

$$\phi_{yy} + \frac{2F\sqrt{K}}{\nu} \phi_y \phi_{yy} - \frac{K}{\mu} \left(\rho_\infty g \beta \sin \frac{x}{R} T_y - \sigma B^2 \phi_{yy} + \frac{\mu}{\epsilon} \phi_{yyy} \right) = 0 \quad (4)$$

$$\phi_y T_x - \phi_x T_y = \alpha T_{yy}, \quad (5)$$

with the boundary conditions:

$$\left\{ \begin{array}{l} \psi_y(x, 0)=0, \psi_x(x, 0)=0 \\ T(x, 0)=T_\infty+(1-\cos\frac{x}{R})^\lambda \\ \psi_y(x, \infty)=2U_\infty \sin\frac{x}{R}, \psi_{yy}(x, \infty)\rightarrow 0 \\ T(x, \infty)\rightarrow T_\infty \end{array} \right\} \quad (6)$$

Introducing the following transformation.

$$\eta = Pe_s^{1/2} \xi^{-1} y/s \quad (7)$$

$$\phi(\xi, \eta) = \alpha Pe_s^{1/2} \sin\frac{x}{R} \xi^{-1} f(\xi, \eta) \quad (8)$$

$$\theta(\xi, \eta) = \frac{T(x, y) - T_\infty}{T_w - T_\infty}, \quad (9)$$

where $s = R / \left(1 + \cos\frac{x}{R}\right)$ and $\xi = Pe_s^{1/2} / (Pe_s^{1/2} + Ra_s^{1/2})$.

Applying the above transformation to the governing equations, Eqs. (4) and (5), the result is

$$\xi^{-2} f'' - \xi^{-2} (1 - \xi)^2 \theta' + R^* [(f')^2] - L^* f^{IV} + M^* f''' = 0 \quad (10)$$

$$\begin{aligned} \theta'' + \frac{1 + \lambda(1 - \xi)}{2} f \theta' - \lambda f' \theta \\ = \frac{\lambda}{2} \xi (1 - \xi) \left[\theta' \frac{\partial f}{\partial \xi} - f' \frac{\partial \theta}{\partial \xi} \right]. \end{aligned} \quad (11)$$

The boundary conditions (Eq. (6)) in terms of the transformed variables are

$$\left\{ \begin{array}{l} f(\xi, 0) + \lambda(\xi - 1) \left[\xi \frac{\partial f(\xi, 0)}{\partial \xi} - f(\xi, 0) \right] = 0 \\ f'(\xi, 0) = 0; \theta(\xi, 0) = 1 \\ f'(\xi, \infty) = \xi^2; f''(\xi, \infty) = 0; \theta(\xi, \infty) = 0 \end{array} \right\}. \quad (12)$$

In the above equations,

$R^* = F \sqrt{K} (2U_\infty / \nu) \sin\frac{x}{R}$; is the inertial parameter,

$L^* = Da_s Pe_s / \epsilon$; is the boundary parameter,

$Da = K/s^2$; is the Darcy number,

$Pe_s = 2U_\infty s / \alpha$; is the Peclet number,

$Ra_s = kgB(T_w - T_\infty)s / \nu \alpha$; is the Rayleigh number,

and

$M^* = Ha^2 / [Pe_s^{1/2} + Ra_s^{1/2}]$ is the magnetic parameter, where $Ha = [\sigma / (\mu / \epsilon)]^{1/2} B s$ is the Hartmann number that measures the strength of the magnetic field effect.

The above transformation covers the entire regime of mixed convection including the pure forced and pure natural convection limits through the usage of the parameter ξ ; $\xi = 0$ represents the limit of pure natural convection and $\xi = 1$ represents the limit of pure forced convection. Thus running the solution between $\xi = 0$ to $\xi = 1$, one can obtain the solution for the entire mixed convection regime.

3. Method of Solution

The transformed governing equations with their boundary conditions are solved using the finite difference technique of the Keller-Box method, as described by Cebeci and Bradshaw⁽¹⁴⁾. The resulting

algebraic differential equations system is solved using uniform grid size in both directions; $\Delta \xi = \Delta \eta = 0.05$ are chosen, below which the results are not affected. The numerical solution is assumed to converge when the difference between the values of the maximum shear stress at the wall during the successive iterations is less than 10^{-5} .

To verify the solution, a second-level local non-similarity method is used. The governing equations are arranged as follows.

$$m_1 f'' + m_2 \theta' + m_3 (f')' + m_4 f^{IV} = 0 \quad (13)$$

$$m_5 \theta'' + m_6 f \theta' + m_7 f' \theta = m_8 [\theta' G - f' \phi] \quad (14)$$

$$\begin{aligned} m_1 G'' + \frac{\partial m_1}{\partial \xi} f'' + m_2 \phi' + \frac{\partial m_2}{\partial \xi} \theta' + 2m_3 f' G'' \\ + 2m_3 f'' G'' + \frac{\partial m_3}{\partial \xi} (f')^2 + \frac{\partial m_4}{\partial \xi} f^{IV} + m_4 G^{IV} = 0 \end{aligned} \quad (15)$$

$$\begin{aligned} m_5 \phi'' + \frac{\partial m_5}{\partial \xi} \theta'' + m_6 f \phi' + m_6 \theta' G + \frac{\partial m_6}{\partial \xi} f \theta' \\ + m_7 f' \phi + m_7 \theta G' + \frac{\partial m_7}{\partial \xi} f' \theta = \frac{\partial m_8}{\partial \xi} [\theta' G - f' \phi]. \end{aligned} \quad (16)$$

The boundary conditions for the above system of equations are

$$\left\{ \begin{array}{l} m_9 f(\xi, 0) + m_{10} G(\xi, 0) = 0 \\ f'(\xi, 0) = 0; \theta(\xi, 0) = 1 \\ f'(\xi, \infty) = \xi^2; f''(\xi, \infty) = 0; \\ \theta(\xi, \infty) = 0 \\ m_9 G(\xi, 0) + m_{11} f(\xi, 0) = 0 \\ G'(\xi, 0) = 0; \phi(\xi, 0) = 0 \\ G'(\xi, \infty) = 2\xi; G''(\xi, \infty) = 0; \\ \phi(\xi, \infty) = 0 \end{array} \right\}, \quad (17)$$

where $G = \frac{\partial f}{\partial \xi}$, $\phi = \frac{\partial \theta}{\partial \xi}$ and the coefficients m 's are certain known functions of ξ .

The above system of equations is solved and the result is compared with the solution of the finite difference technique.

4. Results and Discussions

The results are presented in terms of the Nusselt number expression, $Nu_s (Pe_s^{1/2} + Ra_s^{1/2})^{-1}$, and the wall shear stress, $f''(\xi, 0)$, at different values of magnetic parameter M^* . All results are presented for the case of isothermal boundary condition $\lambda = 0$ for convenience. The solution covers the entire regime of mixed convection by varying the value of ξ from 0 to 1.

Figures 2 and 3 shows the effects of the magnetic field on the velocity and temperature profiles. It is known that the magnetic field has a retardation effect on the flow velocity and reduces heat transfer; this effect is obvious where the gradients of the velocity and temperature profiles become less at the wall. The

Fig. 2 The velocity profiles

Fig. 3 The temperature profiles

thickening of the velocity and temperature profiles is due to the presence of the magnetic field.

In Figs. 4, 5 and 6, the effects of the magnetic field on the wall shear stress are shown; $f''(\xi, 0)$ are plotted against ξ , the mixed convection parameter. In Fig. 4, the magnetic field starts to have a significant effect at values of ξ greater than 0.4, i.e., the magnetic field has a negligible effect in the free convection-dominated regime, and has its largest effect on the wall shear stress at the pure forced convection limit, because the magnetic force is proportional to the flow velocity. Increasing the value of the inertial parameter increases the wall shear stress as shown in Fig. 5, however it seems that the effect of the magnetic field at different values of the inertial parameter on the wall shear stress is minimal. The boundary parameter effect on the wall shear stress is shown in Fig. 6; increasing its value results in decreasing the wall shear stress. The boundary parameter represents the effects of the no-slip boundary condition on the porous medium flows. The magnetic field clearly has its

Fig. 4 Local shear stress versus ξ Fig. 5 Local shear stress versus R^* Fig. 6 Local shear stress versus L^*

largest effects at larger boundary parameter values since this parameter value is proportional to the flow velocity. At low values of boundary parameter $L^* < 2.0$, different values of the magnetic parameter will not have a significant effect on the value of wall shear stress.

Fig. 7 Local Nusselt number expression versus ξ Fig. 8 Local Nusselt number expression versus R^* Fig. 9 Local Nusselt number expression versus L^*

The effects of the magnetic field on the heat transfer coefficient, i.e., the Nusselt number expression, are presented in Figs. 7, 8 and 9. Increasing the mixed convection parameter ξ increases the Nusselt number expression with the largest value at $\xi = 1$. The magnetic field starts to show a significant effect

on the Nusselt number expression at $\xi > 0.4$. At low values of ξ , i.e., in the free convection-dominated regime, different values of M^* do have a negligible effect on the Nusselt number expression values, as shown in Fig. 7. Figure 8 shows that the effects of the magnetic field on the Nusselt number expression at lower values of inertial parameter is more pronounced, although at higher values of the inertial parameter the Nusselt number expression value is larger. On the other hand, the magnetic field effect on the Nusselt number expression is larger at larger values of the boundary parameter, as shown in Fig. 9. The boundary parameter has the effect of decreasing the value of the Nusselt number expression.

Figures. 4 and 7 show a comparison between the results obtained using the finite difference technique and that using the local nonsimilarity method. The results are double-checked and verified against each other. The special case of mixed convection without magnetic effect ($M^* = 0$) is comparable to the results of Hussain⁽¹²⁾.

5. Conclusion

Increasing the magnetic field has the effect of decreasing the local wall shear stress and reducing the value of the Nusselt number expression in the boundary layer flow about a circular horizontal cylinder embedded in a porous medium saturated with an electrically conducting fluid. The effect is more pronounced in the forced convection-dominated regime than in the free convection regime and at higher values of the boundary parameter of the non-Darcian model.

For this problem, the local nonsimilarity solution of the second-order compared very well with the solution using the finite difference technique.

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