Particle Cluster Tracking Algorithm in Particle Image Velocimetry*

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The cross-correlation tracking technique is widely used to analyze image data, in particle image velocimetry (PIV). The technique assumes that the fluid motion, within small regions of the flow field, is parallel over short time intervals. However, actual flow fields may have some distorted motion, such as rotation, shear and expansion. Therefore, if the distortion of the flow field is not negligible, the fluid motion cannot be tracked well using the cross-correlation technique. The author proposed a new particle tracking technique, based on the particle cluster matching using linear affine transformation. The algorithm can be applied to flow fields which exhibit characteristics such as rotation, shear and expansion. The algorithm is based on pattern matching of particle clusters between the first and second image. The deformation of the cluster pattern is expressed by the linear affine transformation. The parameter of the transformation can be determined using the least squares technique from the particle positions. The residue of the particle deformation is taken as the pattern matching parameter. The smallest residue pattern in the second image is the most probable pattern match to the correspondent original pattern in the first image. Therefore by finding the best matches, particle movements can be tracked between the two images. The effectiveness of the proposed technique was verified with synthetic data of three-dimensional flow. It demonstrated a high degree of accuracy for the three-dimensional calculation.

Key Words: Particle Image Velocimetry, Particle Clusters Matching, Cross-Correlation Technique

1. Introduction

Particle image velocimetry (PIV) is a superior measurement technique for studying fluid flows. Its primary advantage is the ability to capture spatial velocity distributions simultaneously and noninvasively.

The fluid flow is visualized by seeding with small particle tracers. Then the flow visualization images are analyzed to obtain the velocity distribution of the flow. The cross-correlation technique is widely used to calculate the seed movement^{(1),(2)}. The technique assumes that the fluid motion, within small regions of the flow field, is parallel over short time intervals. However, actual flow fields may have some distorted motion, such as rotation, shear and expansion. Therefore, if the distorted motions are not negligible, the cross-correlation technique cannot track the flow.

The author proposed a spring model particle

tracking technique⁽³⁾. The algorithm can be applied to flow fields which exhibit characteristics such as rotation, shear and expansion. In the spring model technique, the smallest deformation pattern, i.e., spring force pattern, was selected as the corresponding pattern. However, the information of pattern deformation was not used in the pattern tracking, since the spring force, F , had less physical meanings.

Dracos et al.⁽⁴⁾ derived the affine transformation for tracking the 3D LIF data. They calculated the coefficients of the affine transformation based on the 3D deformation of the LIF pattern, resulting in the vorticity tensor and the stress tensor. Because the LIF data filled in the whole volume, it is easy to calculate these tensors. However, in the PIV images, the particles were sparsely distributed, resulting in difficulties in the application of their technique to PIV.

Ishikawa et al.⁽⁵⁾ also proposed a particle pattern matching technique. The vector gradient tensor for the fluid motion was calculated using the particle positions in the cluster. Then the cross-correlation technique was applied by considering the tensor. They demonstrated the technique in a

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two-dimensional field.

In this study, the improved technique of three-dimensional particle tracking was proposed, based on the particle cluster matching. The three-dimensional tensor was obtained using the particle position information, by assuming the linear affine transformation. Then, the cluster matching was performed by the residue of tensor calculation. The stress tensor was directly calculated in the affine transformation calculation.

2. Particle Cluster Matching

In the cross-correlation technique, the cross-correlation between the two images of particles was calculated. The new technique is based on the pattern matching of the particle cluster between two consecutive images. The particle cluster with 15 to 20 particles was considered. The pattern deformation in the affine transformation is expressed as,

$$\mathbf{X}_1 = \mathbf{A}\mathbf{X}_0 + \mathbf{X}_d \quad (1)$$

That is,

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} x_d \\ y_d \\ z_d \end{pmatrix} \quad (2)$$

where $\mathbf{X}_0$, (x_0, y_0, z_0) and $\mathbf{X}_1$, (x_1, y_1, z_1) denote the coordinate system in the first and second images, respectively. The origin of the coordinate was assumed to be the center particle of the selected cluster in the first image. From the displacement vector between the first and second images, (x_d, y_d, z_d) , the velocity is obtained. When the matrix $\mathbf{A}$ is the unit tensor, the deformation is only the parallel motion.

Because of the continuity, the relation, $\nabla \cdot \vec{u} = 0$, should be satisfied in the affine transformation, i.e., $a_{11} + a_{22} + a_{33} = 3$. In this study, three kinds of fluid motion are considered i.e., parallel displacement, rotation and shear. Then the deformation matrix, $\mathbf{A}$, is rewritten as follows,

$$\begin{aligned} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} &= \begin{pmatrix} 1 + \sigma_{11} & \sigma_{12} + \tau_{12} & \sigma_{13} + \tau_{13} \\ \sigma_{12} - \tau_{12} & 1 + \sigma_{22} & \sigma_{23} + \tau_{23} \\ \sigma_{13} - \tau_{13} & \sigma_{23} - \tau_{23} & 1 + \sigma_{33} \end{pmatrix} \\ &= \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} + \begin{pmatrix} 1 & \tau_{12} & \tau_{13} \\ -\tau_{12} & 1 & \tau_{23} \\ -\tau_{13} & -\tau_{23} & 1 \end{pmatrix} \end{aligned} \quad (3)$$

where $\sigma_{11} + \sigma_{22} + \sigma_{33} = 0$.

The 1st and 2nd term denote the stress and rotational tensor, respectively. The tensor $\mathbf{A}$ has 8 parameters. These parameters are determined using the least squares technique. Therefore, the particle cluster should contain more than 9 particles. The residual, R , is defined as,

$$R_{i,j} = \sum_k^N |X_{j,k} - \mathbf{A}X_{i,k} - \mathbf{X}_{d,i,j}|^2 \quad (4)$$

where $X_{j,k}$ and $X_{i,k}$ denote the location of particle (k) in the particle cluster (i) for the first image and that in (j) for the second image, respectively.

In this study, the residual, R , was taken as the pattern matching parameter. When the cluster in the second image does not correspond to the original cluster in first image, the particle pattern deformation will be quite large, resulting in a larger residual value.

The least squares technique only gives the arguments which exhibit the least residual value. This means, the deformation parameter does not correspond to the residual value. The deformation parameter could not be used as the pattern matching parameter, because the deformation pattern might be small in non-corresponding clusters. For example, even though the obtained affine matrix is a unit matrix, i.e., contains no deformation, the residual value could be extremely large when the clusters do not correspond. So if the deformation parameter in the affine matrix was used as the pattern matching parameter, there will be a lot of incorrect matching.

For the correspondent clusters, the residual value was almost zero, although the deformation might be very large. Therefore, the corresponding particle clusters could be determined using the information of residual value, R . By applying the present technique, the corresponding particle clusters were determined uniquely.

In the calculation, the information of scalar stress was directly obtained. In this study, the scalar stress was defined as,

$$S = \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2 + \frac{1}{2}(\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2) \quad (5)$$

The correspondent cluster was checked by the scalar stress, S . The scalar stress in a certain cluster should be similar to that of the surrounding clusters. If the scalar stress is considerably different from the surrounding cluster stress, the cluster matching might be incorrect.

When the three-dimensional particle positions were obtained from the HPIV technique or the stereo camera technique, the particle cluster matching was carried out using the information of particle locations. For the target particle cluster in the first image, the residue of the affine transformation for every possible target particle cluster in the second image were calculated. Then, the smallest residue particle cluster corresponds to the target particle cluster in the first image.

Using this technique, even when rotation, shear and expansion are applied to the flow, the particles can be correctly tracked. Therefore, the particle

cluster can be easily tracked correctly between the images.

3. Verification

In order to verify the proposed technique, velocity distributions were reconstructed from synthetic particle data.

Three-dimensional flow fields were calculated by a numerical simulation code. Using the simulated flow, the movement of the particles was calculated. Then, the flow was reconstructed from the particle information, using the present technique. To compare the results, the flow was also reconstructed using a cross-correlation technique⁽²⁾ and a spring model technique⁽³⁾ with the same particle information.

Figure 1 shows the simulated three-dimensional laminar flow field with large circulating flow calculated by a finite element method (FEM) code. Five hundred particles were inserted into the flow field. The size of the images was $200 \times 200 \times 200$ pixels. The particle deformation is the parameter of the synthetic

Fig. 1 Reference three-dimensional flow

Fig. 2 Three-dimensional velocity distributions reconstructed by cross-correlation

images. In this simulation, the maximum displacement between two images was about 100 pixels, i.e., 50% of the image width, while the minimum displacement was almost zero. Large shear existed at the entrance and bottom regions, causing the particle tracking to be very difficult.

Using the particle location information, the velocity field was reconstructed with the cross-correlation⁽²⁾, spring model⁽³⁾ and present technique. Figures 2, 3 and 4 show the results of the reconstructed flow field, respectively. The cross-correlation technique results had a low degree of accuracy because of the large deformation, i.e., rotation and shear. Only the region where the movement was almost parallel was correctly reconstructed.

Table 1 shows the number of reconstructed vectors. 484 vectors should have been calculated, since 16 of the 500 inserted particles flowed out through the outlet. Using the cross-correlation technique, only 54% of the vectors were found. The ratio of erroneous vectors to calculated vectors was 23%.

Fig. 3 Three-dimensional velocity distributions reconstructed by spring model

Fig. 4 Three-dimensional velocity distributions reconstructed by affine transformation

Table 1 Results of reconstructed vectors

Technique		CC ^{#1}	SM ^{#2}	AT ^{#3}
Correct Vectors [A]		260	387	370
Erroneous Vectors [B]		89	59	70
Particle to be tracked [C]		484		
Yield [A/C]		54%	80%	76%
Error ratio [B/(A+B)]		26%	13%	16%

#1: Cross correlation technique⁽²⁾#2: Spring model technique⁽³⁾

#3: Affine transformation technique

Fig. 5 Shear distributions on y - z plane ($x=100$)

On the other hand, the spring model technique could reconstruct the flow field with a good degree of accuracy. In this simulation, 80% of the vectors could be correctly obtained by the spring model. The ratio of erroneous vectors was only 13%. So, the spring model is superior for measuring a flow field with rotation and shear motion.

The affine transformation technique also reconstructs the velocity vector with high accuracy. However, there were several erroneous vectors in the inlet jet region. The error vector ratio and yield by the affine transformation was less than those of the spring model.

Since the affine transformation was a linear transformation, the deformation should be expressed by the linear equation. However, in the present case, especially near the inlet, the deformation was too large to assume the linear deformation, resulting in the error of the affine transformation. For the affine transformation approach, the deformation should be expressed as a linear equation. Therefore the larger deformation could not be tracked. The spring model was found to be the better tracking technique under the larger deformation, since it does not assume the linear deformation.

Using the affine transformation, the shear stress distribution could be directly obtained. Figure 5 shows the cross-sectional distribution of scalar shear

S defined in Eq.(5). The cross section is the y - z plane at the $x=100$ pixel. The shear at the inlet jet region ($y<20$ and $z<20$) was large, compared to that at the other location. However, the actual shear at the inlet jet region was much larger than the reconstructed shear. Because of the larger deformation, the particle tracking contained numerous errors in this region. In order to track the particles under such a larger deformation condition, the nonlinear deformation should be taken into account.

4. Conclusion

A new tracking algorithm for three-dimensional PIV has been proposed, which can be applied to a flow field with rotation, shear and expansion. The corresponding particle clusters between the first and second images are determined by the affine transformation. Using the residue of the least squares technique, the pattern matching between the clusters is carried out.

The effectiveness of this technique was verified with synthetic data of three-dimensional flow field. The technique can correctly track the particle pattern in the relatively lower deformation region.

When the deformation of the particle pattern between two images is very large, the pattern deformation could not be expressed by the affine transformation, i.e., linear transformation, resulting in mis-tracking. However, the spring model technique⁽³⁾ was found to be more effective even under the larger deformation condition, because the spring model does not assume linear transformation.

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