Experimental Study on Drag Reduction Using Flexible Tubes*

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Friction drag reduction of pipe flow using a flexible tube was experimentally investigated and compared with a rigid tube. It was found that the coefficient of friction of the flexible tube was lower than that of the rigid tube. Furthermore, when the thickness of the flexible tube was decreased and the Reynolds number was increased, the rate of drag reduction increased, except when using a very thin flexible tube. To clarify these phenomena more precisely, the fluctuating velocity on the center axis of the tube, the fluctuating pressure on the inner wall surface of the tube and the fluctuating displacement on the outer wall surface of the tube were measured. From the power spectrum distribution and the overall band energy obtained by analyzing these waves, it was clarified that there exists an interrelation between the turbulent energy and the drag reduction and a negative correlation between the fluctuating pressure and the fluctuating displacement.

Key Words: Pipe Flow, Turbulent Flow, Fluctuating Pressure, Fluctuating Velocity, Fluctuating Displacement, Drag Reduction, Flexible Tube, Friction Drag

1. Introduction

Friction drag reduction is one of the most important engineering problems and a number of experimental and theoretical investigations^{(1)~(3)} have been carried out. The usual methods employed for reducing friction drag include Tom's effect^{(4),(5)}, riblet^{(6),(7)}, LEBUs (Large Eddy Brakeup Devices)^{(8),(9)} and compliant wall⁽¹⁰⁾. With regard to Tom's effect, the existence of long-chain polymer structures which suppress the burst is known to be associated with the energy creation and dissipation. The riblet reduces the turbulent friction drag by weakening the vortices in the viscous sublayer and by restricting the streamwise vortices on the wall. As reported by Walsh⁽⁶⁾, many investigations were carried out and the

flow mechanism was clarified by the study which focussed on the coherent structure of turbulence. The LEBUs method has the effect of stabilizing the small scale instabilities in the wall.

With regard to the compliant wall, investigations^{(11)~(19)} have been performed, since reported by Kramer⁽¹⁰⁾. For the purpose of fluid transportation, a compliant wall may be called a flexible tube. It is unclear whether a flexible tube can reduce friction drag^{(11),(15),(16),(18)}. The data have not yet clearly confirmed the effectiveness of a flexible tube. Only one method for evaluating the friction drag reduction has been utilized so far, that is, by measuring the fluctuating velocity characteristics on the center axis of a flexible tube. To our knowledge, with the exception of the transition region⁽¹⁹⁾, the coefficient of friction as a macroscopic measure which shows the friction drag in the pipe for the fully developed turbulent flow region has not yet been reported. To clarify the different mechanisms of drag reduction, the fluctuating displacement characteristics of the compliant wall were measured using an eddy-current type gap sensor^{(18),(19)}. Arakawa et al.⁽¹⁹⁾ obtained remarkable results showing the existence of a negative correlation

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between the fluctuating displacement characteristics and the fluctuating velocity characteristics on the center axis of the tube. However, this research was only concerned with the effects of the pipe length where the flow transits from laminar to turbulent. To our knowledge, the effects of the thickness of the tube in the turbulent flow region have not yet been examined.

In this study, we examined friction drag reduction using a flexible tube. We also aimed at developing a flexible tube with a friction drag lower than that of a rigid tube from a fluid dynamic perspective, while clarifying the flow mechanism. The coefficient of friction of a flexible tube was measured in the fully developed turbulent flow region and compared with that of a rigid tube. The experiments were carried out using four different tube thicknesses of tube constructed of the same materials, wherein numerous geometrical and mechanical parameters affected the drag reduction of flexible tubes. To clarify the causes of these different behaviours, it was necessary to measure the flow characteristics near the wall. However, because of the small diameter of the tube and the instantaneous displacements of the wall, it was very difficult to measure them, therefore, as mentioned above, the fluctuating velocity on the center axis of the tube was measured. In this experiment, the fluctuating velocity was measured using a miniature I-type hot-film sensor and the interrelation between the fluctuating velocity and the coefficient of friction was examined. The interrelation of the characteristics of fluctuating displacement and the flow near the wall was also examined. In this research, the fluctuating pressure at the wall surface was measured instead of the fluctuating velocity, because of the difficulty in measuring fluctuating velocity. The measurements were carried out, in spite of the difficulties in attaching the pressure sensor directly on the wall, because of the passive vibration of the flexible tube. In addition, although it is preferable to measure the fluctuating displacement on the inner wall surface, for ease of measurement, the measure-

ment was taken on the outer wall surface using a noncontact optical fiber gap sensor. The frequency analysis of these fluctuating properties was determined using a FFT analyzer and the correlation between them was examined.

2. Nomenclature

- D_i : inner diameter of tube
- D_o : outer diameter of tube
- d : fluctuating displacement
- E : overall band energy
- f : frequency
- P_m : mean pressure
- p : fluctuating pressure
- Re : Reynolds number
- S : power spectrum
- t : time interval
- V_m : mean velocity
- v : fluctuating velocity
- α : friction drag reduction rate
- α_A : friction drag reduction rate due to the variation of flow sectional area
- α_F : friction drag reduction rate due to the effect of flexible tube
- λ : coefficient of friction
- λ_A : coefficient of friction after the reduction due to the variation of flow sectional area subtracted from the value of the rigid tube
- λ_R : coefficient of friction of rigid tube
- Subscripts
 - d : displacement
 - p : pressure
 - v : velocity

3. Experimental Apparatus and Methods

The schematic of the experimental apparatus is shown in Fig. 1. The water as a working fluid flows through the flow control valve ① and the turbulence is suppressed in the surge tank ②. After it is accelerated and regulated by the bellmouth ④, the water flows into the measuring section of the test tube ⑤, which is maintained in a horizontal position, and the water

Fig. 1 Schematic of experimental apparatus

Table 1 Dimensions of test tubes

Type	Di mm	Do mm
Rigid Tube	10.0	20.0
Flexible Tube A	10.0	18.0
Flexible Tube B	10.0	14.0
Flexible Tube C	10.0	13.0
Flexible Tube D	10.0	12.0

then flows into the volumetric flow meter ⑩. In the case where a flexible tube is used as a test tube, the flow is affected by its tension and deflection. These influences are made negligible by connecting the acrylic tube ⑨ to the end of the flexible tube in order to support the flexible tube with its natural length and by placing sponges ⑧ the equidistance apart. The center axis of the test tube is mounted 50 mm below the water level of the water tank ⑦. A silicon tube as a flexible tube (Poisson ratio : 0.45, Young's modulus : $2.5 \times 10^6 \text{ N/m}^2$) and the acrylic tube as a rigid tube with an inner diameter kept constant at 10 mm are used. The inner and outer diameters of the tubes tested are shown in Table 1. The outer diameters of the silicon tubes are 18, 14, 13 and 12 mm, and these tubes are shown as flexible tubes: A, B, C and D, respectively. In the case of flexible tubes, the inner diameter at rest is used as a reference length, whereas the inner diameter increases as the water begins to flow in the tube. Here, the mean velocity in the tube is measured by the volumetric flow meter and the Reynolds number is calculated by using these values.

Usually, the coefficient of friction of the rigid tube is measured by the static pressure difference between two points. The wall of the flexible tube vibrates, therefore, it is difficult to accurately detect the mean pressure. In this experiment, to improve the accuracy of measurement, the mean pressure difference between the surge tank (pressure gage ③) and the exit of the tube is measured. In this case, as the influences of the bellmouth and the entrance length are included, the length of the tube is long enough at 6 m to make such effects negligible. However, the influence which causes the difference at the transition point might be included in the measuring value.

With regard to the fluctuating characteristics, the fluctuating velocity is detected by an I-type hot-film sensor (miniature platinum film sensor dia. : $25 \mu\text{m}$, sensor length : 0.25 mm, frequency response : 10 kHz) mounted on the center axis of the tube. To compensate for the temperature of the hot-film anemometry, another probe is mounted in the surge tank. The fluctuating pressure is measured by the pressure sensor ③ (frequency response : 6~10 kHz) attached to

Fig. 2 Detecting sections of pressure and displacement

the wall directly in contact with the acrylic resin ⑤, as shown in Fig. 2(a). The output signals are processed by a personal computer, after being amplified by a dynamic strain meter. By attaching the pressure sensor to the acrylic resin, the aspect of the vibration system involving the flexible tube is different from that of the flexible tube by itself. However, the qualitative tendency does not change. The fluctuating displacement is detected by an optical fiber sensor (frequency response : 20 kHz, sensitivity : $0.013 \mu\text{m/mV}$, measuring range from the sensor : $65 \sim 125 \mu\text{m}$) placed on the outer wall surface. This method is more effective than an eddy current method or the laser beam method, because the displacement of the flexible tube which is dipped in water and is in isolation can be measured with precision by the noncontacting method. The flexible tube is supported by the sponge blocks ⑥ to prevent the tube from shifting during the measurements, as shown in Fig. 2(b). It is considered that the qualitative tendency does not change even when no supports are present, whereas the mode of the vibration system involving the flexible tube is affected slightly due to restriction caused by the supports.

All the measurements of the fluctuating characteristics were carried out at the measuring location ⑥ 2 m downstream from the bellmouth, as shown in Fig. 1. The frequency analysis of the fluctuating velocity, pressure and displacement was performed using the FFT analyzer. When the discrete frequency components were observed, the power level included the specified frequency band in the power spectrum distribution of the fluctuating component which is called a partial band energy, is picked up as the equivalent

value in the fluctuating energy. The analyzing function of such fluctuating properties is loaded in the FFT analyzer.

4. Experimental Results and Discussion

4.1 Coefficient of friction

The relation between the coefficient of friction λ and the Reynolds number Re is shown in Fig. 3. Due to the fact that the coefficient of friction of the rigid tube conforms to the Blasius experimental equation, the availability of this measuring method is confirmed. It was previously reported that the plastic deformation of the flexible tube occurs to some extent as the flow rate increases. Then, the Reynolds number range up to 3.0×10^4 is used. As is seen from this figure, the values of λ for all flexible tubes are lower than those of the rigid tube. When the thickness of the flexible tube is decreased and the Reynolds number is increased, the value of λ decreases. Whereas, for the flexible tube D, the value of λ does not decrease.

Due to the fact that the flow sectional area increases as the flow rate in the flexible tube increases, when the above mentioned reference length is used to obtain the drag coefficients, the drag appears to decrease dramatically. Here, λ_A denotes the friction coefficient after the reduction due to the variation of flow sectional area is subtracted from the value for the rigid tube λ_R . The drag reduction rate normalized by λ_R can be expressed by

$$\frac{\lambda_R - \lambda}{\lambda_R} = \frac{\lambda_R - \lambda_A}{\lambda_R} + \frac{\lambda_A - \lambda}{\lambda_R}, \quad (1)$$

where the term on the left hand side is the friction drag reduction rate α , the first term on the right hand side is the drag reduction rate α_A due to the variation of the flow sectional area and the second term is the drag reduction rate α_F due to the effect of the flexible

tube. The value of λ_R is estimated by the Blasius experimental equation and $\lambda_A^{(14)}$ by

$$\lambda_A = \frac{2D_i P_A}{\rho V_m^2 L}, \quad (2)$$

where D_i denotes the inner diameter of tube, ρ is the density of fluid, V_m is the mean velocity, L is the pipe length and P_A is expressed by

$$P_A = \frac{2\varepsilon}{D_i \left(1 - \frac{\nu}{2}\right)} \left\{ \left[\left(1 - \frac{\nu}{\varepsilon} T\right)^{6-\beta} + \frac{1}{2\varepsilon} (6-\beta) \left(1 - \frac{\nu}{2}\right) \gamma D_i L \right]^{1/(6-\beta)} - \left(1 - \frac{\nu}{\varepsilon} T\right) \right\}. \quad (3)$$

In Eq. (3), ν denotes a Poisson ratio of the tube, T is the tension of the tube, β is constant ($=1/4$) and γ is the pressure gradient of the rigid tube, and ε is also expressed as follows:

$$\varepsilon = E\delta \left[1 + \frac{\delta^2}{3D_i^2(1-\nu^2)} \right], \quad (4)$$

where E denotes the Young's modulus and δ is the thickness of the tube. Figure 4 shows the relation between the drag reduction rate α_F due to the effects of the flexible tube which is calculated using the above equation and the diameter ratio $[1 - (D_i/D_o)]$. The value of the rigid tube is plotted against the diameter ratio 1.0. When the diameter ratio is higher than 0.23, the value of α_F increases with a decrease in the thickness of the flexible tube and an increase in the Reynolds number. When the diameter ratio is lower than 0.23 or as in the case of the flexible tube D, the value of α_F does not increase and the effects of the Reynolds number do not appear. It was found that there exists an optimum thickness of the tube to minimize the friction drag in this experimental condition.

Fig. 3 Relations between coefficient of friction and Reynolds number

Fig. 4 Drag reduction rate

4.2 Relation between the fluctuating velocity characteristics on the center axis of the tube and drag reduction

To investigate the causes of these different behaviors, the fluctuating velocity on the center axis of the tube was measured and the relation between the drag reduction and this value was examined. Figure 5 shows the instantaneous wave shapes of the fluctuating velocity of the rigid tube and the flexible tube A, B, C and D, respectively. The Reynolds number is shown at $Re=2.5 \times 10^4$ because a power drop of the hot-film sensor occurred when $Re=3.0 \times 10^4$. As is seen from this figure, the fluctuating velocity of all flexible tubes is lower than that of the rigid tube. The power spectrum distributions of the fluctuating velocity S_v are obtained by the FFT analyzer and shown in Fig. 6. The values of S_v of all flexible tubes are lower than for those of the rigid tube. The thinner the thickness of the tube, the lower the value of S_v becomes, especially in the low frequency range, whereas for the flexible tube D the value of S_v increases. To examine the power spectrum more precisely, the relation between the overall band energy of the fluctuating velocity E_v normalized by the mean velocity V_m and the diameter ratio is shown in Fig. 7. The value of E_v/V_m shows the lowest value at the diameter ratio of 0.23, independent of the Reynolds number. As a result, it can be said that the tendency of the reduction of the

Fig. 5 Wave shapes of fluctuating velocity

value of E_v/V_m or turbulent energy corresponds to that of the drag reduction previously mentioned and there exists a strong relation between them.

4.3 Relation between fluctuating pressure and displacement characteristics on the wall surface

The fluctuating pressure and displacement on the wall surface were measured and the relation between the characteristics of flow and displacement was examined. The instantaneous wave shapes of the fluctuating pressure of the tubes tested at $Re=3.0 \times 10^4$ are shown in Fig. 8. As is seen from this figure, the fluctuating pressure of all flexible tubes is lower than that of the rigid tube. The thinner the wall thickness of the tube, the lower the fluctuating pressure becomes for the wave is concerned. The power spectrum distributions of the fluctuating pressure S_p are

Fig. 6 Power spectrum distributions of fluctuating velocity

Fig. 7 Overall band energy of fluctuating velocity

Fig. 8 Wave shapes of fluctuating pressure

Fig. 9 Power spectrum distributions of fluctuating pressure

evaluated by the FFT analyzer, as shown in Fig. 9. The values of S_p for all flexible tubes are lower than those for the rigid tube and the thinner the thickness of a flexible tube, the lower the power spectrum becomes in the whole range of frequencies, whereas

Fig. 10 Overall band energy of fluctuating pressure

Fig. 11 Wave shapes of fluctuating displacement

for the flexible tube D, in the frequency range lower than 200 Hz, the power spectrum increases. The increment of S_p in this low frequency range corresponds to the phenomena of the drag reduction rate α_F of the flexible tube D showing a lower value than that for the flexible tube C, as shown in Fig. 4. The relation between the overall band energy of the fluctuating pressure E_p normalized by the mean pressure P_m and the diameter ratio is shown in Fig. 10. From this figure, it is seen that the lower the diameter ratio, the lower the value of E_p/P_m becomes for all Reynolds numbers except for the flexible tube D. Furthermore,

the reduction rate of E_p/P_m increases as the Reynolds number increases.

The instantaneous wave shapes of fluctuating displacement of all flexible tubes at $Re=3.0 \times 10^4$ are shown in Fig. 11. The thinner the thickness of the tube, the larger the displacement of the tube becomes. For example, in the case of the flexible tube D the maximum displacement is about $5.6 \mu\text{m}$ and the thickness of the viscous sublayer in the rigid tube is estimated⁽²⁰⁾ to be $19.6 \mu\text{m}$, hence it is seen that the wall vibrates about 29% of its thickness. The power spectrum distributions of fluctuating displacement S_d are also shown in Fig. 12. The thinner the thickness of the flexible tube, the higher the value of S_d becomes in the whole range of frequencies. Therefore, for the fluctuating displacement, the aforementioned phenomena of the flexible tube D in the lower range of fre-

Fig. 12 Power spectrum distributions of fluctuating displacement

Fig. 13 Overall band energy of fluctuating displacement

quencies lower than 200 Hz cannot be observed. The relation between the overall band energy of fluctuating displacement E_d normalized by the inner diameter of the tube D_i and the diameter ratio is shown in Fig. 13. From this figure, it is seen that the lower the diameter ratio and the higher the Reynolds number, the higher the value of E_d/D_i becomes.

The relation between the fluctuating pressure shown in Fig. 10 and the fluctuating displacement shown in Fig. 13 was examined. For all Reynolds numbers, the lower the diameter ratio, the lower the overall band energy of fluctuating pressure becomes, except for the flexible tube D, in which the fluctuating displacement increases. Moreover, from the fact that these changing rates increase as the Reynolds number increases, it is shown that there exists a negative correlation between them. Therefore, the displacement of the passive wall increases as the thickness of the tube decreases and has the effect of lowering the fluctuating pressure. It is considered that the decrease in the turbulent energy in the vicinity of the wall, inferred from the fluctuating pressure, corresponds to that of the drag reduction.

The overall band energy of the fluctuating pressure of the flexible tube D is higher than that of the flexible tube C and a negative correlation is not found between the fluctuating pressure and the displacement. As previously mentioned, the fluctuating pressure was analyzed both for lower (≤ 200 Hz) and higher (> 200 Hz) frequency ranges. The partial band energies E'_p of the lower and higher frequencies are shown in Fig. 14 at $Re=3.0 \times 10^4$. From this figure, it is seen that the partial band energy of the higher frequency range shows a tendency opposed to the tendency of the overall band energy. Therefore, from the fact that a negative correlation was observed in

Fig. 14 Partial band energy of fluctuating pressure

the higher frequency range for the flexible tube D, the effect of the drag reduction is also clarified. On the other hand, the negative correlation is not maintained in the lower frequency range. Phenomena peculiar to this range appear to increase the fluctuating pressure. This may be inferred from the fact that the friction drag increases as the effect of the roughness of the wall surface increases. As previously mentioned, the displacement width is smaller than $19.6\text{ }\mu\text{m}$, even for the flexible tube D, and this effect does not appear. This condition holds for the rigid wall, however, and it is considered that when the wall of the flexible tube vibrates, the effect analogous to that of the wall roughness appears. From this hypothesis, the effect of the wall roughness is increased by the fluctuating pressure. It is considered that the tendency of increasing the turbulent energy in the vicinity of the wall inferred by this fluctuating pressure corresponds to that of the drag increase.

As the wall of the flexible tube passively vibrates, it is very difficult to detect the flow characteristics. Hence, the causes of the above mentioned different behavior patterns of the drag reduction are clarified by the given data. To understand the phenomenon more precisely, and to investigate the Reynolds stress near the wall, it is necessary to solve both problems in a physical experiment and in a numerical calculation. This might become important in the near future. With regard to the experiment, it is necessary to measure the flow characteristics near the wall by very expensive large scale facilities. On the other hand, the numerical simulation of the moving boundary problem such as a passive wall is not as easy to solve.

5. Conclusions

The friction coefficient of the flexible tube in the turbulent flow region was experimentally examined using four tubes of various thicknesses (silicon tube: inner dia. 10 mm) and compared with that of a rigid tube. To clarify the differences between them from the results of the frequency analysis of fluctuating velocity on the center axis of the tube, the fluctuating pressure on the inner wall surface of the tube and the fluctuating displacement on the outer wall surface of the tube, the following conclusions were obtained.

(1) When the thickness of the flexible tube is decreased and the Reynolds number is increased, the drag reduction rate increases in the range of thicknesses larger than 1.5 mm. On the contrary, due to the fact that the drag reduction rate decreases when the thickness is equal to 1.0 mm, it was found that there exists a most optimum thickness of the tube to minimize the friction drag in this experimental condition.

(2) The overall band energy of the fluctuating velocity on the center axis of tube or the turbulent energy becomes lower as the thickness of the tube decreases and shows the lowest value at a thickness of 1.5 mm. In this case there exists a strong relation between the friction drag and the turbulent energy.

(3) From the change of the overall band energy of the fluctuating pressure and displacement near the wall, it is clear that there exists a negative correlation between them. For the case of the thinnest flexible tube, the partial band energy in the high frequency range ($>200\text{ Hz}$) shows a negative correlation. The displacement of the passive wall increases effectively as the thickness of tube decreases and has the effect of causing the fluctuating pressure to decrease. On the other hand, as the power spectrum of the fluctuating pressure in the low frequency range ($\leq 200\text{ Hz}$) increases when the tube thickness is 1.0 mm, it is seen that the drag reduction rate does not necessarily increase.

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