Bursting Phenomena in a Turbulent Square-Duct Flow*

(Generation Mechanisms of Turbulent Wall Skin Friction)

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Near-wall velocity measurement has been performed at $y^+ = 2.6$ in a fully developed turbulent square-duct flow in order to evaluate fluctuating wall skin friction. Turbulence structures, which induce high wall shear rates, have been identified using simultaneous flow visualization. It is shown that quasi-streamwise vortical structures play a dominant role in high-friction generation, and that they can be classified into the 'cyclonic' or the 'anticyclonic' vortex, which is tilted in the spanwise direction with its vorticity parallel or antiparallel, respectively, to the mean shear vorticity. The generation of high wall shear rates is often seen on the sweep sides of the cyclonic and anticyclonic streamwise vortices. In addition to the experiments, the long-wavelength instability of a spanwise vortex filament in a background shear flow has been investigated using the Biot-Savart law with a suitable cutoff in order to demonstrate the generation mechanism of streamwise vortices. It is found that the unstable modes lead to the generation of streamwise vorticity, and that the most unstable mode has a spanwise wavelength of the order of 100 wall units in the near-wall region.

Key Words: Turbulent Flow, Turbulence, Shear Flow, LDV, Flow Visualization, Bursting Phenomena, Wall Skin Friction, Streamwise Vortex, Vortex Filament, Linear Stability Analysis

1. Introduction

Wall-bounded turbulent flows, represented by a turbulent boundary layer and a duct flow, are well known to lead to much higher wall skin friction compared with corresponding laminar states. In the case of a turbulent duct flow, since wall skin friction is proportional to a pressure drop along a duct and the pressure drop is then proportional to energy dissipation in the whole duct, the elucidation of generation mechanisms of turbulent wall skin friction is useful not only for engineering applications but also for

understanding production and dissipation processes of turbulence kinetic energy.

It has been observed that in the near-wall region of turbulent shear flows, e.g., turbulent boundary layers and two-dimensional channel flows, most of the turbulence kinetic energy is produced during the so-called bursting^{(1), (2)}. Much effort has been devoted to the examination of near-wall bursting, which has led to the conclusion that near-wall quasi-streamwise vortical structures cause the bursting through lifts of low-speed fluids away from a wall, i.e., ejection, and intrusions of high-speed fluids toward a wall, i.e., sweep⁽³⁾⁻⁽⁵⁾. Comparing ejection and sweep motions induced by a streamwise vortex, the former is considered to suppress the generation of turbulent wall skin friction while the latter is considered to contribute to it. Recently, in their investigation of the relationship between streamwise vortices and skin friction using the DNS (direct numerical simulation) data base, Kravchenko et al.⁽⁶⁾ suggested that the sweep induces high-friction generation. Orlandi and Jiménez⁽⁷⁾ introduced an idealized quasi-two-dimensional model

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flow to point out that the ejection weakens wall shear stress but the sweep intensifies it, and consequently the existence of the streamwise vortex can lead to a higher wall skin friction compared with the laminar state. No experimental demonstration has, however, been performed to relate streamwise vortical motions with high-friction generation because of the difficulty in measurement of instantaneous wall shear stress. In addition, although several attempts have been made to give a theoretical explanation for the generation of near-wall streamwise vortical motion using the direct resonance theory⁽⁸⁾, the optimal perturbation theory⁽⁹⁾ and the concept of a minimal flow unit⁽¹⁰⁾, we must admit that the knowledge of the actual dynamical process involved in its generation is still lacking.

The main objectives of this study are to experimentally demonstrate the direct relationship of streamwise vortical motion with high-friction generation and to theoretically explore the generation mechanism of streamwise vortical motion. In the experiments, near-wall LDV (laser-Doppler velocimeter) measurement as well as simultaneous dye flow visualization was performed at the wall bisector in a fully developed turbulent square-duct flow in order to evaluate instantaneous wall shear stress and to detect turbulence structures that induce high wall shear stress. A new phenomenological approach is also made to the early stages of the generation of streamwise vortical motions, which is based on a linear stability theory for a rectilinear vortex filament in a background shear flow.

2. Experimental Apparatus and Procedures

The experiments were performed in the straight square duct previously described by Ayukawa et al.⁽¹¹⁾. The square duct is nearly 9 m in length and 100 mm in width. The test section is located 6.8 m downstream of the duct inlet, in which the velocity field is fully developed⁽¹¹⁾. The mean flow parameters are listed in Table 1. Water was used as the working medium, and the Reynolds number Re , based on one-half of duct width $H/2$ and mean centerline velocity U_c , was set around 5 000. In spite of the presence of a secondary flow, turbulence statistics⁽¹¹⁾, mean streak spacing⁽¹²⁾ and streamwise vortical structures⁽¹³⁾ in the vicinity of the wall bisector have been observed to be quite similar to those for boundary layers and two-dimensional channel flows. In Table 1, u_τ represents local friction velocity, the value of which was obtained from the near-wall velocity measurement on the wall bisector (see below).

The two-component LDV system was used to obtain the streamwise and spanwise velocity components, u and w . This LDV system is composed of the

Table 1 Mean flow parameters

Kinematic viscosity ν	$1.10 \times 10^{-6} \text{ m}^2/\text{s}$
Duct width H	100mm
Mean centerline velocity U_c	10.4cm/s
Friction velocity u_τ	5.79mm/s
Reynolds number $Re = (U_c H/2)/\nu$	4 730

argon-ion laser supply Spectra-Physics 2017, the optical part Dantec 60 X series, and the signal processing part Dantec 57 X 20, 57 X 35. The measuring volume for the velocity components is estimated to be $46 \mu\text{m}$ in diameter and $387 \mu\text{m}$ in length. The measurement point was located at a distance $y^+ = 2.6$ from the bottom wall along the bottom wall bisector [hereafter $()^+$ denotes a dimensionless value normalized by kinematic viscosity ν and friction velocity u_τ]. Since this measurement location belongs to the lower part in the linear sublayer ($0 \leq y^+ < 4$), instantaneous wall shear stress τ_w is expected to be evaluated by using measured streamwise velocity u as

$$\tau_w \approx \mu \frac{u}{\delta y}, \quad (1)$$

where μ is the coefficient of viscosity, and $\delta y (= 0.5 \text{ mm})$ is the distance from the wall to the measurement point. The friction velocity is also evaluated as

$$u_\tau \approx \sqrt{\nu \frac{\bar{u}}{\delta y}} \quad (2)$$

[hereafter $()$ denotes a time-averaged value]. The value of u_τ obtained from Eq. (2) is tabulated in Table 1. On comparing this value with the one estimated using Clauser's chart based on the wall law: $\bar{u}^+ = 2.44 \ln y^+ + 4.9$, we observed a 2.1% discrepancy between them.

Fluorescein, the specific gravity of which was 1.00, was used for visualization of near-wall turbulence structures. The 1-mm-wide slit was located on the bottom wall of the duct $500\nu/u_\tau$ upstream of the measurement point, through which the dye was injected at a very small flow rate of $330 \text{ mm}^3/\text{s}$. Sublayer flow structures were illuminated in the region $y^+ \leq 25$ by an argon-ion laser light sheet, and then the visualized structures were recorded from above the LDV measurement point with the high-speed video camera Nac HSV 200 (frame intervals were $1.5\nu/u_\tau^2$). In the experiments, visual and LDV data were synchronized and taken simultaneously, and they were registered on a video tape and a magnetic disk, respectively. The sampling frequency of the LDV ranged from 100 Hz to 200 Hz, and the total number of recording samples was 270 000.

3. Turbulence Structures Inducing High Wall Skin Friction

In this section we shall discuss the turbulence structures that generate turbulent wall skin friction by making use of flow visualization results. We especially take note of dye-filled low-speed streaks⁽¹²⁾ in the visual data. The visualized low-speed streaks were examined when they appeared within $150v/u_\tau$ upstream and $100v/u_\tau$ downstream of the LDV measurement point in the streamwise direction and within $50v/u_\tau$ of the measurement point in the spanwise direction, as well as when instantaneous wall shear stress τ_w satisfied the high-friction condition that

$$\frac{\tau_w - \bar{\tau}_w}{\tau_{w \text{ rms}}} > 1, \quad (3)$$

where $\tau_{w \text{ rms}} (=0.0162 \text{ N/m}^2)$ is an rms value of the wall shear stress.

In Fig. 1 are conceptually illustrated visualized turbulence structures associated with high skin friction at the measurement point. During high-friction generation, four cases [Fig. 1: types (a)–(d)] were observed: type (a) a single low-speed streak appears; type (b) a pair of low-speed streaks appears; type (c) a pocket-like structure (Falco's pocket⁽¹⁴⁾) without the dye appears; and type (d) no low-speed streak appears, around the measurement point. The ratios of each turbulence structure to the total are listed in Table 2. It can be seen that type (a) comprises over 60% of the total and thus type (a) turns out to be a dominant contributor to high-friction generation.

Table 2 Ratios of each turbulence structure

Type	Ratio
(a) Single low-speed streak	0.64
(b) Pair of low-speed streaks	0.22
(c) Pocket	0.03
(d) No low-speed streak	0.11

Fig. 1 Schematic drawing of visualized structures

As commonly accepted, a low-speed streak is formed through the pumping action of a single quasi-streamwise vortex⁽¹⁴⁾. This implies, in conjunction with the above result, that a single streamwise vortex plays a dominant role in high-friction generation. In a previous paper⁽¹⁵⁾, three of the present authors have pointed out that the low-speed streak (and so the rotation axis of the streamwise vortex) often has a spanwise inclination with respect to the streamwise direction. Two kinds of spanwise inclinations are considered to exist as shown in Fig. 2: one is the case that the spanwise vorticity component of the inclined streamwise vortex is parallel to the mean shear vorticity, and another is the case that it is anti-parallel. Hereafter we call the former and the latter a 'cyclonic' case and an 'anticyclonic' case, respectively. In the following, we take into account these inclinations of the streamwise vortices in the discussion of the turbulence structure, type (a), that plays a central role in high-friction generation. We fix the orientation of the spanwise inclination of each streamwise vortex according to the inclination angle of the low-speed streak, type (a), that is, positive or negative (Fig. 3). The orientation of the vorticity of each streamwise vortex is, on the other hand, fixed according to whether the spanwise velocity component w at the measurement point is positive or negative during the passage of the low-speed streak, i.e.,

Fig. 2 Cyclonic and anticyclonic inclinations of streamwise vortices

Fig. 3 Inclination angle of low-speed streaks

Table 3 Ratios of cyclonic/anticyclonic structures

Cyclonic case	Anticyclonic case
0.46	0.54

Table 4 Ratios of low-speed streaks upstream/downstream of measurement point

	Upstream	Downstream
Cyclonic case	0.71	0.29
Anticyclonic case	0.34	0.66

the streamwise vortex. Kravchenko et al.⁽⁶⁾ reported, using the DNS data base, that the central axis of the streamwise vortices that induce high wall skin friction is located around $y^+ = 12$. Thus, the present measurement point of w ($y^+ = 2.6$) turns out to be located under the central axis of the vortices, so that we can obtain the vorticity orientation from the sign of w (see Fig. 2). Note that the streamwise vortex has a cyclonic (or anticyclonic) inclination in the case of $w\theta > (<) 0$ in the present configuration.

Table 3 shows the ratios of cyclonic and anticyclonic structures to the total, i.e., type (a). No attention had been paid to the cyclonic and the anticyclonic inclinations as yet, but the present result tells us that the ratios of cyclonic and anticyclonic structures are nearly the same. Table 4 also shows the ratios of whether the low-speed streak is located upstream or downstream of the measurement point in the cyclonic and the anticyclonic case. In the cyclonic case, 71% of the low-speed streaks are located upstream of the measurement point while in the anticyclonic case, 66% of the low-speed streaks are located downstream of it, which means that high skin friction is often observed downstream (or upstream) of the low-speed streaks for the cyclonic (or anticyclonic) case. It can be seen from Fig. 2 that the cyclonic (or anticyclonic) streamwise vortex induces lifts of low-speed fluids away from the wall, i.e., the low-speed streak or ejection, upstream (or downstream) of it. Hence, sweep, i.e., intrushes of high-speed fluids toward the wall, is expected to be induced by the cyclonic (or anticyclonic) vortex downstream (or upstream) of the low-speed streak, which is consistent with the above observation (Table 4). These results lead us to the conclusion that the sweep, induced by the streamwise vortical motion, generates high wall skin friction. This conclusion supports Kravchenko et al.'s⁽⁶⁾ numerical result.

In a previous paper⁽¹⁵⁾, three of the present authors have suggested, through their experimental and theoretical work, that the anticyclonic (or

Fig. 4 Definition of stability problem of vortex filament

cyclonic) streamwise vortex causes the bursting event of $du/dt > (<) 0$, which is detected by the VITA (variable-interval time-averaging) technique⁽¹⁶⁾. Also in the above results, the sweep is spatially located upstream (or downstream) of the ejection in the anticyclonic (or cyclonic) case, which implies that the bursting event of $du/dt > (<) 0$ is observed at a spatially fixed measurement point around the anticyclonic (or cyclonic) streamwise vortex. Thus, we can say that the suggestion by the previous paper⁽¹⁵⁾ is now actually proved.

4. Generation Mechanism of Streamwise Vortical Motions

In section 3 we have demonstrated that the sweep induced by streamwise vortical motions generates high wall skin friction. However, the elucidation of the origin of streamwise vortical motions is indispensable for understanding the generation mechanisms of turbulent wall skin friction. In this section we shall make a phenomenological approach to the early stages of the generation of streamwise vortical motions through a theoretical analysis of the long-wavelength instability of a rectilinear vortex filament to infinitesimal disturbances in a background shear flow.

4.1 Linear stability analysis of a vortex filament in a shear flow

The problem to be considered here is a vortex filament embedded in a shear flow, as shown in Fig. 4. e_x , e_y and e_z are the streamwise, wall-normal and spanwise unit vectors, and x , y and z are used to represent the coordinates corresponding to their directions, respectively. a is the vortex core radius, which is necessary to estimate cutoff length δ below, and Γ the strength of the vortex filament. $U_{\text{ext}} = U(y)e_x$ is the background velocity field. Note that in the absence of perturbations, the vortex is parallel to the z -direction. The instantaneous flow due to the vortex filament is described by the Biot-Savart law with a cutoff as

$$\mathbf{u}(\mathbf{x}, t) = -\frac{\Gamma}{4\pi} \int_{C[\delta]} \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \times d\mathbf{x}' + \mathbf{U}_{\text{ext}}(\mathbf{x}), \quad (4)$$

where $\mathbf{x} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$, and the notation $\int_{C[\delta]}$ means that a length 2δ centered on $\mathbf{x}' = \mathbf{x}$ is omitted from a line integral along C defining the vortex filament. δ is referred to as the cutoff length. If we estimate δ properly, then we shall be able to take into account the effects of the internal structure of the vortex filament on its motion. Given the fluid velocity $\mathbf{u}$, the equation of motion of the vortex for each point $\mathbf{X} = X\mathbf{e}_x + Y\mathbf{e}_y + Z\mathbf{e}_z$ on the filament is of the form

$$\frac{d\mathbf{X}(\mathbf{X}_0, t)}{dt} = \mathbf{u}[\mathbf{X}(\mathbf{X}_0, t), t], \quad (5)$$

where

$$\mathbf{X}(\mathbf{X}_0, t)|_{t=0} = \mathbf{X}_0. \quad (6)$$

We note that the above treatment of the background shear flow is an approximation that neglects the effects of the vortex filament on the background flow. In their estimate based on the vorticity equation, Aref and Flinchem⁽¹⁷⁾ found that the dominant interaction term is the advection of the filament by the background flow and the neglected terms are much smaller for the case of the vortex diameter being negligible compared with the characteristic length scale of the background flow. In the present case, a similar estimate shows that this approximation is valid for $|a(dU/dy)/U| \ll 1$ and $|a^2(d^2U/dy^2)/U| \ll 1$. We now consider the rectilinear vortex filament perturbed by a sinusoidal disturbance

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} U(y_0)t \\ y_0 \\ z \end{bmatrix} + \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ 0 \end{bmatrix} \exp(ikz - \sigma t), \quad (7)$$

where k represents the axial (spanwise) wavenumber, σ the complex angular frequency and y_0 the y -location of the undisturbed vortex. $\tilde{x}$ and $\tilde{y}$ denote infinitesimal complex amplitudes ($|\tilde{x}| \ll 1, |\tilde{y}| \ll 1$). Substituting Eq. (7) into Eq. (5) and linearizing the integrand with respect to $\tilde{x}$ and $\tilde{y}$, we obtain

$$-i\sigma\tilde{x} = \frac{\Gamma}{2\pi} \omega(k\delta)k^2\tilde{y} + U'\tilde{y}, \quad (8.a)$$

$$-i\sigma\tilde{y} = -\frac{\Gamma}{2\pi} \omega(k\delta)k^2\tilde{x}, \quad (8.b)$$

where

$$U' = \left. \frac{dU}{dy} \right|_{y=y_0}, \quad (9)$$

$$\omega(\xi) = \frac{1}{2} \left(\frac{\cos \xi - 1}{\xi^2} + \frac{\sin \xi}{\xi} - C_i(\xi) \right). \quad (10)$$

ω is Crow's self-induction function⁽¹⁸⁾, and C_i is the integral cosine function. The physical meanings of each term in Eqs. (8) are as follows. The first terms in Eqs. (8.a) and (8.b) yield the self-induced rotation by the filament itself, and the second term in Eq. (8.a) originates from the advection by the background

shear flow. Because we can show that $\omega(k\delta) \geq 0$ for $k\delta \leq 1$, it turns out that the directions of the self-induced rotation and of the filament vorticity itself are opposite to each other. This opposition leads to an important consequence in the instability discussed below. Equations (8) are similar to those obtained by Pierrehumbert⁽¹⁹⁾ who studied the same stability problem using the LIA (localized-induction approximation). In the present case, however, the self-induced rotation rate is $O(-k^2 \ln k)$ for $k\delta \ll 1$ while in the LIA it is $O(k^2)$. The factor $-\ln k$, which has appeared in this study, represents the nonlocal effect neglected in the LIA.

In order to discuss the instability of the vortex filament, we should estimate cutoff length δ properly. We do so by following the methods of Crow⁽¹⁸⁾ and Moore and Saffman⁽²⁰⁾. Assume that the axial vorticity is uniform on any cross section of the vortex core along the filament and that no axial flow exists therein. When we take the value of δ so that the induction velocity, which is calculated by the Biot-Savart integral in Eq. (4) for a circular filament, coincides with the translational velocity⁽²¹⁾ of a circular vortex ring, we obtain

$$\delta = \frac{1}{2} e^{1/4} a. \quad (11)$$

In this estimate of δ , the higher order of the product of the curvature and the core radius has been dropped, so that the cutoff method used here is accurate to $O[(ka)^2]$. Substituting Eq. (11) into Eq. (8), the dimensionless eigenvalues and the corresponding eigenvectors are given by

$$\frac{\pi a^2 \sigma}{|\Gamma|} = \pm \frac{1}{2} [\phi(ka)(\phi(ka) - 2\beta)]^{1/2}, \quad (12)$$

$$\frac{\tilde{y}}{\tilde{x}} = \mp i \frac{\Gamma}{|\Gamma|} \phi(ka) [\phi(ka)(\phi(ka) - 2\beta)]^{-1/2}, \quad (13)$$

where

$$\phi(\xi) = \xi^2 \omega\left(\frac{1}{2} e^{1/4} \xi\right), \quad (14)$$

$$\beta = -\frac{\pi a^2 U'}{\Gamma} \quad (15)$$

ϕ corresponds to the self-induced rotation rate. β represents the ratio of circulation of the vortex to that of the background shear, and is only one stability parameter. It can be shown that ϕ is not negative for $ka \leq 1$, and therefore it follows from Eq. (12) that the vortex can be unstable for $\beta > 0$. The physical mechanism of this instability is explained as follows. When $\beta > 0$, the vorticity of the background shear flow has the same sign as that of the vortex filament. Recall that the direction of the filament vorticity itself is opposite to that of the self-induced rotation. Hence, in the case of $\beta > 0$, the direction of the background rotation by the shear flow turns out to be opposite to

that of the self-induced rotation. As a consequence, the background rotation suppresses the stabilizing effect of the self-induced rotation, which leads to the instability via a background purely straining motion by the shear flow. The unstable band of ϕ is $0 < \phi < 2\beta$, and the unstable modes lie on a plane tilted toward the streamwise direction. Therefore the unstable modes yield the streamwise vorticity. The maximum instability occurs at

$$\phi(ka) = \beta. \quad (16)$$

In this case the background rotation cancels out with the self-induced rotation, and so the eigenvector is pointed in the principal direction of the purely straining motion, which is tilted 45° toward the streamwise direction. The resulting stability diagram is shown in Fig. 5. The dashed line denotes the most unstable β for a fixed value of ka .

4.2 The most unstable spanwise wavelength of a vortex filament

The vortex instability discussed in section 4.1 will be considered for the background shear flow that has the same velocity profile as a wall-bounded turbulent shear flow. We should first estimate the values of velocity gradient U' , vortex core radius a and vortex strength Γ . We have estimated the value of U' from Kim et al.'s⁽²²⁾ DNS data. The values of a and Γ will, on the other hand, be estimated as follows. Robinson⁽²³⁾ demonstrated the existence of near-wall spanwise vortical structures using the DNS data base for a turbulent boundary layer. He also presented several kinematic properties of the spanwise vortices. In Robinson's⁽²³⁾ results, the variation of the vortex diameter against the vortex height from the wall can be well represented by a linear relation scaled by the Kármán constant $\kappa = 0.41$, and over 40% of the spanwise vortices have circulation around $\Gamma/(2\pi\nu) =$

-30 . Thus, we estimate the core radius and strength of the disturbed vortices from Robinson's⁽²³⁾ results as

$$a^+ = \frac{1}{2} \kappa y^+, \quad (17)$$

$$\Gamma^+ = -60\pi, \quad (18)$$

where y is the vortex height from the wall. Although the vortical structures detected by Robinson⁽²³⁾ are conjectured to have already evolved into a highly three-dimensional shape, e.g., hairpin-like or arch-like vortices, their original shape is expected to be nearly two-dimensional⁽²⁴⁾. We then examine the instability of a spanwise rectilinear vortex with the same radius and strength as the real spanwise vortical structure. For the cases considered here, the above neglected effects of the vortex filament on the background flow, i.e., $|a(dU/dy)/U|$ and $|a^2(d^2U/dy^2)/U|$, are estimated to be of the order of 10^{-1} and 10^{-2} , respectively. Thus, we anticipate that these effects are not serious at least during the early period of vortex evolution.

We next introduce an image vortex into the stability problem in order to take account of the effects of a wall, especially the wall impermeability. Introducing the image vortex, Eqs. (8) are rewritten in the forms

$$-i\sigma\tilde{x} = -\frac{\Gamma}{2\pi D^2}\tilde{y} - \frac{\Gamma}{2\pi D^2}\kappa(kD)\tilde{y} + \frac{\Gamma}{2\pi}\omega(k\delta)k^2\tilde{y} + U'\tilde{y}, \quad (19.a)$$

$$-i\sigma\tilde{y} = -\frac{\Gamma}{2\pi D^2}\tilde{x} + \frac{\Gamma}{2\pi D^2}\psi(kD)\tilde{x} - \frac{\Gamma}{2\pi}\omega(k\delta)k^2\tilde{x}, \quad (19.b)$$

where D is the distance between the undisturbed real and image vortices, and

$$\kappa(\xi) = \xi K_1(\xi), \quad (20)$$

$$\psi(\xi) = \xi^2 K_0(\xi) + \xi K_1(\xi) \quad (21)$$

are Crow's first and second mutual-induction functions⁽¹⁸⁾, respectively. K_0 and K_1 are modified Bessel functions of the second kind. The dimensionless eigenvalues and corresponding eigenvectors for Eqs. (19) are respectively given by

$$\frac{\pi a^2 \sigma}{|\Gamma|} = \pm \frac{1}{2} [(\phi(ka) - a^2 \psi(a^{-1}ka) + a^2)(\phi(ka) - 2\beta - a^2 \kappa(a^{-1}ka) - a^2)]^{1/2}, \quad (22)$$

$$\frac{\tilde{y}}{\tilde{x}} = \mp i \frac{\Gamma}{|\Gamma|} (\phi(ka) - a^2 \psi(a^{-1}ka) + a^2) \times [(\phi(ka) - a^2 \psi(a^{-1}ka) + a^2)(\phi(ka) - 2\beta - a^2 \kappa(a^{-1}ka) - a^2)]^{-1/2}, \quad (23)$$

where $a = a/D$ and a takes a constant value of $\kappa/4$ in the present case.

The above stability parameter β is plotted as a function of y^+ in Fig. 6. The vortex can be unstable for a positive value of β , as discussed in section 4.1. As y^+ is increased, β increases monotonically. However, β is of the order of 10^{-1} near the wall, which

Fig. 5 Contour plot of growth rate in ka and β

Fig. 6 Plot of stability parameter β versus y^+ Fig. 7 Growth rate versus ka for various y^+ -locations

means that the magnitude of vorticity of the vortex is much larger than that of vorticity and strain of the mean shear. For such weak strain, the growth rate predicted by the Biot-Savart cutoff theory is in excellent agreement with an exact solution at a small wavenumber⁽²⁵⁾.

Figure 7 shows the variations of growth rate $\text{Im } \sigma^+$ with ka at different y -locations. The maximum growth rate becomes larger in the near-wall region, and there the vortex may deform rapidly. As y^+ is increased, the most unstable dimensionless wavenumber ka increases and the unstable band of ka becomes broad. This means that the vortex apart from the wall is unstable to the disturbances that have a wavelength comparable to the vortex radius ($ka \approx 1$). It is difficult to quantitatively discuss the instability around $ka=1$. The maximum instability, however, occurs at $ka < 0.5$ when the vortex is located at $y^+ \leq 70$. Therefore, in the near-wall region, the maximum instability allowed by the cutoff theory is within its range of validity.

In Fig. 8 is shown the variation of the most unstable spanwise wavelength λ^+ with y^+ . In this figure, the solid and the dotted-dashed line denote the results without and with an image vortex, respectively, and

Fig. 8 The most unstable wavelength λ^+ versus y^+

the open circles represent the experimental data⁽²⁶⁾ for the mean streak spacing. Vortex radius a^+ increases with y^+ , which implies that λ^+ increases with y^+ at fixed β . Stability parameter β , on the other hand, increases with y^+ , which implies that λ^+ decreases with y^+ at fixed a^+ . The variation of λ^+ has been determined by these two competitive effects. It can be seen that the resulting dependence of λ^+ on the distance from the wall is quite similar to the experiment. Moreover, the most unstable mode has a spanwise wavelength of about 100 wall units in the buffer region ($4 < y^+ < 40$), which is consistent with the commonly accepted value of the mean streak spacing. Next, considering the effect of an image vortex on the instability, we found that it affects the asymptotic behavior of λ^+ at $y^+ \rightarrow 0$ (see solid and dotted-dashed lines in Fig. 8). In the absence of the image vortex, λ^+ can be shown to increase as $\lambda^+ \propto \sqrt{-\ln y^+}$ for $y^+ \rightarrow 0$. When the image vortex is introduced, on the other hand, λ^+ can be shown to approach zero as $\lambda^+ \propto y^+$ for $y^+ \rightarrow 0$. However, both wavelengths with and without the image vortex are nearly the same at $y^+ > 10$, and the effect of the image vortex on the instability is confined only to the vicinity of the wall. Therefore, wall impermeability may not be necessary for the generation of a streamwise vortical motion. The present analytical result without the image vortex is compared with representative experimental data reported by Kasagi⁽²⁷⁾ for the mean streak spacing, as shown in Fig. 9. At $10 < y^+ < 60$, the experimental data have the same trend as the present analysis although there exist some deviations in the experiments. At $y^+ > 60$, the comparison between experiments and analysis may be difficult because of a wide scatter of the experimental data.

The above results suggest that the vortex instability discussed here may represent the early stages of the generation of streamwise vortical motions in wall-bounded turbulent shear flows. At early stages of the instability, the vortex filament evolves into an

Fig. 9 Comparison of λ^+ with experimental data

undulant structure on a plane tilted 45° toward the streamwise direction. With time, the vortex filament is conjectured to be turned toward the streamwise direction near the wall by a large velocity gradient⁽²⁴⁾. As a consequence, the near-wall vortex filament induces a streamwise vortical motion that generates a low-speed streak as well as sweep. A sweep motion in turn increases the wall shear rate through intrushes of high-speed fluids toward a wall, which leads to high wall skin friction. Finally, we would like to state that the formation of near-wall spanwise vortical structures, which may be the origin of streamwise vortical motions, has been interpreted in terms of a statistical theory for roll-up of internal shear layers by three of the present authors⁽²⁸⁾.

5. Conclusions

Near-wall velocity measurement and simultaneous dye flow visualization have been performed at the vicinity of the wall $y^+ = 2.6$ on the wall bisector in order to evaluate instantaneous wall shear stress and to investigate turbulence structures that induce high wall skin friction. It is demonstrated that single near-wall streamwise vortical structures play a dominant role in high-friction generation. These streamwise vortices are inclined toward the spanwise direction away from the streamwise direction, and can be classified into two types of vortices according to whether their spanwise vorticity component has the same or opposite sign as that of the mean shear. They appear at nearly the same ratios, and induce high wall skin friction through intrushes of high-speed fluids toward the wall, i.e., sweep motion.

In order to examine the generation mechanism of streamwise vortical motions, we have analyzed the long-wavelength instability of a straight vortex filament in a shear flow using the Biot-Savart cutoff theory. It is shown that the vortex filament can be

unstable for the case where the strength of the vortex filament has the same sign as the circulation of the shear flow, and that the unstable mode leads to the generation of the streamwise vorticity. For the shear flow with the same velocity profile as a wall-bounded turbulent flow, the most unstable spanwise wavelength λ^+ is nearly 100 in wall unit. Moreover the variation of λ^+ with the distance from the wall exhibits a trend similar to the experimental data for mean streak spacing. These results suggest that the present vortex filament instability may represent the early stages of the generation of streamwise vortical motions in wall-bounded turbulent shear flows.

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