Numerical Study on the Unsteady Flows through the Bifurcation of the Porcine Renal Artery*

Masahide NAKAMURA**, Margot R. ROACH***, Neil F. MACLEAN***, Toshinori EBA** and Shinya SHOJI**

Three-dimensional steady and unsteady flows through the bifurcation of the porcine renal artery were analyzed numerically using the finite-element method. As the bifurcation geometry plays a primary role in determining the hemodynamical variables, the present bifurcation geometry was precisely developed in accordance with the measured data. In the present calculations, the arterial wall was assumed to be rigid and the non-Newtonian effect of blood was not taken into consideration. The calculated results showed that the present bifurcation geometry had an effect of suppressing the flow separation. In addition, it was shown that the present bifurcation geometry flattened the spatial distributions of wall shear stress, which has important implications for the concept of Wall Shear Stress Gradient.

** Key Words:** Biofluid Mechanics, Computational Fluid Mechanics, Unsteady Flow, Three-Dimensional Flow, Separation.

1. Introduction

It is well known that arterial diseases such as atherosclerosis tend to develop preferentially at bifurcations. The localization of atherosclerotic lesions near the arterial bifurcation has led several researchers to suggest that fluid mechanics may play an important role in this phenomenon[10]. Therefore, hemodynamics near the arterial bifurcation has attracted the interest of many researchers and many theoretical and experimental studies have been carried out.

It is clear that arterial geometry plays a crucial role in determining the hemodynamical variables (e.g. flow pattern, wall shear stress, pressure distribution). Therefore, the detailed information about the geometry of arterial bifurcations is indispensable to a detailed hemodynamical analysis. However, there is little information about the geometry of arteries, particularly near bifurcations. For this reason, Roach and her colleagues have conducted the detailed geometrical analyses[20,21] on the bifurcation of the porcine renal artery in recent years. They showed that the geometry of bifurcation had two important features.

(1) At the apex, the cross section is elliptical with a large aspect ratio. In other words, the ratio of the long radius to the short radius is much greater than 1.0 at the apex.

(2) The cross-sectional area increases monotonically up to the apex and decreases monotonically distally from the apex. The cross-sectional area reaches its maximum at the apex.

These two features must have significant effects on the hemodynamical variables near the bifurcation. However, theoretical or experimental studies including these two geometrical features have not yet been carried out. In this study, the three-dimensional unsteady blood flows through the bifurcation of the porcine renal artery are analyzed numerically. The geometry of the bifurcation was determined from the measured data[21]. Therefore, the present analysis
includes these two geometrical features and we investigate the effects of these geometrical features on the hemodynamical variables (e.g. flow pattern, wall shear stress, pressure distribution). Moreover, we qualitatively consider the role of these geometrical features in the development of arterial diseases (e.g. atherosclerosis) on the basis of the calculated results.

2. Numerical Methods

2.1 Geometry

The present bifurcation model was developed based on the data of Roach and MacLean(3), which were obtained using a sledge microtome. Their data showed that the physiological parent trunk and daughter branches of the porcine renal artery lay in the same plane and the geometry of bifurcation was almost symmetrical. Therefore, only one fourth of the bifurcation area was numerically analyzed. The shape of cross-sectional area was approximated by an ellipse. The long radius and short radius of each cross section was determined from the measured data(3).

Subsequently, the flow domain was divided into a number of simply shaped regions called elements. In this study, we employed a slice-cut method(3) to define the elements. The application range of this method is not very wide. However, this method is easy to use and we determined that this method is suited to our purposes. Figure 1 shows the element division of the flow domain. In this figure, the z axis is a coordinate parallel to the inlet flow with its origin at the apex, and the x and y axes are coordinates perpendicular to the z axis. The flow domain exhibits symmetry with respect to the planes of x=0 and y=0. The number of total nodes is about 20,000. This element division is fine enough to reveal the complex geometry of the bifurcation. As mentioned above, the geometrical features of this model are characterized by the geometrical parameters of the cross section at the apex, which are listed in Table 1.

2.2 Basic equations and numerical calculation methods

In this study, we assumed that blood was an incompressible, homogeneous, Newtonian fluid. As the non-Newtonian effect of blood does not play an important role in a high shear rate region(6), this assumption is reasonable for our analysis. Therefore, the basic equations (Navier-Stokes equations and equation of continuity) in orthogonal coordinates system are written as

\[
\begin{align}
\frac{\partial u_i}{\partial x_i} &= 0 \\
\rho(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}) &= -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2} \quad (i, j = 1, 2, 3)
\end{align}
\]

where, \( t \) is time, \( x_i \) is the coordinate component \( (x_1 = x, x_2 = y, x_3 = z) \), \( u_i \) is the component of velocity vector in the \( x_i \) direction, \( p \) is the pressure, \( \rho \) is the density and \( \mu \) is the viscosity coefficient. In this study, the density and viscosity coefficients used are 1050 kg/m³ and 3.5×10⁻³ Pa·s respectively. These values were determined with reference to the values of typical human blood.

The applied boundary conditions are written as follows.

a. At the wall, the no-slip condition is applied, i.e., the components of velocity vector are set at zero.

b. At the plane of symmetry, the conditions of zero normal velocity gradient and zero cross flow are applied.

c. At the outlet, the condition describing zero traction force is applied. In addition, the pressure is set at zero.

d. At the inlet, the following velocity distributions are applied.

\[
u_1 = u_2 = 0
\]

Table 1 Geometrical parameters of the cross section at the apex and inlet

<table>
<thead>
<tr>
<th></th>
<th>inlet</th>
<th>apex</th>
</tr>
</thead>
<tbody>
<tr>
<td>shape of cross section</td>
<td>circle</td>
<td>ellipse</td>
</tr>
<tr>
<td>radius = 1.55mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cross-sectional area</td>
<td>7.1mm²</td>
<td>10.1mm²</td>
</tr>
</tbody>
</table>

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\[ u_0 = K \cdot h(t) \cdot \left[1 - \frac{(r/R)^6}{3} \right] \]
\[ r = (x^2 + y^2)^{1/2}, \quad R = 1.5 \text{ mm} \]

Additional explanations are presented here. The blood flow in arteries has a very complex nature. Therefore, we cannot obtain the exact velocity profiles at the inlet. However, we expected the blood flow to have a blunt profile because of the unsteadiness effect and entrance region effect. For these reasons, Eqs. (3) and (4) were employed as velocity profiles at the inlet. The function \( h(t) \) in Eq. (4) denotes the time course of the flow rate at the inlet, which is shown in Fig. 2. The waveform of the flow rate in Fig. 2 is the same as that used by Daly\(^{19}\). In this figure, \( T_0 \) is the period of pulsation and the value of \( T_0 \) is set at 0.36 sec. The value of \( K \) in Eq. (4) is determined to set the peak Reynolds number at 600, a typical Reynolds number in human arteries\(^{8}\). We should note that the Reynolds number is defined by the trunk diameter and the section-averaged axial velocity in the trunk. Moreover, we should note that the numerical calculation results are barely affected by the index of power in Eq. (4).

In this study, three-dimensional steady blood flows through the bifurcation of the porcine renal artery were also analyzed. In the steady flow analysis, the applied boundary conditions at the inlet are written as follows.

\[ u_i = u_0 = 0 \]
\[ u_0 = K \left[1 - \frac{(r/R)^6}{3} \right] \]
\[ r = (x^2 + y^2)^{1/2}, \quad R = 1.5 \text{ mm} \]

It should be noted that equations similar to Eqs. (3) and (4) were used as inlet boundary conditions by other researchers\(^{9}\).

The finite-element computer code FIDAP Ver. 7.5 was utilized for this calculation and an eight-node isoparametric brick element was used to solve the Navier-Stokes equations and the equation of continuity. The numerical calculations were carried out on an HP 9000 715/64 workstation. The required CPU time for one pulsation cycle was about 40 hours.

Moreover, the numerical results showed that three pulsation cycles were required to ensure the complete convergence of the solutions.

Finally, let us consider the effect of wall flexibility. The effect of wall flexibility on the hemodynamical variables near the wall was studied by many researchers\(^{10,11}\). However, the role of wall flexibility is still unknown and there is much left to be studied. Therefore, we adopted a rigid model for the present. The effect of wall flexibility is a subject for future study.

3. Results and Discussion

3.1 Flow pattern

The velocity vector plots of the unsteady flow in the plane of \( y = 0 \) are shown in Fig. 3. In this figure, the results at \( t/T_0 = 3/6 \) (just before the peak systole), \( t/T_0 = 4/6 \) (deceleration phase) and \( t/T_0 = 5/6 \) (just after the end systole) are shown in Figs. 3(a), 3(b) and 3(c) respectively. Moreover, the velocity vector plots of the steady flow in the plane of \( y = 0 \) is shown in Fig. 4. We should note that the scale of velocity vector is varied according to the plots. From Fig. 3(a), we can determine that flow separation does not
occur at this time (just before the peak systole). From Fig. 3(b), however, we can determine the separation bubble near the outer wall of the bifurcation at this time (deceleration phase). Furthermore, from Fig. 4, we can determine that flow separation does not occur under the condition of steady flow. It should be noted that the instantaneous Reynolds number at $t/T_b = 3/6$ (Fig. 3(a)) is about 600 and the Reynolds number of steady flow (Fig. 4) is also about 600.

Numerical studies on the viscous flow through the arterial bifurcation have been carried out by many researchers. For example, the steady flow through the symmetric bifurcation was analyzed by Yung et al.\(^{(12)}\). They found the separated flow region along the outer wall of the bifurcation at a low Reynolds number of less than 600\(^{(12)}\). This result is clearly in conflict with the present result. The reason of this contradiction can be explained by the differences of bifurcation geometry. We should note that the geometrical features mentioned above are not included in the model adopted by Yung et al.\(^{(12)}\). Therefore, we concluded that the present bifurcation geometry has the effect of suppressing the flow separation. The pressure drop of the flow through the bifurcation is strongly affected by the flow separation. Therefore, this result will have important implications from the engineering viewpoint. Moreover, it is well known that the separated flow region promotes the formation of thrombus\(^{(11)}\). Therefore, this result will also have important implications from the biomechanical viewpoint. Subsequently, let us consider the reason that the present bifurcation geometry has the effect of suppressing the flow separation. We now introduce the concept of hydraulic equivalent diameter\(^{(13)}\). The hydraulic equivalent diameter $D_{eq}$ is considered to be a characteristic length of the cross-sectional area with a complicated shape. This parameter is defined as

$$D_{eq} = 4S/L$$  \hspace{1cm} (5)$$

where, $S$ is the cross-sectional area and $L$ is the wetted perimeter. The calculation of the hydraulic equivalent diameter is easily carried out using the data shown in Table 1. The calculated results show that the hydraulic equivalent diameter is 2.2 mm (apex) and 3.0 mm (inlet). We should note that the hydraulic equivalent diameter at the apex is shorter than the one at the inlet. Therefore, it is expected that the present bifurcation denotes a behavior similar to that of the converging tube. The reason that the present bifurcation geometry has the effect of suppressing the flow separation will be explained by this discussion.

Next, let us consider the existing results of experimental studies. The experimental studies on the flows through the arterial bifurcation were carried out by many researchers. We refer to the results of Walburn and Stein\(^{(14)}\) as an example. In their experiments, the velocity profiles were measured with a laser Doppler anemometer. Under the condition of steady flow, they showed that flow separation did not occur at the Reynolds numbers of 400, 500, 1,000 and 1,500. Their results agree with our results discussed above.

Figure 3 shows that the flow separation occurs easily during the deceleration phase as compared with that of the acceleration phase. The unsteady flows with the separated flow region were studied by several researchers. For example, the numerical study on the unsteady flows through the axisymmetric stenosis was carried out by Nakamura and Sawada\(^{(15)}\). The geometry adopted in their analysis differs remarkably from the geometry adopted in the present analysis, but they also showed that the flow separation occurred easily during the deceleration phase. Therefore, we can conclude that the flow separation occurs easily during the deceleration phase in many cases.

As mentioned above, the pressure drop of the flow through the bifurcation is strongly affected by the flow separation. So, suppression of the flow separation has important implications from the engineering viewpoint. The present study may offer an important suggestion for the development of a high performance (low pressure drop) bifurcation tube.

Next, let us consider the velocity vector in the cross section of the artery. Figure 5 shows the velocity vector plots of the unsteady flow in the plane of $z = 0$. In this figure, the results at $t/T_b = 3/6$ (just before the peak systole), $t/T_b = 4/6$ (deceleration phase) and $t/T_b = 5/6$ (just after the end systole) are shown in Figs. 5(a), 5(b) and 5(c). Figure 6 shows the velocity vector plot of the steady flow in the plane of $z = 0$. In these figures, the left edge is the outer wall of the bifurcation and the right edge is the inner wall of the bifurcation. Moreover, we should note that the scale of the velocity vector is varied according to the plots. Figures 5 and 6 show that the direction of these velocity vectors are almost parallel to the $x$ axis. Of course, the reason for this result is explained by the
shape of the cross section. We should note that the velocity profiles shown in these figures and the velocity profiles of the Hele-Shaw flow resemble each other. Moreover, we should note that the Hele-Shaw flow has a similar property to that of the potential flow. After all, the present velocity profiles shown in these figures are expected to have a similar property to that of the potential flow. Of course, the flow separation does not occur in the potential flow. The reason that the present bifurcation geometry has the effect of suppressing the flow separation can also be explained by this discussion. We can now locate the low velocity region near the outer wall of the bifurcation in Fig. 5(b). The cause of this low velocity region will be explained by the flow separation that occurred during the deceleration phase.

### 3.2 Wall shear stress

The spatial distributions of wall shear stress are shown in Figs. 7 and 8. Figure 7 shows the results for unsteady flow and Fig. 8 shows the results for steady flow. In these figures, the apex of the bifurcation lies in the position of $z = 0$, and the upper panel of Fig. 7 shows the results along the outer wall and the lower panel of Fig. 7 shows the results along the inner wall. First, let us consider the results shown in the upper panel of Fig. 7. As mentioned above, flow separation does not occur at $t/T_0 = 3/6$ (just before the peak systole). So, we cannot locate the region of negative wall shear stress at this time. However, flow separation occurs at $t/T_0 = 4/6$ (deceleration phase). So the sign of wall shear stress becomes negative at this time. Thus, the wall shear stress along the outer wall denotes the complicated changes over time. However,
we can determine from the upper panel of Fig. 7 that the spatial gradient of the wall shear stress is fairly mild. This point will be discussed in detail later in this paper. Next, let us consider the results shown in the lower panel of Fig. 7. This panel shows the existence of sharp peaks at $z=0$ (apex). However, the peaks of wall shear stress appear only at $t/T_{0}=3/6$ and $4/6$. Moreover, we find that the region where the wall shear stress denotes a rapid spatial change is limited in the vicinity of $z=0$. On the other hand, we can determine from Figs. 7 and 8 that the spatial distribution of the wall shear stress is strongly affected by the unsteadiness of the bulk flow. Namely, these figures show that the unsteadiness of the bulk flow has the effect of flattening the spatial distribution of the wall shear stress.

Thus, it was shown that the present bifurcation geometry had the effect of flattening the distribution of the wall shear stress. Subsequently, let us consider the meaning of this result. As mentioned above, it has been suggested by many researchers that fluid mechanics might play an important role in the onset and development of atherosclerosis. Therefore, several fluid dynamic parameters were proposed as the initiating factors. Among these parameters, the concept of Wall Shear Stress Gradient (WSSG) has recently attracted special interest and it is suggested that this parameter is the strongest indicator of abnormal hemodynamics influencing atherogenesis\(^{49}\). This parameter is given by

$$WSSG = |\frac{\partial \tau_w}{\partial s}|$$

where, $\tau_w$ is the wall shear stress and $s$ is the length along the wall. If this concept is generally accepted, we find that the flatness of the distribution of the wall shear stress has the effect of preventing the onset and development of atherosclerosis. Therefore, it is easily expected that the present bifurcation geometry has the effect of preventing the onset and development of atherosclerosis by decreasing the value of WSSG. However, the concept of WSSG is only a hypothesis and there is much left to study. Therefore, we cannot draw a conclusion and more detailed studies should be conducted in the future.

In connection with the arterial geometry, we should consider the results of Kamiya et al.\(^{17}\). They showed that the geometry of the arterial tree was determined to keep the value of wall shear stress constant. This result implies that the geometry of the arterial tree is designed to decrease the value of WSSG. Clearly, the results of Kamiya et al.\(^{17}\), the present results mentioned above, and the concept of WSSG are compatible.

3.3 Pressure distribution

The contour plots of the pressure in the plane of $y=0$ are shown in Figs. 9 and 10. Figure 9 shows the results of unsteady flow and Fig. 10 shows the result of steady flow. In these figures, $\Delta p$ is the difference of contour lines. We should note that the value of $\Delta p$ varies according to the plots. Figures 9 and 10 show that the pressure distribution denotes complex behavior patterns near the apex. Therefore, let us consider the pressures at two representative points (P1 and P2) in order to grasp an understanding of the basic behavior of the pressure distribution near the apex. The positions of P1 and P2 are shown in Fig. 11. Point P1 is equal to the apex and the point P2 lies on the outer wall of the bifurcation. The $x$ component of point P2 is equal to the $x$ component of point P1. The change
of pressures at point P1 and P2 over time are shown in Fig. 11. In this figure, "st" denotes the result of steady flow. From this figure, we find that the pressure at points P1 and P2 denote the complex behavior patterns. However, we should note that the pressure at point P1 is higher than the pressure at point P2 in general. Of course, the reason for this result is explained by the flow impingement at the apex. Therefore, we find that the apex lies in the local maximum point of pressure. Moreover, Fig. 11 shows that the instantaneous peak pressure obtained from the unsteady flow analysis is higher than the pressure obtained from the steady flow analysis. These results indicate that the high pressure point will be observed in the vicinity of the apex. As this high pressure point effect to expand the arterial wall, it is easily expected that a saccular aneurysm could easily occur in the vicinity of the apex. This result is compatible with the well-known clinical facts\(^{18}\).

4. Conclusions

To understand the effects of the bifurcation geometry on the hemodynamical variables near the apex, three-dimensional steady and unsteady blood flows through the bifurcation of the porcine renal artery were numerically calculated. The geometry of bifurcation was developed in accordance with the measured data. The calculated results showed that the present bifurcation geometry had an effect of suppressing the flow separation near the apex. Moreover, it was shown that the present bifurcation geometry had an effect of flattening the distribution of wall shear stress.

It is well known that local disturbed flow patterns, characterized by the abnormal distribution of wall shear stress and the separated flow region, may play an important role in the onset and development of atherosclerosis and thrombosis. Therefore, these results have important implications from a biomechanical viewpoint and suggest that the present bifurcation geometry may have an effect of preventing the onset and development of some arterial diseases.

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References

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