Study of a Flapper-Nozzle System for a Water Hydraulic Servovalve*

Eizo URATA** and Yohichi NAKAO**

We discuss the characteristics of a flapper-nozzle system for water hydraulic servovalves. High-pressure water at the supply port is used for the first time as the working fluid for the hydrostatic bearings supporting the spool. Spool valve adhesion induced by poor lubrication with water is thus avoided. The fluid is then led to the ends of the spool and is used as the working fluid of the flapper-nozzle system. In this flapper-nozzle system the circumferential clearance of the spool becomes a laminar restriction that replaces the fixed orifice used in conventional servovalves. The linearity in the pressure-displacement relationship of the new flapper-nozzle system is better than that of conventional fixed orifice systems.

Key Words: Flapper-Nozzle System, Nozzle, Servovalve, Valve, Fluid Power, Water Hydraulic Servovalve, Water Hydraulics, Hydrostatic Bearing

1. Introduction

The purpose of this work is to improve the flapper nozzle system driving the spool of the servovalve. The work is performed as a basic study for the water hydraulic servovalve developed by the authors. A sliding valve for water hydraulic control(1) requires the prevention of leakage. When we reduce clearances between moving parts in order to prevent leakage, the parts often stick. Servovalves can be driven anyway when we use stronger driving force, however, wear induced by friction will degrade valve characteristics within a short period. Although the conventional hydrostatic support of spools(2) can resolve the problem of stick and wear, flow through the bearing increases the leakage.

A flapper-nozzle system is widely used as the first stage amplifier in a two-stage servovalve. This also requires a certain amount of flow loss. To reduce total flow loss, we lead a part of flow from the hydrostatic bearing to the ends of the spool, and further to the nozzle. Thus we make the nozzle flow a part of the bearing flow(3)-(6). The flapper-nozzle system incorporating this design principle has other advantages over conventional flapper-nozzle systems. In this paper we will present the characteristics of our flapper-nozzle system. Since our system is assumed to be used in a servovalve, the study is conducted considering spool motion(6).

Nomenclature

- $c_s$: discharge coefficient of the throttle of hydrostatic bearing
- $c_n$: nozzle discharge coefficient
- $d_s$: diameter of hydrostatic bearing throttle
- $d_c$: diameter of conduit between spool and nozzle
- $d_n$: nozzle diameter
- $d_f$: fixed orifice diameter
- $h_s$: spool circumferential clearance, return side
- $h_n$: spool circumferential clearance, nozzle side
- $H$: neutral clearance between nozzle and flapper
- $L_s$: spool land length, return side
- $L_n$: spool land length, nozzle side
- $p_{l}, p_{r}$: spool end pressure, left and right sides
- $p_{bn}, p_{bm}$: bearing pocket pressure, left and right sides
- $p_{n}, p_{m}$: pressure at nozzle entrance, left and right sides

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at the end of the spool. The pressure drops accompanying the flow become
\[ p_n - p^* = \frac{\xi_n \rho}{2 \sqrt{\frac{4g}{\pi d^2}}} \left( \frac{2}{\pi} \right)^2, \] \[ p_s - p_n = \frac{\xi_s \rho}{2 \sqrt{\frac{4g}{\pi d^2}}} \left( \frac{2}{\pi} \right)^2. \]

We proceed with the following calculation assuming that four bearing throttles are mounted on the circumference with equal angular spacings. The fluid from the supply port flows through the hydrostatic bearings and enters the circumferential clearance of the spool. At the clearance, the flow is divided into parts flowing to the spool end and to the tank port. The flow division is expressed by
\[ \pi c_{sd} \text{sinn}(p_n - p_{sb}) \left( \frac{2}{\pi} \right)^2 \frac{p_n - p_{sb}}{\rho} \]
\[ = \frac{\pi Rh^2}{6 \mu L_s} (p_n - p_s) + \frac{\pi Rh^2}{6 \mu L_s} p_{sb}. \] \[ (4) \]
The left side of Eq. (4) expresses not only normal flow but also an inverse flow due to a fast spool motion.

The flow rate through the nozzle is the sum of flow through the clearance and the displacement flow induced by spool motion. Using the coordinate of spool displacement in Fig. 1, we have
\[ q = \frac{\pi Rh^2}{6 \mu L_s} (p_n - p_i) - \frac{\pi Rh^2}{6 \mu L_s} \frac{dz}{dt}. \] \[ (5) \]

Similar equations are obtained for variables relating to the right side of the valve. We can obtain the formula for the right side corresponding to Eqs. (1) – (5) by changing the subscript l to subscript r, x to \(-x\), and \(dz/dt\) to \(-dz/dt\).

Using Eqs. (1) – (5) and the listed nondimensional parameters, we can express the flow division by
\[ \text{sgn}(1 - P_{sb}) \sqrt{1 - P_{sb}} = (a_s + a_0) P_{sb} - a_0 P_s, \] \[ (6) \]
and the nozzle flow by
\[ a_0 (P_{sb} - P_s) = \frac{d \xi}{dt} + \frac{a_0 (1 - \xi^2) \sqrt{P_s}}{\sqrt{1 + a_0 \xi^2 (1 - \xi^2)}}. \] \[ (7) \]

Equation (7) expresses the continuity relation of the fluid in the space behind the spool. Equations for right-side variables, i.e., \(P_r\) and \(P_{rb}\), are obtained by replacing \(\xi\) and \(d \xi/dt\) in the above expressions with \(-\xi\) and \(-d \xi/dt\), respectively.

\[ \text{sgn}(1 - P_{sb}) \sqrt{1 - P_{sb}} = (a_s + a_0) P_{sb} - a_0 P_r, \] \[ (8) \]
and
\[ a_0 (P_{sb} - P_r) = - \frac{d \xi}{dt} + \frac{a_0 (1 + \xi^2) \sqrt{P_r}}{\sqrt{1 + a_0 \xi^2 (1 + \xi^2)}}. \] \[ (9) \]

For the analysis of servovalve dynamics, it is often required to determine \(P_l\) and \(P_r\) as output variables under given \(\xi\) and \(d \xi/dt\). It is sufficient, however, to determine only \(P_l\) (or \(P_r\)) because \(P_l\) and \(P_r\) are symmetric, as shown above. Therefore, we will calculate \(P_l\) in the following.

The values of \(P_l\) and \(P_{sb}\) at an operating point are obtained as the crossing points of curves expressed by.

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**Fig. 1 Flapper nozzle system with hydrostatic bearing**

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Eqs. (6) and (7). The operating point \((P_a, P_b)\) always lies on curve (6) since this equation does not contain \(x\) or \(dz/dt\). Figure 2 shows a \(P_t-P_a\) plane. In the region \(P_a > P_t\), the fluid flows from the bearing to the valve end, hence the direction of flow is normal. The region \(P_a \geq 1\) does not exist physically.

In the region under the line \(P_a = P_t\), the flow direction in the clearance is from the valve chamber to the bearing, hence the flow direction is reverse. This state is realized when the spool motion establishes a pumping action due to its inertia. Such a reverse flow may only occur with a resonant state of the valve. A high velocity of the spool may induce cavitation when the flow from the clearance is not sufficient to fill the valve chamber. This state corresponds to the region where \(P_t < 0\). \(P_a\) at the boundary of this region is calculated substituting \(P_t = 0\) in Eq. (6). Since \(P_t \geq 0\), the state \(P_a < 0\) does not exist physically. \(P_a > 1\) implies reverse flow from the bearing to the pressure source, which is possible only when \(P_t > P_a\). In summary, the solution of Eqs. (6) and (7) must satisfy

\[ P_t \geq 0 \] with \(0 < P_a < 1\), and \(P_t > P_a > 1\).

The latter relation corresponds to the possible reverse flow.

3. Characteristic of Pressures

Using Fig. 3, we explain pressure in the valve chamber. The pressure \(P_a\), i.e., the value at the crossing point of curve (6) to the line \(P_t = 0\), is given by

\[ P_a = \frac{2}{1 + \sqrt{1 + 4(a_t + a_0)^2}} < 1. \]  

At the normal flow boundary, we have \(P_t = P_a = P_e\). Substituting these into Eq. (6), we have

\[ P_a = \frac{2}{1 + \sqrt{1 + 4a_t^2}} < 1. \]  

When the spool end pressure reaches the supply pressure, namely, when \(P_t = 1\), \(P_a\) at this state \((P_t)\) is given by

\[ P_t = \frac{2}{1 - 2a_t(a_t + a_0) + \sqrt{1 - 2a_t(a_t + a_0)^2 + 4(a_t + a_0)^2}}. \]

These three values are in the following order and illustrated in Fig. 3:

\[ 0 < P_a < P_t < P_e \]

The curves expressed by Eqs. (6) and (7) intersect in the region \(P_t < 0\) when \(P_a < (d^2z/dt^2)/a_0\). For this case, we have to set

\[ P_t = 0, P_a = P_e. \]  

The reverse flow limit at the bearing is given by \(P_a = 1\). For this state we have

\[ \frac{dP_a}{dP_t} = 0, P_t = 1 + \frac{a_t}{a_0} = P_e. \]  

We seldom observe this state, although we must include this condition in our numerical calculation program.

4. Numerical Calculation

4.1 Calculation procedure

To apply numerical methods for solving nonlinear simultaneous equations (6) and (7), we must consider the region of existence of the solution explained above. To solve the above equations the fixed point iteration is not effective because it often diverges with this system. The Newton method fails often because an approximate value easily falls into regions where \(P_t\) or \(P_a\) does not exist physically. The regular falsi method and the secant method are effective for solving the system. However, a large step size for trial calculation by these methods often leads to the region of false solutions and gives an erroneous result.
Although the bisection method gives a true solution, its convergence is very slow. To avoid a solution that falls into the region of false solutions, we adopt the following procedure.

Rearranging Eq. (6), we have

$$G = U + 2a_0(a_0 + a_s)P,$$

$$P_a = \frac{G + \sqrt{G^2 - 4(U + a_0 P)^2}(a_0 + a_s)^2}{2(a_0 + a_s)^2} = F_i(P_i),$$

(15)

where

$$U = -1, \text{ when } P_i < 1 + a_0/a_s,\$$

$$U = 1, \text{ when } P_i > 1 + a_0/a_s.$$

Rearranging Eq. (7), we have

$$P_a = \frac{1}{a_0} \frac{d\xi}{dt} + \frac{a_0(1-\xi)}{\sqrt{1 + a_0 a_s(1-\xi)^2}} + p_i = F_i(P_i).$$

(16)

Let

$$F(x) = F_i(x) - F_s(x).$$

(17)

Substituting 0 and 1 into Eq. (18) we have

$$F(0) = P_a - (1/a_0)(d\xi/dt) \text{ when } P_i = 0;$$

$$F(1) = p_i - F_s(1) \text{ when } P_i = 1.$$

The solution set for $F(0) \leq 0$ is $P_i = 0, P_a = P_s$. When $F(0) > 0$, we can find the unique solution of $F(P_i) = 0$ that satisfies $P_i > 0$. This is the nozzle pressure at the operating point.

4.2 Flapper displacement, spool velocity and pressure

The following numerical examples show how the operating point is determined using $\xi$ and $d\xi/dt$. Figure 4 shows the curve defined by Eq. (6) and the nine curves defined by Eq. (7). The nine curves correspond to the combinations of parameters $\xi = 1, 0, 1$ and $d\xi/dt = -0.1, 0, 0.1$ that are indicated to each curve. The operating points are given by the values at intersections of these curves and the curve for Eq. (6). All curves for Eq. (7) pass through the point $P_i = 0, P_a = (d\xi/dt)/a_0$, although the curves are changed by parameter $\xi$. Curves for $P_i$ are not shown because they can be expressed by curves of $P_i$ with inverted signs of $\xi$ and $d\xi/dt$.

Equation (7) expresses a parabolic curve, except when $\xi = 1$ which corresponds to a straight line. The operating point must lie between the curves expressing $\xi = 1$ and $\xi = -1$, since these two extremities correspond to contact between the flapper and nozzle. For a known value of $\xi$, we can define a parabolic curve lying between the above two extremities (refer Fig. 5). For a change of $d\xi/dt$, all parabolic curves simultaneously shift parallel in the vertical direction.

When the input signal has a frequency lower than the resonance frequency of the servovalve, the operating point falls in the normal operating region because $\xi$ and $d\xi/dt$ are in the same phase. When the input signal has a frequency higher than the resonance frequency $\xi$ and $d\xi/dt$ are in inverted phases, hence the operating point can lie in the reverse flow region. When $d\xi/dt$ becomes larger than in the examples shown in Fig. 4, then the curves for Eq. (7) move upward and do not intersect with the curve of Eq. (6). This loss of the intersection corresponds to cavitation in the valve chamber.

Figure 5 shows extracted curves for which $\xi$ and $d\xi/dt$ take the same sign. This also illustrates the $P_i$ values for a low frequency motion of the flapper. The curve of $P_i$ for $\xi = 1$ and $d\xi/dt = 0.1$ also represents the curve of $P_i$ for $\xi = -1$ and $d\xi/dt = -0.1$. For the curves in Fig. 5, we can see that the values of $P_i$ and $P_s$ are neither symmetric nor skew symmetric at any instant.

To explain the symmetric character of left- and right-side variables noted in Section 2, the nozzle and bearing pressures for left and right sides are calculated as functions of $\xi$ and illustrated in Fig. 6.
discussing the symmetric property of pressure, we note that \( P_B \) and \( P_N \) deviate slightly from 1, whereas \( P_l \) and \( P_r \) change substantially with \( \xi \). As seen from Fig. 6 (b), the pressures on the two sides are symmetric with respect to \( \xi = 0 \), when spool velocity \( d\xi/dt \) is zero. This symmetric property of pressure is broken when \( d\xi/dt \) is not zero. However, the curves in symmetry with those on Fig. 6 (a) appear on Fig. 6 (c) as curves with the inverted sign of \( d\xi/dt \). On comparing these curves, we expect that the nonlinearity in this system can be approximated by a linear variable that is obtained as the difference between the variables on the two sides, when they belong in the normal operation region.

The pressure curves are determined by nondimensional parameters \( a_a \), \( a_b \), \( a_c \) and \( a_d \). Hence the curves are influenced by physical dimensions through these parameters. In the following, we will briefly explain how physical parameters influence the system variables.

Figure 7 shows the curves of the chamber pressure \( P_l \) as a function of \( \xi \). For these curves, \( d\xi/dt = 0 \). In this figure the parameters for the broken curve are standard values. The parameters are perturbed around the standard and the results are drawn as bold curves. The following trends are observed from these curves:

1. A viscosity decrease accompanying temperature rise induces a rise of the chamber pressure.
2. A small nozzle diameter induces a rise of the chamber pressure.
3. A high supply pressure induces higher rise of the chamber pressure than the linear proportion.
4. An increase of spool clearance induces pressure rise, and the curve approaches a straight line.
5. An increase of spool radius induces a rise of the chamber pressure.
6. An increase of the flapper gap at the neutral position induces a decrease of the chamber pressure.

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**Fig. 6** Pressure vs flapper displacement

**Fig. 7** Influence of parameters
5. Comparison with a Conventional Flapper Nozzle System

In this section we will compare our system with a conventional system. The conventional and most widely used flapper nozzle system has a circular restriction as the first stage throttle of a two-stage servovalve [27]. Figure 8 shows illustrations of the conventional system (type B) and present system (type A).

Pressures in the type B system can be expressed by the following relations.

\[
\begin{align*}
\rho^* & = \frac{\rho}{\rho - \rho^*} \left( \frac{4}{\pi \rho_d} \right)^{\frac{1}{2}} \\
\rho_{\text{m}} - \rho^* & = \frac{\rho}{\rho - \rho^*} \left( \frac{4}{\pi \rho_d} \right)^{\frac{1}{2}} \\
\rho - \rho_m & = \frac{\rho}{\rho - \rho^*} \left( \frac{4}{\pi \rho_d} \right)^{\frac{1}{2}} \\
\rho_s - \rho_m & = \frac{\rho}{\rho - \rho^*} \left( \frac{4}{\pi \rho_d} \right)^{\frac{1}{2}}
\end{align*}
\]

From these relations we have

\[
P_r = \frac{1 + (C_1 + C_2)(1 - \xi)^2}{1 + (C_1 + C_2 + C_3)(1 - \xi)^3},
\]

where

\[
C_1 = 16 \xi \left( \frac{H C_s}{d_e} \right)^2, \quad C_2 = 16 \xi \left( \frac{H d C_s}{d_e} \right)^2,
\]

\[
C_3 = 16 \xi \left( \frac{H d C_s}{c d_t} \right)^2.
\]

The pressures at the spool end of type A and B systems are compared in Fig. 9 where the curves for \( P_r \) and \( \Delta P/2 = (P_r - P_t)/2 \) are shown. \( \Delta P \) is the driving force of the spool; at the same time it is the output of the flapper–nozzle system. Therefore, a linear relation between \( \Delta P \) and \( \xi \) is an important basis for the linear relationship between the input and output of a servovalve.

For the example shown in Fig. 9, \( d_t \) for the type B system is selected so that its range for \( \Delta P \) with \( \xi = 1 \) and \( \xi = -1 \) is the same range as for \( \Delta P \) of the type A system. When the nozzle is blocked by the flapper surface, the \( P_r \) curve for type B has a zero gradient at

<table>
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<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
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<td>2.50E-05</td>
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Fig. 9 Pressure characteristics of type A and type B systems \((H=60 \mu m, d_s=1.5 \text{ mm}, h_s=10 \mu m, h_b=25 \mu m, R=6 \text{ mm}, d_t=0.33 \text{ mm}, \mu=0.8 \text{ cP})\)

\( \xi = 1 \); the \( P_r \) curve for type A has a negative gradient at \( \xi = -1 \). Therefore, the linearity of the curve for type A is better than that for type B.

A numerical inspection of the linearity is given in Table 1. In this table \( \lambda \) represents the mean gradient of \( \Delta P/2 \) calculated using the least squares method; \( \lambda_0 \) represents the gradient at the equilibrium point, and S.D. represents the mean deviation given by

\[
\lambda_0 = \frac{\partial (\Delta P/2)}{\partial \xi}_{\xi=0},
\]

\[
(\text{S. D.)}^2 = \frac{1}{2} \int (\Delta P/2 - \lambda \xi)^2 d\xi.
\]

The stability of the servovalve depends on the rate of change of variables near the operating point. A two-stage servovalve using a flapper–nozzle system as its first stage amplifier approaches the stability limit when \( \lambda_0 \) is increased. Type B has \( \lambda_0 \) larger than \( \lambda \), whereas Type A has \( \lambda \) smaller than \( \lambda_0 \). Hence we can conclude that type A is better than type B in terms of stability.

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Table 2  Experimental conditions

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<th>Case 2</th>
<th>Case 3</th>
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<tr>
<td>$h_b$ (μm)</td>
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<td>38.5</td>
<td>43.5</td>
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</table>

Common parameters:
- $\mu$ (Pa.s) = 0.9 x 10^{-3}
- $p_b$ (bar) = 32.5
- $d_a$ (mm) = 1.5
- $d_b$ (mm) = 1.00
- $R$ (mm) = 5.05
- $L_s$ (mm) = 28.5
- $L_i$ (mm) = 10
- $C_m = 0.7$, $C_{ap} = 0.7$, $\xi = 0.1$, $\xi = 0.5$

An experiment is performed for various nozzle diameters and spool dimensions.

Figure 11 shows an example of experimental results. The circumferential clearance $h_a$ has a small influence on the pressure characteristic. It is known that the nozzle discharge coefficient depends on the flapper configuration and the Reynolds number of the flow. We prefer to use the following measured result for this experiment from a practical viewpoint to estimate the theoretical values:

$$c_m = c_{m0}[(5/7) + (2/7)(H-x)/d_a/10]$$
for $(H-x) < d_a/10$;

$$c_m = c_{m0} = 0.7$$ for $(H-x) \geq d_a/10$.

Experimental conditions for the theoretical calculation shown in Fig. 11 are listed in Table 2. Since the nozzle dimensions are small, a high machining accuracy is required to achieve the same value of the discharge coefficient. Therefore, the experiment was performed for several different nozzles and spools to prove the reproducibility.

Figure 11 shows the characteristic of the system explained in the previous section. The deviation between the theoretical and experimental results is sufficiently small. Hence we can conclude that the theory is applicable for the design and simulation of the flapper-nozzle system.

7. Conclusions

The spool in a two-stage servovalve is supported by a set of hydrostatic bearings. The fluid from the bearings is used as the working fluid of the flapper-nozzle system in the servovalve. Thus the flow loss in the valve is decreased. In the new system, the circumferential clearance forms a laminar restriction upstream of the nozzle that substitutes for the fixed orifice in the conventional flapper-nozzle system.

As the result, linearity between flapper displacement and the spool driving pressure is improved, i.e., the gradient of the spool driving pressure near the neutral position is modified to a moderate value. Since this system involves laminar restriction, its characteristic is influenced by the change of fluid viscosity. When water is used as the working fluid, however, the influence of viscosity becomes small. Hence the flapper-nozzle system treated in this paper
is suitable for water hydraulic systems.

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References