Axisymmetric Interfacial Oscillations of a Magnetic Liquid Column*

Motoyoshi OHABA**, Tatsuo SAWADA***,
Shin-hachiro SAITO****, Seiichi SUDO*****
and Takahiko TANAHASHI***

This paper presents the stability and dynamic characteristics of an axisymmetrically oscillating magnetic liquid column subjected to a uniform magnetic field in the presence of gravity. Initially, the axisymmetric mode response of a magnetic liquid column is analyzed using potential theory based on a magnetic field perturbation method. Then, an experiment is described in which a liquid column is formed between Helmholtz coils and vibrated vertically to produce axisymmetric oscillations by an electodynamic exciter. The surface frequency response of the column is measured optically using a high-speed video camera, a strobovision-analyzer and a stroboscope. The results show that the magnetic field exerts a stabilizing effect on the column and the resonance frequency increases for increased liquid column aspect ratio (diameter/height). Both the experimental interfacial response and resonance frequency show good agreement with the theoretical prediction.

Key Words: Magnetic Fluid, Magnetohydrodynamics, Liquid Column, Axisymmetric Mode, Oscillation, Stability, Frequency Response

1. Introduction

A material processing technique known as the "Floating Zone Technique⁽¹⁾" is expected to be a useful technique for crystal growth in a microgravity environment. This technique involves the formation of a liquid column between a feed rod and a growth crystal. Unfortunately, the stability of such a column is easily affected by vibrational disturbances such as machine movements and crew activities⁽²⁾. The vibrational disturbances also produce interfacial oscillations which may produce internal flows within the

column and thus directly affect the quality of the crystal produced. As such, the dynamic characteristic of the column is an important area that must be studied to develop control methods for stabilizing liquid columns⁽³⁾.

In order to perform terrestrial simulations, a method called the "Neutral Buoyancy Method⁽⁴⁾" has been used. In this method a liquid column is produced in an other immiscible liquid of equal density. However, this method is complicated by the interfacial viscosity and the inertial effects of the surrounding liquid⁽⁵⁾. In seeking an alternative to the Neutral Buoyancy Method we noted that a magnetic liquid, that is, a stable colloidal dispersion of magnetic particles, is quickly magnetized in response to a magnetic field. Thus, this type of liquid has wide industrial potential as a functional liquid (smart material)⁽⁶⁾. In previous studies^{(7),(8)}, we proposed a new simulation method using a magnetic liquid for terrestrial simulation of a liquid column. In particular, we examined the static shape and the lateral resonance frequency of a magnetic liquid column in a non-uniform magnetic field both theoretically and experimentally.

In the present paper, we develop the study of the

* Received 13th July, 1997

** Department of Control System Engineering, Toin University of Yokohama, 1614 Kurogane-cho, Aoba-ku, Yokohama 225-8502, Japan

*** Department of Mechanical Engineering, Keio University, 3-14-1 Hiyoshi, Kouhoku-ku, Yokohama 223-8522, Japan

**** Physics Laboratory, Toin University of Yokohama, 1614 Kurogane-cho, Aoba-ku, Yokohama 225-8502, Japan

***** Department of Mechanical Engineering, Iwaki Meisei University, 5-5-1 Iino, Chuoudai, Iwaki 970-8551, Japan

Magnetic Liquid Method by examining axisymmetric oscillations of a magnetic liquid column in a uniform magnetic field in the presence of gravity. In the theoretical analysis, we introduce the velocity potential and the resonance frequency using linear potential theory, and consider the magnetic field effect on the static and dynamic stabilities. In the experiment, a cylindrical magnetic liquid column is formed in a magnetic field created by Helmholtz coils and the interfacial frequency response is examined at various aspect ratios as the column is vibrated vertically. The experimental results are in quantitative agreement with the theoretical prediction. As such, the magnetic field effects can be allowed for to provide an accurate simulation of a liquid column in a microgravity environment.

Nomenclature

- a : radius
 A : aspect ratio $= 2a/h$
 B : magnetic flux density
 f_0 : forcing frequency
 g : gravitational acceleration
 h : height
 H_0 : non-perturbed magnetic field intensity
 H : magnetic field intensity
 I_m : modified Bessel function of the first kind of order $m(=0, 1)$
 K_m : modified Bessel function of the second kind of order $m(=0, 1)$
 M : magnetization
 n : mode number along liquid column axis
 N_B : Bond number $= \rho g h^2 / \sigma$
 N_M : magnetic Bond number $= (\mu_1 - \mu_0) h H_0^2 / 2\sigma$
 p : pressure
 u : unit vector normal to liquid interface
 Z_0 : forcing amplitude
 $\eta(z, t)$: interfacial displacement
 χ : wave number along liquid column axis
 μ_0 : permeability of vacuum
 μ : permeability of fluid
 σ : interfacial tension
 ρ : liquid density
 $\phi(r, z, t)$: velocity potential
 $\psi(r, z, t)$: perturbed magnetic potential
 ω : forcing circular frequency
 ω_{on} : axisymmetric resonant circular frequency
- Subscripts
 1 : liquid column
 2 : surrounding gas

2. Theoretical Analysis

2.1 Flow field formulation

We analyze the interfacial axisymmetric oscillations

Fig. 1 Analytical model

tion of a cylindrical magnetic liquid column of height h and radius a . The column is formed between two non-magnetic flat solid bases as shown in Fig. 1. A uniform magnetic field H_0 is applied in the z -axis direction. The column is vibrated vertically according to $Z(t) = Z_0 e^{i\omega t}$, where Z_0 is the forcing amplitude and ω the circular frequency, which can be complex. The equations are formulated in a cylindrical coordinate system (r, θ, z) which is fixed relative to the solid bases, and with the z -axis directed upward along the liquid column axis. The surface displacement is assumed to be small so that the pressure fluctuation of the surrounding gas may be neglected. We further assume that the magnetic liquid is non-viscous and incompressible. If the liquid motion begins from a quiescent state, the flow is irrotational, and a velocity potential $\phi(r, \theta, z, t)$ may be defined such that the continuity equation is satisfied by the Laplace equation

$$\nabla^2 \phi = 0. \quad (1)$$

In this coordinate system, the axial velocities at the ends of the column, $z = \pm h/2$, are zero and the solid boundary conditions are given by

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = \pm h/2. \quad (2)$$

Further assuming that the liquid motion is axisymmetric, $\phi(r, \theta, z, t)$ does not depend on the angle θ , and we seek the velocity potential $\phi(r, z, t)$ which satisfies the Laplace equation and the solid boundary conditions in Eq. (2) in the form

$$\phi(r, z, t) = \sum_{n=1}^{\infty} C_n I_0(\chi r) \sin \chi z e^{i\omega t}, \quad (3)$$

where

$$\chi = (2n-1)\pi/h, \quad n=1, 2, \dots,$$

C_n is an unknown constant, $I_0(\chi r)$ the modified Bessel function of the first kind of order zero, and χ the wave number of the interfacial wave in the z -direction.

Next, the equation of the liquid interface is described by

$$r = a + \eta(z, t), \quad (4)$$

where the interfacial displacement $\eta(z, t)$ is

$$\eta(z, t) = \sum_{n=1}^{\infty} A_n \sin \kappa z e^{i\omega t}, \quad (5)$$

and A_n is an unknown constant.

Based on the linearized kinematical boundary condition, the normal component of the liquid velocity at the interface of the column must be equal to the rate of the displacement, thus

$$\frac{\partial \phi}{\partial r} = \frac{\partial \eta}{\partial t} \quad \text{at } r=a. \quad (6)$$

Substituting Eqs.(3) and (5) into Eq.(6), and eliminating C_n , we can write the velocity potential as

$$\phi(r, z, t) = \sum_{n=1}^{\infty} \frac{i\omega A_n I_0(\kappa r)}{\kappa I_1(\kappa a)} \sin \kappa z e^{i\omega t}, \quad (7)$$

where $I_1(\kappa r)$ is the modified Bessel function of the first kind of order one.

Now, the modified Bernoulli's equation for a magnetic fluid, from Rosensweig⁽⁹⁾, including the dynamic pressure by the vertical vibration is

$$\rho \frac{\partial \phi}{\partial t} + \frac{1}{2} \rho |\nabla \phi|^2 + \rho z \left(g + \frac{d^2 Z}{dt^2} \right) + p_1 - \mu_0 \int_0^H M dH = p_2, \quad (8)$$

where ρ is the liquid density, μ_0 the permeability of vacuum, M the magnetization, H the magnetic field intensity parallel to the magnetization M , p_1 and p_2 the pressure in the liquid column and of the surrounding gas respectively. Also, in considering the interfacial tension, the unit vector normal to the interface is

$$\mathbf{u} = \left(\mathbf{e}_r - \frac{\partial \eta}{\partial z} \mathbf{e}_z \right) / \left[1 + \left(\frac{\partial \eta}{\partial z} \right)^2 \right]^{1/2}, \quad (9)$$

where $\mathbf{e}_r$ and $\mathbf{e}_z$ are the unit normal vectors of the r and z -axis respectively. Using the linearized mean curvature of the interface obtained from Eq.(9), the pressure difference ($p_1 - p_2$) at a same height between the two sides of the interface due to the interfacial tension σ is given by

$$p_1 - p_2 = \sigma \left[\frac{1}{a} - \frac{1}{a^2} \left(\eta + a^2 \frac{\partial^2 \eta}{\partial z^2} \right) \right]. \quad (10)$$

The liquid pressure p_1 can also be expressed by a linearized Eq. (8):

$$p_1 = p_2 - \rho \frac{\partial \phi}{\partial t} - \rho z \left(g + \frac{d^2 Z}{dt^2} \right) + \mu_0 \int_0^H M dH. \quad (11)$$

Substituting Eq.(11) into Eq.(10), we have the mechanical boundary condition

$$\rho \frac{\partial \phi}{\partial t} + \frac{\sigma}{a} - \frac{\sigma}{a^2} \left(\eta + a^2 \frac{\partial^2 \eta}{\partial z^2} \right) + \rho z \left(g + \frac{d^2 Z}{dt^2} \right) - \mu_0 \int_0^H M dH = 0 \quad \text{at } r=a. \quad (12)$$

2.2 Magnetic field perturbation

Next we consider the magnetic field perturbation at the interface for evaluating the effect of the magnetic field. In this case, since the inner fluid (the magnetic liquid column) and the outer fluid (the surrounding gas) are non-conductive, Maxwell's equa-

tions can be described

$$\nabla \times \mathbf{H} = 0, \quad (13)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (14)$$

where $\mathbf{B}$ is the magnetic flux density. From Eq.(13), $\mathbf{H}$ has a magnetic potential $\varphi(r, z, t)$ such that $\mathbf{H} = -\nabla \varphi$. In particular, the non-perturbed magnetic potential can be described as $H_0 z$ and the perturbed magnetic potential of the first order as $\psi(r, z, t)$ so that the magnetic potential $\varphi(r, z, t)$ may be represented as

$$\varphi(r, z, t) = H_0 z + \psi(r, z, t). \quad (15)$$

Further, if we assume the inner and outer fluids have constant permeabilities, $\psi(r, z, t)$ satisfies the Laplace equation from Eq.(14):

$$\nabla^2 \psi(r, z, t) = 0, \quad (16)$$

and the perturbed magnetic potentials $\psi_1(r, z, t)$, $\psi_2(r, z, t)$ may be expressed

$$\psi_1(r, z, t) = \sum_{n=1}^{\infty} C_{11} I_0(\kappa r) \cos \kappa z e^{i\omega t}, \quad (17)$$

$$\psi_2(r, z, t) = \sum_{n=1}^{\infty} [C_{21} I_0(\kappa r) + C_{22} K_0(\kappa r)] \cos \kappa z e^{i\omega t}, \quad (18)$$

where the subscript 1 indicates the liquid column and 2 the surrounding gas, $K_0(\kappa r)$ is the modified Bessel function of the second kind of order zero, and C_{11} , C_{21} , and C_{22} are unknown constants. These magnetic potentials must satisfy the magnetic field boundary conditions which express the continuity of the normal magnetic flux density and of the tangential magnetic field at the interface; they are

$$\mathbf{u} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0, \quad (19)$$

$$\mathbf{u} \times (\mathbf{H}_1 - \mathbf{H}_2) = 0 \quad \text{at } r=a. \quad (20)$$

Thus, the interfacial condition of Eq.(19) for the perturbed potentials becomes

$$\mu_1 \left(\frac{\partial \psi_1}{\partial r} - H_0 \frac{\partial \eta}{\partial z} \right) = \mu_2 \left(\frac{\partial \psi_2}{\partial r} - H_0 \frac{\partial \eta}{\partial z} \right), \quad (21)$$

where μ_1 and μ_2 are the permeabilities of the inner and outer fluids. And from Eq.(20) we get

$$\psi_1 = \psi_2 \quad \text{at } r=a. \quad (22)$$

A further condition is that the perturbed potential of the magnetic field at a large distance from the liquid interface $r = a_0 (a_0 \gg a)$ is almost zero,

$$\psi_2 = 0 \quad \text{at } r = a_0 (a_0 \gg a). \quad (23)$$

Using the conditions in Eqs.(21)-(23), we get the perturbed constants

$$C_{11} = A_n (\mu_1 - \mu_2) [I_0(\kappa a) K_0(\kappa a_0) - I_0(\kappa a_0) K_0(\kappa a)] H_0 / D_0, \quad (24)$$

$$C_{21} = A_n (\mu_1 - \mu_2) [I_0(\kappa a) K_0(\kappa a_0)] H_0 / D_0, \quad (25)$$

$$C_{22} = -A_n (\mu_1 - \mu_2) [I_0(\kappa a) I_0(\kappa a_0)] H_0 / D_0, \quad (26)$$

and put

$$D_0 = \mu_1 [I_0(\kappa a) K_0(\kappa a_0) - I_0(\kappa a_0) K_0(\kappa a)] I_1(\kappa a) - \mu_2 [I_1(\kappa a) K_0(\kappa a_0) + I_0(\kappa a_0) K_1(\kappa a)] I_0(\kappa a), \quad (27)$$

where $K_1(\kappa a)$ is the modified Bessel function of the second kind of order one. By using $K_0(\kappa a_0) \approx 0$, and

substituting into Eqs.(24)-(27), we obtain simplified coefficients

$$C_{11}=A_n(\mu_1-\mu_2)H_0K_0(xa)/D, \quad (28)$$

$$C_{21}=0, \quad (29)$$

$$C_{22}=A_n(\mu_1-\mu_2)H_0I_0(xa)/D, \quad (30)$$

where

$$D=\mu_1K_0(xa)I_1(xa)+\mu_2I_0(xa)K_1(xa).$$

For simplicity, these coefficients Eqs.(28)-(30) are used in the following analysis.

Thus, the magnetizations of the inner and outer fluids are linearly related to the applied magnetic field, and the fifth term in the left side of Eq.(12) can be calculated as the first-order perturbation:

$$\mu_0 \int_0^H MdH = (\mu_1 - \mu_0) \left(\frac{1}{2} H_0^2 + H_0 \frac{\partial \phi}{\partial z} \right). \quad (31)$$

2.3 Axisymmetric mode responses

Since the unknown constant A_n can be determined from the perturbed magnetic boundary condition, we use Eq.(12) to obtain the non-perturbed boundary condition

$$\frac{\sigma}{a} - \frac{1}{2}(\mu_1 - \mu_0)H_0^2 + \rho g z = 0, \quad (32)$$

and the perturbed boundary condition

$$\rho \frac{\partial \phi}{\partial t} - \frac{\sigma}{a^2} \left(\eta + a^2 \frac{\partial^2 \eta}{\partial z^2} \right) + \rho z \frac{d^2 Z}{dt^2} - (\mu_1 - \mu_0) H_0 \frac{\partial \phi}{\partial z} = 0. \quad (33)$$

Then z in Eq.(33) can be expanded in a Fourier series as

$$z = \frac{4}{\pi^2} h \sum_{n=1}^{\infty} \frac{(-1)^{(n-1)}}{(2n-1)^2} \sin \pi z. \quad (34)$$

Finally, substituting Eqs.(5), (7), (18), and (34) into Eq.(33), and setting $\mu_2 = \mu_0$, we obtain the unknown coefficient A_n as

$$A_n = \frac{(-1)^{n-1} 4 Z_0 \omega^2 I_1(xa)}{\pi h (\omega_{0n}^2 - \omega^2) I_0(xa)}, \quad (35)$$

where ω_{0n} is the resonant circular frequency of the axisymmetric mode (0, n), that is, the frequency when the denominator of Eq.(35) equals zero. Thus

$$\omega_{0n}^2 = - \frac{\sigma \chi I_1(xa)}{\rho a^2 I_0(xa)} (1 - \chi^2 a^2) + \frac{\chi^2 (\mu_1 - \mu_0)^2 H_0^2 I_1(xa) K_0(xa)}{\rho [\mu_1 K_0(xa) I_1(xa) + \mu_0 I_0(xa) K_1(xa)]}. \quad (36)$$

This frequency Eq.(36) agrees with the resonant circular frequency of a magnetic liquid jet as analyzed by Taktarov⁽¹⁰⁾.

The velocity potential $\phi(r, z, t)$ is obtained by substituting Eq.(35) into Eq.(7):

$$\phi(r, z, t) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 4 Z_0 \omega^2 I_0(\chi r)}{\pi^2 h (\omega_{0n}^2 - \omega^2) I_0(xa)} \times \sin \pi z e^{i\omega t}, \quad (37)$$

furthermore, the interfacial displacement $\eta(z, t)$ is then obtained by substituting Eq.(35) into Eq.(5):

$$\eta(z, t) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 4 Z_0 \omega^2 I_1(xa)}{\pi h (\omega_{0n}^2 - \omega^2) I_0(xa)} \sin \pi z e^{i\omega t}. \quad (38)$$

3. Static Stability in Gravitational Field

We now discuss the static stability in a gravitational field. Let us consider the static pressure difference along the z -axis direction. Substituting the non-perturbed magnetic term of Eq.(31) into Eq.(11), we get

$$p_1 - p_2 = -\rho g z + \frac{1}{2}(\mu_1 - \mu_0)H_0^2. \quad (39)$$

In Eq.(39) the pressure difference ($p_1 - p_2$) decreases with an increase in the z -coordinate. In the case of ($p_1 - p_2$) < 0 the liquid column may disintegrate due to negative liquid pressure. However, the applied magnetic field acts to counteract the negative static pressure and exerts a stabilizing effect on the column.

In order to examine the relation between the magnetic and gravitational field effects on the stable state, we use the Eq.(32) at $z=h$:

$$\frac{1}{2}(\mu_1 - \mu_0)H_0^2 = \frac{\sigma}{a} + \rho g h. \quad (40)$$

A dimensionless form of Eq.(40) can be written

$$N_M = N_B + \frac{2}{A}, \quad (41)$$

where N_M is the magnetic Bond number, N_B the Bond number, and A the aspect ratio as shown in Eqs.(42)-(44), respectively.

$$N_M = \frac{1}{2\sigma}(\mu_1 - \mu_0)hH_0^2, \quad (42)$$

$$N_B = \frac{\rho g h^2}{\sigma}, \quad (43)$$

$$A = \frac{2a}{h}. \quad (44)$$

From Eq.(41), the magnetic Bond number N_M that maintains a magnetic liquid column stably in a gravitational field is proportional to the Bond number N_B and inversely proportional to the aspect ratio A . It is also clear that a gravitational field only affects the static stability and not the resonance frequency of the column in Eq.(36).

4. Magnetic Field Effect on Dispersion Relation

Next we consider the effect of the magnetic field on axisymmetric oscillations. In the absence of the magnetic field, substituting $H_0=0$ into Eq.(36), we get

$$\omega_{0n}^2 = - \frac{\sigma \chi I_1(xa)}{\rho a^2 I_0(xa)} (1 - \chi^2 a^2). \quad (45)$$

The term ω_{0n}^2 in Eq.(45) may have both positive and negative values. At $\chi a > 1$, ω_{0n} is real and the corresponding oscillation is stable. On the contrary, at $\chi a < 1$, ω_{0n} has two imaginary values both positive and negative; in the positive case, the interface displacement Eq.(38) decreases whereas, in the negative case, the displacement may increase as a function of time and therefore the oscillating column will become unstable and will finally break at the corresponding forcing frequency. This case is known as the sausage

instability of liquid columns.

A graph illustrating the marginal stability defined by the dispersion relation of the dimensionless quantity $s^2 \equiv (\rho a^3 / \sigma) \omega_{0n}^2$ of Eq.(36) is shown as a function of κa in Fig. 2. The physical values are $\mu_1 = 1.5 \mu_0$, $\rho = 1390 \text{ kg/m}^3$, $\sigma = 0.026 \text{ N/m}$, the liquid column radius $a = 2.0 \text{ mm}$, and height $h = 10.0 \text{ mm}$. In the absence of the magnetic field, the magnetic Bond number $N_M = 0$, the column is unstable in the range $\kappa a < 1$. For the axisymmetric first mode $n=1$ the column is unstable when $\pi a < h$, that is, the liquid column becomes unstable as the circumference of the column decreases to less than two times the column height, i.e., the wavelength of the first mode.

In a magnetic field, the unstable range decreases as the magnetic Bond number increases. For example, the unstable range for $N_M = 27$ decreases to almost half the range of the case $N_M = 0$. That is, the magnetic field along the interface exerts a stabilizing effect on the axisymmetric oscillations and suppresses breaking. We note that the dynamic interfacial instability above occurs only in the axisymmetric oscillation, not in the non-axisymmetric oscillation⁽⁷⁾.

Figure 3 shows the theoretical dimensionless maximum interfacial displacement $\eta_{\max}/Z_0$ of the liquid column as a function of the excitation frequency f_0 for various values of the magnetic Bond number N_M . These responses are calculated using the real part of Eq.(38) for the parameters: $h = 10.0 \text{ mm}$, $a = 4.0 \text{ mm}$, $Z_0 = 0.50 \text{ mm}$, and the same physical values used in Fig. 2. The interfacial frequency response shifts to higher-frequency regions and the resonant range increases with increasing applied magnetic field.

5. Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in Fig. 4. The system consists of a pair of Helmholtz coils, an electrodynamic exciter, and optical displacement measuring instruments. The Helmholtz coils supporting the liquid column have an inner diameter 50 mm, an outer diameter 140 mm, and an iron core 12 mm. To produce the same wetting condition at both ends of the column, acrylic plastic plates, 10 mm thick are glued onto the pole faces of the Helmholtz coils. A coil current is imposed by a DC power supply, and the magnetic field intensity is controlled by varying the coil current. The finite cylindrical liquid column, height $h = 9.78 \text{ mm}$, was made by injecting the magnetic liquid on the lower plate, and increasing the coil current.

An electrodynamic exciter (EMIC, F-400B/A-E07) is used to generate a sinusoidal excitation with a controlled frequency and amplitude. In the frequency response experiment, the surface displacement is

Fig. 2 Dispersion relation for axisymmetric mode of magnetic liquid column in uniform magnetic field

Fig. 3 Theoretical interfacial frequency response of axisymmetric oscillating magnetic liquid column for $A=0.40$ and $Z_0/h=0.03$

Fig. 4 Schematic diagram of experimental apparatus

measured optically using a high-speed video camera (NAC, HSV-400) and a strobo-vision analyzer (SUGAWARA, SVA-2) with a CCD camera and a stroboscope. These systems provide stop motion pictures of the oscillating liquid column for analysis. The

resonance frequencies are determined based on the corresponding forcing frequency which produces the maximum liquid surface response. The magnetic liquid used in the experiments is kerosene-based HC-50 (Taihou Industrial Co., Ltd.) having specific density 1.390; viscosity 10.5 mPa·s; liquid/air interfacial tension 0.026 N/m (25°C); and saturated magnetization 37.8 kA/m.

6. Experimental Results and Discussion

In Fig. 5 the static stability limit of the liquid column in a gravitational field is shown by the magnetic Bond number N_M as a function of the aspect ratio A for the Bond number $N_B=50.1$. The theoretic-

Fig. 5 Threshold of magnetic Bond number N_M in static stability of liquid column for Bond number $N_B=50.1$

Fig. 6 Sequence photographs of axisymmetric first mode (0,1) shape of an oscillating liquid column for $A=0.580$, $f_0=12.6$ Hz, $N_M=180$, and $Z_0=0.60$ mm

Fig. 7 Sequence photographs illustrating axisymmetric progressive mode wave near the first mode (0,1) resonance point for $A=0.580$, $f_0=13.0$ Hz, $N_M=180$, and $Z_0=0.60$ mm

cal solid curve calculated using Eq.(41) gradually decreases with increasing A . The experimental values denote the threshold of the magnetic Bond number N_M supporting the liquid column stably. The experimental results have a somewhat sharper gradient than that of the theory. In this experiment, when decreasing the magnetic field, we observed the column shape deforming like a cone with the top at the upper end, and the size becoming larger in higher aspect ratios. Thus, the discrepancy with the theory is due to the non-uniformity in the liquid column diameter along the z -axis.

The axisymmetric first mode (0,1) shape of an oscillating magnetic liquid column is shown in the sequence photographs in Fig. 6. This mode corresponds to the aspect ratio $A=0.580$, the forcing frequency $f_0=12.6$ Hz, the magnetic Bond number $N_M=180$, and the forcing amplitude $Z_0=0.60$ mm. It can be seen that the liquid column has a node near the center and antinodes at the ends. When the forcing frequency was increased, the axisymmetric standing wave changes to progressive one near the mode (0,1) resonance point at $f_0=13.0$ Hz as shown in the sequence photographs in Fig. 7. This kind of wave is not predicted theoretically.

Figure 8 shows the theoretical and experimental

Fig. 8 Interfacial frequency response for the first mode (0,1), $A=0.632$, $N_M=188$, and $Z_0=0.30$ mm

Fig. 9 Resonance frequency f_r as a function of aspect ratio A for the first mode (0,1), $N_M=188$, and $Z_0=0.30$ mm

dimensionless maximum interfacial frequency responses $\eta_{\max}/Z_0$ of the mode (0, 1) for $A=0.632$, $N_M=188$, and $Z_0=0.30$ mm. As the forcing frequency f_0 increases, the discrepancy between the theoretical and experimental results increases. This may be caused by the viscous dissipation of the magnetic liquid in the magnetic field.

Figure 9 shows the resonance frequency f_r of the axisymmetric first mode (0, 1) for various values of aspect ratio A of the column. The experimental conditions are $N_M=188$ and $Z_0=0.30$ mm. Both the theoretical and the experimental results indicate that the resonance frequency is a function of aspect ratio A . Generally, the resonance frequency of the first mode is in quantitative agreement with the theory.

7. Conclusions

We have examined theoretical and experimental aspects of the axisymmetric oscillation of a cylindrical magnetic liquid column supported by a magnetic field in the presence of gravity. In particular, we find that the static and dynamic stabilities of the liquid column are enhanced by the uniform magnetic field along the column axis. The surface frequency response shifts to higher frequency regions, and the resonant range widens with increasing applied magnetic field. The resonance frequency increases with an increase in aspect ratio of the liquid column. The experimental results agree well with the theoretical predictions. Future studies will be directed towards a full analysis of the dynamic behavior of an oscillating magnetic liquid column.

References

- (1) Keck, P.H. and Golay, M.J.E., Crystallization of Silicon from a Floating Liquid Zone, *Phys. Rev.*, Vol. 89 (1953), p. 1297.
- (2) Hamacher, H., Fitton, B. and Kingdon, J., *Fluid Sciences and Materials Science in Space*, (Ed. Walter, H.U.) (1987), p. 10-13, Springer-Verlag.
- (3) Fujimori, Y., Space Dream Factory in 21st Century, *Jour. Jpn. Soc. Mech. Eng.* (in Japanese), Vol. 100, No. 941 (1977), p. 405-409.
- (4) Plateau, J.A.F., *Statique Expérimentale et Théorique des Liquides Soumis aux Seules Forces Moléculaires*, Vol. 2 (1873), Gauthier-Villars, Paris.
- (5) Martinez, I., Haynes, J.M. and Langbein, D., *Fluid Sciences and Materials Science in Space*, (Ed. Walter, H. U.) (1982), p. 73, Springer-Verlag.
- (6) Kamiyama, S., Application of Functional Fluids in a Magnetic Field, *Jour. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 96, No. 899 (1993), p. 896-899.
- (7) Ohaba, M., Sawada, T., Sudo, S., Saito, S. and Tanahashi, T., Vibrational Characteristics of a Magnetic Liquid Bridge Subject to a Collinear Magnetic Field, *Nonlinear Electromagnetic Systems*, (1996), p. 410-413, ISO Press.
- (8) Ohaba, M., Saito, S., Sudo, S., Sawada, T. and Tanahashi, T., Magnetic Field Effects on a Laterally Vibrating Magnetic Liquid Column, *Transport Phenomena in Materials Processing and Manufacturing*, ASME HTD-Vol. 336/FED-Vol. 240 (1996), p. 207-213.
- (9) Rosensweig, R. E., *Ferrohydrodynamics*, (1985), p. 125, Cambridge University Press.
- (10) Taktarov, N. G., Disintegration of Streams of Magnetic Fluid, *Magnetohydrodynamics*, Vol. 11 (1975), p. 156-158.