Slip of Newtonian Fluids at Solid Boundary*

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A real fluid does not slip at the surface of a solid boundary. Most experimental results of a Newtonian fluid satisfy with this condition. If a real fluid can slip freely over the surface of a solid boundary, how can we deduce the slip velocity. The purpose of this study is to experimentally clarify fluid slip velocity of Newtonian fluids at the duct wall. Velocity profiles of tap water and 20 wt% glycerin solution flowing in a square duct with a highly water repellent wall were measured by means of a conical hot film anemometer. Consequently, the velocity profile with the slip is measured, and the laminar drag reduction phenomena and the friction factor formula for a square duct with fluid slip at the wall have been obtained.

Key Words: Fluid Mechanics, Newtonian Fluids, Square Duct Flow, Boundary Condition, Velocity Profile, Fluid Slip, Coefficient of Sliding Friction

1. Introduction

A real fluid does not slip at the solid boundary. Most experimental results of a Newtonian fluid satisfy with this condition, i.e., the condition of no-slip at the surface of a solid body. Although fluid slip appears to be important in most non-Newtonian flow problems, it is not important in most Newtonian fluid flow problems. Goldstein⁽¹⁾ reviewed for fluid slip and stated in his publication that Navier⁽²⁾ proposed that slipping at the point of contact with the solid wall is resisted by a force proportional to the relative velocity of the liquid. But the Navier's hypothesis has not been verified by the experiment at present. Although fluid slip is a basic interesting problem in fluid mechanics, fundamental data are still lacking, and it is not sufficiently grasped in the experimental study.

On the other hand, we⁽³⁾ reported a new laminar drag reduction phenomenon of Newtonian fluids in square and rectangular ducts with highly water-repellent wall. Generally speaking, it can be considered

that a highly water-repellent wall reduce the skin friction. The characteristics of the drag reduction of the highly water-repellent wall is not clarified. Thus it needs to research for the fluid slip phenomena at the highly water-repellent wall

The purpose of this study is experimentally to clear fluid slip phenomena of Newtonian fluid at a solid boundary by using highly water-repellent wall. Velocity profiles of tap water and 20 wt% glycerin solution flowing through a square duct were measured by a conical hot film anemometer, and the variation of the velocity profiles were presented in sufficient detail to explain the slip velocity.

As the results, it was shown that the Navier's hypothesis is verified and the friction factor formula substituted the physical constant of the sliding coefficient obtained in this experiment agree well with the experimental data.

Nomenclature

$2a$: width, height of duct
 L : length of duct
 m : hydraulic mean depth ($=ab/a+b$)
 p : pressure
 Re : Reynolds number ($=4m\bar{u}/\nu$)
 u : velocity in the z direction
 $\bar{u}$: mean velocity

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- U : maximum velocity
 u_s : slip velocity
 β : coefficient of sliding friction
 ε : aspect ratio
 λ : friction factor
 μ : viscosity
 ν : kinematic viscosity
 ρ : density
 τ_w : wall shear stress
 x, y, z : rectangular coordinate

2. Experimental Apparatus

Experiments were carried out to measure the velocity profiles of Newtonian fluids in $15 \times 15 \times 2000$ mm square horizontal duct with highly water-repellent wall. A range of Reynolds number is from about 1000 to 1980. It is laminar flow range in the square duct flow.

Figure 1 shows the pipeline system. The system consisted of components: inlet section, test section, outlet section, pump, valve, tank and a velocity measuring device. The inlet lengths of the square ducts were sufficient for the flow to be fully developed over the test lengths. Test fluid is circulated in the pipeline by a pump. The rate of flow was measured by a digital counting scale. Test duct is square duct with highly water-repellent wall. A square duct made of acrylic resin wall was tested for comparison with the exact solution of Newtonian fluid for the non-slip condition. They are 140° and 80° in the contact angle of the highly water-repellent wall and acrylic resin wall, respectively.

Tested fluids were tap water and 20 wt% glycerin solution. Viscosity of these solutions was measured by a Ubbelohde viscometer. Fluid temperature was measured by a thermometer at the outlet of the duct. The constant temperature of the fluid was controlled by an electrical heater for about 25°C .

Figure 2 shows a conical hot film anemometer apparatus. The velocity profiles were obtained by a ϕ 1.5 mm diameter of conical hot film anemometer. The calibration of the anemometer was done by measuring the flow rate and the velocity of laminar flow at the center axis of a smooth circular pipe. The uncertainties were analyzed in the experimental results of the velocity profile. The reported values are the best estimates for the experimental results, and with 95% confidence, the true values are believed to lie within $\pm 4.6\%$ of the presented values for $Re=1550$.

3. Analysis

In steady-state laminar flow of an incompressible fluid in a square duct which the coordinate system is represented in Fig. 3, the Navier-Stokes equation is

written as

$$\mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = \left(\frac{dp}{dz} \right) \quad (1)$$

If fluid slip occurs at the duct wall, the boundary condition for Eq. (1) are

$$u(-a, y) = u_s; u(a, y) = u_s$$

$$u(x, 0) = u_s; u(x, 2a) = u_s$$

where, u_s is the slip velocity at the duct wall.

The exact solution of Navier-Stokes equation for a square duct flow with no-slip condition had been published by Cornish⁽⁴⁾. By referring it, we obtain the velocity profile for it.

$$u = \frac{1}{\mu} \left(-\frac{dp}{dz} \right) \left[\frac{y}{2}(2a-y) - \frac{16a^2}{\pi^3} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^3} \times \sin\left(\frac{n\pi y}{2a}\right) \frac{\cosh\left(\frac{n\pi x}{2a}\right)}{\cosh\left(\frac{n\pi}{2}\right)} \right] + u_s \quad (2)$$

By using the Navier's hypothesis for fluid slip, the

Fig. 1 Experimental pipeline system

Fig. 2 Hot film anemometer

Fig. 3 Coordinate system for square duct

boundary condition is given as follows

$$\tau_w = \beta u_s = \mu \left| \frac{\partial u}{\partial y} \right|_{y=0} \quad (3)$$

where τ_w and β are the wall shear stress and the sliding coefficient, respectively.

By substituting Eq. (2) into Eq. (3), the slip velocity at the wall is calculated.

$$u_s = \frac{a}{\beta} \left(-\frac{dp}{dz} \right) \left[1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \frac{\cosh\left(\frac{n\pi x}{2a}\right)}{\cosh\left(\frac{n\pi}{2}\right)} \right] \quad (4)$$

By substituting Eq. (4) into Eq. (2), we can calculate the volumetric flow rate and the mean velocity. Therefore, the friction factor for laminar rectangular duct flow was obtained as

$$\lambda = \frac{64}{Re} \frac{3/2}{(1+\varepsilon)^2 \left[\left(1 + \frac{3\mu}{\beta a} \right) - \frac{192}{\pi^5} \varepsilon \sum_{n=1,3,5,\dots}^{\infty} \left\{ \frac{1}{n^5} \left(1 + \frac{n^2 \pi^2 \mu}{4\beta a} \right) \tanh\left(\frac{n\pi}{2\varepsilon}\right) \right\} \right]} \quad (5)$$

The physical significance of $\beta = \infty$ is no-slip at the solid boundary, Eq. (5) agree with the exact solution.

4. Experimental Results

Figure 4 shows an example of the velocity profiles of tap water obtained in this study. We can see that fluid slip occurs at the highly water repellent wall duct, as shown by the dotted line. It was seen from the experimental data for $Re = 1000 - 1980$ that the extrapolated slip velocity u_s increases as Reynolds number increases. In Fig. 4, the solid line represents the calculated result using the solution of Eq. (2) under no slip conditions, and the indicated by the dotted line, which represents the experimental data, both correspond to the same constant pressure gradient $(dp/dz) = 10.8 \text{ Pa/m}$.

Because the values of mean velocity for the cases of acrylic resin and the highly water-repellent wall square ducts are 0.116 m/s and 0.094 m/s , respectively, a drag reduction ratio of about 23.4% in this Reynolds number range can be explained by the existence of the slip in the velocity profiles. The value of drag reduction ratio obtained by Watanabe, K. et al.⁽³⁾ agrees well with the calculated value from the experimental result on the velocity profiles obtained in this study.

Fig. 4 Comparison of velocity profiles of acrylic resin and highly water-repellent duct

Figures 5(a) and (b) show the velocity profiles for tap water and 20% glycerin solution in the duct with highly water-repellent wall, respectively. As above mentioned, the slip velocity increases with the increase of Reynolds number. The solid lines, velocity profiles obtained by Eq. (2) were in good agreement with the experimental data.

At a given Reynolds number, we can calculate the wall shear stress by using the pressure drop data. Thus it is to decide the physical constant of the coefficient of sliding friction by Eq. (3).

(a) Tap water
Relationship between wall shear stress and slip velocity

(b) 20 wt% Glycerin solution
Velocity profiles of highly water-repellent duct

Fig. 5

Fig. 6 Relationship between wall shear stress and slip velocity

Fig. 7 Comparison of experimental data with Eq. (4) for slip velocity

Figure 6 shows the relationship between the wall shear stress τ_w and the slip velocity u_s . The wall shear stress is proportional to the slip velocity in this experiment. The slip velocity increases as Reynolds number and viscosity increase.

According to Eq. (3) the coefficient of sliding friction β is given by the slope of line in Fig. 6. The value of the coefficient of sliding friction obtained in this study are $\beta=1.56(\text{Pa}\cdot\text{s}/\text{m})$ and $\beta=2.56(\text{Pa}\cdot\text{s}/\text{m})$ for tap water and 20 wt% glycerin solution, respectively. By using these values, we can calculate the slip velocity and the friction factor from Eqs. (4) and (5).

Figure 7 shows the comparison of calculated results of Eq. (4) and the experimental data for the slip velocity. It is seen the result agree well with the experimental data because the data fit with the correlation line.

Figure 8 shows the comparison between the experimental data and Eq. (2) for the non-dimen-

Fig. 8 Non-dimensional velocity for Eq. (2)

Fig. 9 Friction factor

sional velocity. The solid line and the dotted line were calculated by the analytical solution given by Eq. (2); $u_s=0$ and Eq. (2), respectively.

On the other hand, by substituting the value of β into the analytical results Eq. (5), we obtain laminar drag reduction that the values of the friction factor are $\lambda=44.3/Re$ and $\lambda=44.2/Re$ for tap water and 20 wt% glycerin solution, respectively.

Figure 9 shows the comparison of experimental data with Eq. (5) for friction factor. Experimental data for the friction factor fit to Eq. (5). The drag reduction ratio of tap water was about 22%. Thus it can be considered that the laminar drag reduction is occurred in a highly water-repellent wall duct by fluid slip.

5. Conclusions

Velocity profiles for Newtonian fluids in the square duct with highly water-repellent wall were measured by a conical hot film anemometer. The results are summarized as

(1) The slip of a Newtonian fluid at the duct wall was clarified experimentally, and Navier's hypothesis was verified.

(2) The calculated results for the slip velocity, Eq.(4) agrees well with the experimental results.

(3) The friction factor formula for the rectangular duct flow, Eq.(5) agree well with the experimental data quantitatively by substituting the sliding coefficient obtained in this study into it.

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