Stability Analysis of Supersonic Cascade Flow by Actuator Disk Methods*

Yukiharu IWAMOTO**, Kazuhiko YOKOTA***
and Yoshinobu TSUJIMOTO***

This paper presents the results of a two-dimensional stability analysis on supersonic flow relative to a cascade, using actuator disk methods. The results show the possibility of existence of two types of destabilizing mode in the supersonic cascade flow. One mode corresponds to conventional rotating stall in subsonic cascade flows. This mode can be unstable both in subsonic and supersonic regions, and its propagation velocity ratio (absolute propagation velocity/rotor speed) is smaller than unity. The other mode is a newly found mode unique to supersonic cascade flows. This mode can become unstable only in the supersonic region, and its propagation velocity ratio is for the most cases larger than unity. Both modes become unstable due to the positive slope of the characteristic curves caused by flow compressibility as in the case of, for example, a shock wave.

Key Words: Compressor, Stability, Supersonic Flow, Cascade, Actuator Disk Method, Shock Wave

1. Introduction

High-performance gas turbines (or jet engines) must employ compression systems that provide high pressure ratio with smaller total number of stages. For this requirement, the rotational speed of recent gas turbines has been increasing.

Accordingly, it is becoming more common that the relative inflow to the cascade is supersonic near the tip of fans or the first stage of compressors. At higher speeds, it is well-known that stable operating range becomes extremely narrow due to surge and choke. Under this circumstances, it is important to understand the characteristics of supersonic compressors in the off-design region as well as at the design point. For this purpose, many studies have been done,

for example, about unstated flutter^{(1),(2)}.

Freeman et al.⁽³⁾ investigated the steady characteristics of supersonic cascades by means of an analytical method using one dimensional flow model. It is indicated that the performance curve of the supersonic cascade has a positively sloped region, which is enlarged at higher rotational speeds. The positive slope of the performance curve for the subsonic cascades is well-known to cause flow instabilities, such as surge or rotating stall. Therefore, the results of Freeman et al. indicate the possibilities of the flow instabilities in supersonic cascades similar to those in subsonic ones. The present study investigates the flow instabilities in supersonic cascades with subsonic axial flow by actuator disk methods. Especially, we focus on the possibility of the flow instabilities which might be associated with shock waves.

2. Nomenclature

Variables are denoted in a frame running with a cascade if not stated otherwise.

a : sound speed

i : imaginary unit

k : wave number in y -direction

* Received 16th January, 1998. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 63, No. 612, B (1997), p. 2667-2675 (Received 20th June, 1996)

** Graduate Student, Graduate School of Engineering Science, Osaka University, 1-3 Machikaneyama, Toyonaka 560-8531, Japan

*** Graduate School of Engineering Science, Osaka University

- M : Mach number
 p : static pressure
 t : time
 u, v : velocities in x -, y -direction
 x, y : cartesian coordinates
 κ : specific heat ratio
 λ : wave number in x -direction
 ρ : density
 χ : stagger angle
 ω : complex angular frequency of fluctuation

Subscripts

- abs : absolute frame
 D : downstream of cascade
 L : just upstream of shock wave
 im : imaginary part
 re : real part
 rel : relative frame
 t : stagnation state
 U : upstream of cascade

Superscripts

- $\sim$: disturbance component
 $'$: disturbance amplitude

3. Theoretical Model - Actuator Disk Method

3.1 Cascade model

When the supersonic cascade with subsonic axial flow is unstated, detached shock waves occur just upstream of the blade leading edges. Figure 1(a) shows the schematic of an unstated supersonic flat plate cascade. In actuator disk methods, this cascade is modeled by an actuator disk with infinitesimal thickness as shown in Fig. 1(b). In the present study, the actuator disk is placed on the y -axis, i.e., $x=0$, as shown in Fig. 1(b). The boundary conditions are applied at the inlet and outlet of the actuator disk. We consider two actuator disk models based on two different relations connecting the conditions at the inlet and outlet of the actuator disk.

3.2 Steady components

Generally, there exist the following relations between upstream and downstream flow variables. The continuity equation given by

Fig. 1 Unstarted supersonic cascade and actuator disk

$$\rho_U u_U = \rho_D u_D, \quad (1)$$

the energy equation (the total enthalpy conservation equation) given by

$$\frac{\kappa}{\kappa-1} \frac{p_U}{\rho_U} + \frac{1}{2} (u_U^2 + v_U^2) = \frac{\kappa}{\kappa-1} \frac{p_D}{\rho_D} + \frac{1}{2} (u_D^2 + v_D^2), \quad (2)$$

and the Kutta's condition given by

$$u_D \sin \chi - v_D \cos \chi = 0. \quad (3)$$

For compressible flows, another condition is needed to determine the relationship between upstream and downstream variables. In the first model (Model I), we use the conservation equation of the momentum in the blade chord direction proposed by Freeman et al.⁽¹⁾:

$$p_U \cos \chi + \rho_U u_U (u_U \cos \chi + v_U \sin \chi) = p_D \cos \chi + \rho_D u_D (u_D \cos \chi + v_D \sin \chi). \quad (4)$$

From Eqs. (1)-(4), we obtain two Mach numbers. These correspond to two cases with and without a shock wave. By selecting the lower Mach number solution, we obtain the solution that represents the flow field downstream of the supersonic cascade with shock waves. Freeman et al. note that this model gives a good prediction of the supersonic cascade characteristics.

Model I implicitly includes a incidence loss in addition to the loss caused by shocks. In the present study, we focus especially on the flow instabilities due to the loss produced by a shock wave. Therefore, we also examine the model (Model II) which includes no loss except that produced by the shock wave. Model II consists of the following equations in addition to Eqs. (1)-(3):

$$p_U \left(1 + \frac{\kappa-1}{2} M_U^2 \right)^{\kappa/(\kappa-1)} = p_L \left(1 + \frac{\kappa-1}{2} M_L^2 \right)^{\kappa/(\kappa-1)}, \quad (5)$$

$$\frac{p_D}{p_L} = \frac{2\kappa}{\kappa-1} M_L^2 - \frac{\kappa-1}{\kappa+1}. \quad (6)$$

The static pressure p_L and Mach number M_L represent those when the flow isentropically turns into the direction of the blade chord, respectively. Equation (6) is the Rankine-Hugoniot equation. Thus, Model II assumes that the flow into the cascade is isentropic and then a normal shock occurs in a pitch.

From Eqs. (1)-(3) and (5), we get two solutions of M_L corresponding to supersonic and subsonic flows after the isentropic turning at the inlet. In the present study, we adopt the supersonic solution ($M_L > 1$) since we assume normal shocks in the cascade.

3.3 Disturbance components

If the amplitudes of the disturbance components are assumed to be infinitesimal, the infinitesimal disturbance components should satisfy the linearized versions of the continuity equation, the two-dimensional Euler equations and the energy equation given

$$\frac{\partial \hat{u}}{\partial t} + u \frac{\partial \hat{u}}{\partial x} + v \frac{\partial \hat{u}}{\partial y} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial x}, \quad (8)$$

$$\frac{\partial \hat{v}}{\partial t} + u \frac{\partial \hat{v}}{\partial x} + v \frac{\partial \hat{v}}{\partial y} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial y}, \quad (9)$$

$$\frac{1}{p}\left(\frac{\partial \bar{p}}{\partial t}+u \frac{\partial \bar{p}}{\partial x}+v \frac{\partial \bar{p}}{\partial y}\right)-\frac{\kappa}{\rho}\left(\frac{\partial \bar{\rho}}{\partial t}+u \frac{\partial \bar{\rho}}{\partial x}+v \frac{\partial \bar{\rho}}{\partial y}\right)=0. \quad (10)$$

$$\exp[i(\omega t - ky) + \lambda x]. \quad (11)$$
$$\exp[-\omega_{\text{im}}t + \lambda_{\text{re}}x] \exp[i(\omega_{\text{re}}t - k_{\text{re}}y + \lambda_{\text{im}}x)]. \quad (12)$$

By substituting the disturbance components with forms of Eq.(11) into Eqs.(7)-(10), we obtain a set of homogeneous first-order simultaneous equations for the infinitesimal amplitudes. For the existence of non-trivial solutions, the determinant of the coefficient matrix must equal to zero. This leads to an algebraic fourth-order equation in terms of λ . Solving the equation with respect to λ , we obtain the following expression of the wave number λ in the x -direction

$$\lambda_{1,2} = \frac{1}{a^2 - u^2} [iu(\omega - kv) \pm a\sqrt{(a^2 - u^2)k^2 - (\omega - kv)^2}], \quad (13)$$

$$\lambda_3 = -\frac{i}{u}(\omega - kv). \quad (14)$$

With the wave number λ_3 , the momentum equations (8) and (9) show that the pressure disturbance becomes zero. This wave number λ_3 represents the disturbances of vorticity and entropy. On the other hand, for the wave numbers $\lambda_{1,2}$, the disturbances appear in pressure and the corresponding velocity field becomes irrotational. Two values of $\lambda_{1,2}$ correspond to the two values of the square root in Eq. (13). One of the two values of $\lambda_{1,2}$ represents the upstream

Fig. 2 Relationship between circular wave front and travelling single disturbance source

First, we consider the case in which the attenuation rate ω_{lm} equals to zero. Focusing on a single disturbance source, the circular wave front of the disturbance from the source propagates with the sound speed a with being convected with the main flow velocity (u, v) . On the other hand, the single disturbance source itself travels on the cascade in the y -direction at the velocity ω_{re}/k . Figure 2 shows the schematic of this state. The total disturbances in the flow field is obtained by superposing all the circular waves generated from all the disturbance sources on the cascade. When the axial, i.e. x -directional, flow velocity is subsonic, a circular wave front has two intersections R1 and R2 with y -axis. The case in which the source on the cascade moves with the same velocity as these intersections is called “resonance”⁽⁴⁾. Under resonance condition, the angular velocity ω_{re} is given by setting the radical sign in Eq.(13) to zero. The cases in which the source moves faster/slower than two intersections R1 and R2 are called super-resonance/sub-resonance, respectively. For super-resonance, the Mach wedge with its vertex at the disturbance source is generated as shown in Fig.2. For $\omega_{\text{lm}}=0$, $\lambda_{1,2}$ become purely imaginary and the slopes of the two sides of the Mach wedge equals to the ones of the isophase surfaces of Eq. (13)⁽⁶⁾, i.e. $\text{Im}[\lambda_{1,2}/k]$, where $\text{Im}[\]$ represents the imaginary part. Thus the two sides of the Mach wedge correspond to the two disturbance propagation waves. One of the two waves extends upstream of the cascade, and the other downstream. Since the disturbance source is located on the cascade, we can choose the upstream and downstream extending wave for the upstream and downstream flows, respectively. Based on this

Table 1 Relationship between $\lambda_{1,2}$ and their amplitude
(amplification or attenuation of disturbance)

	Amplifying ($\omega < 0$) im	Neutrally stable ($\omega = 0$) im	Attenuating ($\omega > 0$) im
Super-resonance			
Sub-resonance			

Note :

Vertical arrows mean time-amplifying or attenuating state.
Horizontal arrows mean the propagating direction of the isophase surface.

considerations, we can choose appropriate waves from the magnitudes of $\text{Im}[\lambda_{1,2}/k]$. For sub-resonance, the Mach wedge is not generated. One of the real parts of $\lambda_{1,2}$ is positive and the other negative. We should assign $\lambda_{1,2}$ so that the disturbances remain finite at $x = \pm\infty$, i.e. choose the wave numbers $\lambda_{1,2}$ of which real parts are positive and negative for the upstream and the downstream regions, respectively.

The above method for $\omega_{im}=0$ can be extended to the case of $\omega_{im} \neq 0$. It should be noted that, although the angle formed by the two isophase surfaces becomes smaller than the Mach wedge vertex angle for the case with $\omega_{im} \neq 0$ at super-resonance, we can choose the appropriate wave numbers $\lambda_{1,2}$ based on the magnitudes of $\text{Im}[\lambda_{1,2}/k]$ in the same manner as for $\omega_{im}=0$. With this method, the two wave numbers approach the ones for the $\omega_{im}=0$ in the limit of $\omega_{im} \rightarrow 0$.

Table 1 illustrates the behaviors of the disturbances with $\lambda_{1,2}$ chosen from the method mentioned above. Table 1 indicates that the amplitudes with approach zero at $x = \pm\infty$ for sub-resonance, but not always for super-resonance. It is because all the disturbances from the sources on the cascade cancel out with one another for sub-resonance, but the cancellation does not occur for super-resonance. In the present study, we denote λ_1 for the upstream wave number and λ_2 for the downstream one.

It is assumed that the upstream region is irrotational. Accordingly, only the disturbance with λ_{1U} exists in the upstream of the cascade. On the other hand, the downstream region has the entropy disturbance with the vorticity shed from the cascade. Accordingly, there exist the disturbances with λ_{2D} and λ_{3D} in the downstream of the cascade. The ratios of the disturbances, $\bar{\rho}$, $\bar{u}$, $\bar{v}$, $\bar{p}$, with each λ are determined as an eigenvector of homogeneous simultaneous linear equations (7)-(10). Then we can choose $\bar{v}_{1U}$ with λ_{1U} in the upstream region, $\bar{v}_{2D}$ with λ_{2D} , $\bar{v}_{3D}$

and $\bar{\rho}_{3D}$ with λ_{3D} in the downstream region as unknown disturbances. These four unknowns are determined by the following four conditions connecting the upstream and downstream flow fields.

Here, we consider the cases in which the conditions of [Eqs. (1)-(4)] or [Eqs. (1)-(3) and (5), (6)] can be applied quasi-statically.

From Eq. (1), the linearized continuity equation for the infinitesimal disturbances is given by

$$(\bar{\rho}_U u_U + \rho_U \bar{u}_U) - (\bar{\rho}_D u_D + \rho_D \bar{u}_D) = 0. \quad (15)$$

From Eq. (2), the linearized total enthalpy conservation equation for the infinitesimal disturbances is given by

$$\left\{ \frac{\kappa}{\kappa-1} \frac{\bar{p}_U}{\rho_U} \left(\frac{\bar{p}_U}{\bar{p}_U} - \frac{\bar{\rho}_U}{\rho_U} \right) + u_U \bar{u}_U + v_U \bar{v}_U \right\} - \left\{ \frac{\kappa}{\kappa-1} \frac{\bar{p}_D}{\rho_D} \left(\frac{\bar{p}_D}{\bar{p}_D} - \frac{\bar{\rho}_D}{\rho_D} \right) + u_D \bar{u}_D + v_D \bar{v}_D \right\} = 0. \quad (16)$$

From Eq. (3), the linearized Kutta's condition for the infinitesimal disturbances is given by

$$\bar{u}_D \cos \chi - \bar{v}_D \sin \chi = 0. \quad (17)$$

For Model I, the linearized version of Eq. (4) is applied:

$$[\bar{p}_U + (\bar{\rho}_U u_U + \rho_U \bar{u}_U)(u_U + v_U \tan \chi) + \rho_U u_U (\bar{u}_U + \bar{v}_U \tan \chi)] - [\bar{p}_D + (\bar{\rho}_D u_D + \rho_D \bar{u}_D)(u_D + v_D \tan \chi) + \rho_D u_D (\bar{u}_D + \bar{v}_D \tan \chi)] = 0. \quad (18)$$

For Model II, the linearized version of Eq. (5) and (6) is applied:

$$\frac{1}{2} \frac{u_U^2 + v_U^2}{\rho_U} \bar{\rho}_U + u_U \bar{u}_U + v_U \bar{v}_U + \frac{1 - \frac{1}{2} M_U^2}{\rho_U} \bar{p}_U - \frac{1 - \frac{\kappa-1}{2} M_D^2}{\kappa M_D^2 - \frac{\kappa-1}{2}} \left[(2M_D^2 - 1) \left(\frac{1}{2} \frac{a_D^2}{\rho_D} \bar{\rho}_D + \frac{u_D \bar{u}_D + v_D \bar{v}_D}{M_D^2} \right) + \frac{1}{2} \frac{1}{\rho_D} \bar{p}_D \right] = 0. \quad (19)$$

Then [Eqs. (15)-(18)] or [Eqs. (15)-(17), (19)] can be

rewritten as a set of homogeneous linear simultaneous equations for the disturbance amplitudes, v'_{1U} , v'_{2D} , v'_{3D} and ρ'_{3D} , given by

$$[M] \begin{bmatrix} v'_{1U} \\ v'_{2D} \\ v'_{3D} \\ \rho'_{3D} \end{bmatrix} = [0], \quad (20)$$

where $[M]$ is the coefficient matrix. For the existence of non-trivial solutions, the determinant of the coefficient matrix should equal to zero, i.e.

$$\det[M] = 0. \quad (21)$$

This is an algebraic equation in terms of ω/k . Since the steady components of the inflow is irrotational, the rotor speed corresponds to the y -directional relative flow velocity v_U . Therefore, the propagation velocity ratio of the disturbances, i.e. the ratio of the propagation velocity to the rotor speed, is given by $1 - \omega_{re}/(kv_U)$. On the other hand, ω_{im}/k represents the temporal attenuation rate of the disturbances. Therefore, if $\omega_{im}/k > 0$, the flow is stable because the disturbances attenuate temporally, but if $\omega_{im}/k < 0$, the flow is unstable because the disturbances amplify temporally.

Equation (21) includes λ_{1U}/k and λ_{2D}/k with square root of the equation of ω/k as shown in Eq. (13). By transposing and multiplying by itself properly, we can eliminate the square root. As a result, we obtain a 12th order polynomial equation of ω/k . To solve this 12th order equation, we used the DKA method and obtained twelve solutions numerically. These twelve solutions include solutions not satisfying Eq. (21), which appear from the process of transposing and multiplication by itself Eq. (21). True solutions should be identified by substituting the twelve solutions into Eq. (21). Based on this method, we can obtain the complete set of the solutions of Eq. (21).

4. Results and Discussions

4.1 Steady performance curve

First of all, we calculated the steady components. We set specific heat ratio γ and stagger χ to be 1.4 and 60 [deg.], respectively.

Figure 3 shows the steady performance map for the flat plate rotor cascade based on Model I. The horizontal and the vertical axes indicate the axial flow velocity ratio to the rotational speed and the pressure ratio of the downstream static pressure to the upstream absolute total pressure, respectively. Figure 3 shows that the positively sloped region of the performance curve is enlarged with the increase of the rotational speed v_U .

As mentioned in section 3.1, Model I includes the implicit incidence loss. Therefore, the positive slope is found also in the region where the rotational speed v_U (inflow Mach number) is very small. To compare

Fig. 3 Performance map for Model I (flat plate cascade, $\chi=60^\circ$)

Fig. 4 Performance map for Model II (flat plate cascade, $\chi=60^\circ$)

with the model which includes only the loss produced by the shock, we also calculated the steady components by Model II.

Figure 4 shows the steady performance map for the flat plate rotor cascade based on Model II. The positively sloped region in Fig. 4 is produced by the shock because there is no loss other than that produced by the shock. Figure 4 also shows that the slopes of the performance curves are positive even in the region where the inlet flow relative to the cascade is subsonic, $v_U/a_{Uabs} < 1$. This is because the supersonic value was selected for M_L in Eq. (6). Hence, the present results are valid only for the cases with larger inflow velocity v_U .

The steady performance maps of Model I and II in Figs. 3 and 4 show that the flow compressibility and the shock produce and enlarge the positively sloped regions.

4.2 Stability analysis

We classified all solutions for ω/k obtained from

Eq. (21) for various cases of u_U/v_U and v_U/a_{tUabs} from the continuity of the values of ω/k . Each solution shows the frequency and the attenuation rate of the corresponding mode. We found 8 and 7 disturbance modes for Model I and II, respectively. Most of them are stable or neutrally stable in all operating conditions. It was found two and one modes become unstable under some conditions for Model I and II, respectively. We focus on them for the purpose of studying the flow instabilities.

Figure 5 shows the stability map for Model I of a mode named mode A on the steady performance map. Mode A exists in the left-side region of the thick solid curve. On the thick broken curve, mode A becomes neutrally stable. Mode A becomes stable in the gray region indicated by A(S), and becomes unstable in the hatched region indicated by A(U).

Figure 5 shows that the performance curves are positively sloped in the region where mode A is unstable. It should be noted that mode A becomes unstable even in the region where there exists no shock waves ($M_{Urel} < 1$) and the rotational speed is low, i.e. v_U/a_{tUabs} is small.

Figures 6 and 7 show the propagation velocity ratio, $1 - \omega_{re}/(kv_U)$, and the attenuation rate, ω_{im}/k , of mode A for $v_U/a_{tUabs} = 0.7$ and 1.3, respectively. These figures indicate that the propagation velocity ratio of mode A is smaller than unity when mode A is unstable. The same tendency is generally observed. On the other hand, the actuator disk analysis of the incompressible flow shows that the positive slope of the performance curve due to the loss causes the rotating stall, of which propagation velocity ratio is smaller than unity⁽⁶⁾. Based on these results, mode A corresponds to the conventional rotating stall also found in the incompressible flow.

Figure 8 shows the stability map for Model I of a mode named mode B. Figure 8 shows that the perfor-

mance curves are positively sloped in the region where mode B is unstable. Comparing Figs. 5 and 8, we can see that there exists the region where both modes A

Fig. 6 Propagation velocity ratio and attenuation rate of mode A ($v_U/a_{tUabs} = 0.7$)

Fig. 7 Propagation velocity ratio and attenuation rate of mode A ($v_U/a_{tUabs} = 1.3$)

Fig. 5 Stability map of mode A for Model I

Fig. 8 Stability map of mode B for Model I

and B occur. Therefore, we can say that they are mutually independent.

Mode B occurs in the region where the rotational

Fig. 9 Propagation velocity ratio and attenuation rate of mode B ($v_U/a_{tUabs}=1.3$)

Fig. 10 Propagation velocity ratio and attenuation rate of mode B ($v_U/a_{tUabs}=1.4$)

Fig. 11 Stability map of mode C for Model II

speed v_U is high. When the inflow is supersonic and the incidence angle equals to zero, i.e. $u_U/v_U = \cot 60^\circ$, the shock wave in the steady flow becomes the normal shock. Even under this condition, there exists mode B. Therefore, we can say that mode B is associated with the shock wave.

Figures 9 and 10 show the propagation velocity ratio and the attenuation rate of mode B for $v_U/a_{tUabs}=1.3$ and 1.4, respectively. These figures indicate that the propagation velocity ratio of mode B is mostly larger than unity where mode B is unstable, except in the low flow rate region.

As mentioned in section 3.2, Model I implicitly includes the incidence loss, it is impossible to conclude that mode B associates with only the loss produced by the shock wave. Therefore, we examine the results for Model II in the same manner as Model I.

Figure 11 shows the stability map for Model II of a mode named mode C. Figure 11 shows that the

Fig. 12 Propagation velocity ratio and attenuation rate of mode C ($v_U/a_{tUabs}=1.0$)

Fig. 13 Propagation velocity ratio and attenuation rate of mode C ($v_U/a_{tUabs}=1.3$)

performance curves are positively sloped in the region where mode C is unstable. Since only the loss produced by the normal shock is included in Model II, mode C is considered as the instability associated with the normal shock.

Figures 12 and 13 show the propagation velocity ratio and the attenuation rate of mode C for $v_u/a_{t_{abs}} = 1.0$ and 1.3, respectively. These figures indicate that the propagation velocity ratio of mode C is always larger than unity where mode C is unstable. This tendency is the same as the one for mode B at high flow rates as shown in Figs. 9 and 10.

When the inflow is supersonic and the incidence angle equals to zero, i.e. $u_u/v_u = \cot 60^\circ$, Model I coincides with Model II. Figures 9 and 13 show that modes B and C agree under incidence free conditions.

The above observations suggest that modes B and C represent the same type of instability. They are considered to be associated with the loss caused by shocks.

5. Conclusions

From the present analysis, the following conclusions are obtained.

(1) The flow compressibility and the shock wave produce and enlarge the positively sloped region of the performance curves.

(2) From the analysis of Model I, it was found that the conventional rotating stall also occurs at supersonic flows. The slope of the performance curve is positive in the onset region, and its propagation velocity ratio is smaller than unity.

(3) From the analysis of Models I and II, it was found that a flow instability associated with the shock wave may also occur. This instability occurs at higher supersonic speeds. The slope of the performance curve is positive in the onset region. Its propagation velocity ratio is larger than unity except in the low flow rate region for Model I, and always larger than unity for Model II.

References

- (1) Doi, H. and Kaji, S., Analysis of Unstarted Supersonic Flutter in Cascade by Semiactuator Disk Theory, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 59, No. 561, B (1993), p. 1573-1579.
- (2) Kobayashi, H., Effects of Shock Waves on Aerodynamic Instability of Annular Cascade Oscillation in a Transonic Flow, Trans. ASME, J. Turbo., Vol. 111, (1989), p. 222-230.
- (3) Freeman, C. and Cumpsty, N. A., A Method for the Prediction of Supersonic Compressor Blade Performance, ASME Paper No. 89-GT-326 (1989).
- (4) Kaji, S. and Okazaki, T., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 35, No. 273, II (1969), p. 1008-1020.
- (5) Kaji, S. and Okazaki, T., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 38, No. 309, II (1972), p. 1023-1033.
- (6) Tsujimoto, Y., Kamijo, K. and Yoshida, Y., A Theoretical Analysis of Rotating Cavitation in Inducers, Trans. ASME, J. Fluids Eng., Vol. 115, (1993), p. 135-141.