Analytical Method for Stirling Engines and Coolers*

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For aiding the design and improving the performance of various Stirling engines and coolers, a Stirling engine thermodynamic and mechanical analysis, SETMA, has been developed and examined. A simple SETMA, whose working space is divided into five control volumes, has been developed to enable easy, accurate prediction of the performance of Stirling machines. In this paper, the SETMA and fundamental equations are given in detail. Several examinations of the applicability of SETMA are discussed, comparing the calculated and experimental results of two types of actual engines. The calculation accuracy can be markedly improved by optimizing the friction-loss factors and heat transfer coefficients and carefully scanning of the dimensions of the working space. For calculations of the thermodynamic properties used by SETMA, it was clarified that the SRK equation of state is more reliable than the ideal-gas equation when analyzing higher-pressure-charged Stirling machines.

Key Words: Stirling Engine, Stirling Cooler, Simulation, Thermodynamic Analysis, Mechanical Analysis, Five Control Volumes, Equation of State

1. Introduction

Beginning with the mid-twentieth century, a wide variety of applicable Stirling engines have been constructed. Their superior advantages have been demonstrated by various R & D projects. The major advantages are high thermal efficiency of 30 to 40%, preferable exhaust characteristics, low noise and vibration, and flexibility of heat sources⁽¹⁾. Stirling engines promise to play an important role in the alleviation of current global environmental and energy problems. Also, Stirling coolers have attracted attention as one of alternative refrigeration systems⁽²⁾. Generally, the Stirling coolers use helium or nitrogen as the working fluid and can realize high coefficients of performance (COP) without fluorocarbon working fluids. Although the fluorocarbons have a long history as safe and stable working fluids in conventional vapor compression systems (refrigerators and heat pumps), some of the fluorocarbons have serious drawbacks concerning ozone depletion and the

global warming effect. The production of fluorocarbons will continue to be limited. To develop high-performance and environmentally-friendly Stirling cycle machines, as for other cycles, requires a broad engineering knowledge. Therefore, useful design methods for the Stirling machines must be established.

In the case of designing and developing a Stirling machine, it is required to optimize the thermodynamic cycle in the working space. However, it is not possible to construct any actual machine without irreversible losses. The cycle work and losses are intricately influenced by the dimensions of the cylinders, the heater, the cooler, the regenerator, and the manifolds to connect these elements of the machine. It is very important to estimate the influences accurately at the early stages of the design process. A reliable mathematical analysis which can predict the machine performance and the losses from the machine dimensions and specifications, is a useful design method.

Furthermore, the analysis can suggest some important improvements for the machine which cannot be confirmed by simple empirical procedures. For example, an improvement of the heat exchange system must take into consideration the complicated interaction of heat transfer characteristics, pressure

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losses, and dead volumes in the heat exchange system. Generally, for an optimal modification process, it is recommended to evaluate the machine both experimentally and analytically. A mechanical analysis which is combined with the thermodynamic (working-space) analysis, is also useful to develop the actual machine with higher mechanical efficiency. Due to each force loading of the mechanical parts, the instantaneous mechanical loss is changed dramatically. To modify the mechanical parts efficiently, the mechanical losses must be calculated with the analyzed pressure curve in the working space which is loading the pistons.

For the purpose of optimizing the Stirling cycles, many types of mathematical analysis models have been proposed⁽⁹⁾. Among them, nodal analysis models are those which divide the working space into several control volumes and can solve a mathematical model which theoretically represents the phenomenon in each control volume. They are more suitable to analyze the Stirling machines with higher accuracy. On the other hand, the nodal analysis models have notable problems due to its minuteness. If the number of the divided volumes is too large, the analysis model becomes complicated and requires a lot of calculation time. The minute modeling of the machines may be an obstruction to finding an optimum dimension of the machine parts. Many nodal models have been published, e.g., GLIMPS⁽⁴⁾, HFAST⁽⁵⁾, MS*2⁽⁶⁾ and the models developed by Azetsu⁽⁷⁾, Yamashita⁽⁸⁾, Sekiya et al.⁽⁹⁾ Most of the proposed nodal models utilize some assumptions to simplify the fundamental equations and to determine some coefficients to represent the actual phenomenon, like as friction-loss factors and heat transfer coefficients. For these models, however, comparisons with experimental data are very limited and the applicability is frequently questionable⁽¹⁰⁾. Also, they do not support direct mechanical analysis.

For aiding design and improving the performance of various Stirling machines, a Stirling Engine Thermodynamic and Mechanical Analysis, SETMA, has been developed and examined since 1982. In this paper, SETMA and the fundamental equations are introduced in detail. The applicability of SETMA is discussed comparing the calculated and experimental results on two types of actual engines. Furthermore, the calculations of the friction factors and heat transfer coefficients and the influence of the thermodynamic information of the working fluids on SETMA are described.

2. Stirling Engine Thermodynamic Analysis Model, SETMA

The present Stirling Engine Thermodynamic and

Fig. 1 Analysis model for Stirling engine

Mechanical Analysis, SETMA, can deal with the thermodynamic analysis for the working space and the mechanical analysis for the driving system as shown in Fig. 1. The newest version of SETMA was written in the ANSI C language and can calculate one analysis with 5 minutes on a 386 PC machine (16 MHz) with a 387 co-processor.

2.1 Thermodynamic analysis module

The source code of SETMA must be constructed not only as a high precision model with adequate fundamental equations but also as an easy model to handle with a simple algorithm. For the current version, the working space is divided into five control volumes: expansion space, heater, regenerator, cooler, compression space. For the fundamental equations of the control volumes, a one dimensional model proposed by Yamashita⁽⁸⁾ was adopted and was modified. It is assumed that the outer-wall temperatures of the cylinders, heater, and cooler are kept constant. The flow pattern of the working fluid is treated as one-dimensional coaxial flow. For the regenerator, the outer wall is insulated sufficiently and a heat transfer between the working fluid and a heat-storage matrix is the only path considered. No leakage of the working fluid from the well-designed piston-ring system is considered. The modified equations for the i -th control volume are derived as follows.

(1) Energy equation

$$c_v d(m_i T_i) / dT + c_p (w_{i*} T_{i*} - w_{(i+1)*} T_{(i+1)*}) = A_{si} \alpha_i (T_{wi} - T_i) - P_i dV_i / dt \quad (i=1, 2, 4, 5) \quad (1)$$

where

A_s : Wetted surface area,

c_p : Isobaric specific heat capacity

c_v : Isochoric specific heat capacity

m : Mass of the working fluid
 P : Pressure
 t : Time
 T : Temperature
 T_w : Wall temperature
 V : Volume
 w : Mass flow rate
 α : Heat transfer coefficient

Subscript

i : Control volumes referred to Fig. 1
 i^* : Boundary between control volumes (1*, 2*, 3*, 4* in Fig. 1).

In the case of neglecting the leakage of the working fluid from the expansion and compression spaces to the piston backspaces, w_0 and w_5 become zero.

(2) Momentum equation

$$|P_i - P_{i+1}| = f_{(i-1)*} w_{(i-1)*}^2 (l_j/2) / (2r_{Hj} < \rho_{(i-1)*} > A_j^2) \quad (i=2, 3, 4, 5) \quad (2)$$

where

A : Cross sectional area of flow pass
 f : Fluid friction factor
 r_H : Hydraulic radius
 l : Heat exchanger length
 R : Gas constant
 ρ : Density, $\langle \rho_{(i-1)*} \rangle = (P_{i-1} + P_i) / (2RT_{wj})$

Subscript

$j=2$ ($i=2, 3$), $j=4$ ($i=4, 5$).

(3) Rule of mass conservation

$$\sum_{i=1}^5 m_i = m_o \quad (3)$$

m_o : Total working gas mass.

(4) Equation of state of a ideal gas

$$P_i V_i = m_i R T_i \quad (4)$$

The temperature of the working fluid in the regenerator, T_3 , is defined by the following equation with the temperatures of the working fluid in the regenerator ends, T_{23} and T_{34} . From the experimental results, the temperature profile in the regenerator is supposed to be linear.

$$T_3 = (T_2 + T_{23} - T_{34} - T_4) / [2 \ln\{(T_2 + T_{23}) / (T_{34} + T_4)\}]$$

(5) Continuity equation

$$dm_i/dt = w_{(i-1)*} - w_{i*} \quad (5)$$

T_{i*} in Eq. (1) is determined by the flow direction of the working fluid.

$$T_{i*} = T_i \quad (w_{i*} \geq 0), \quad T_{i*} = T_{i+1} \quad (w_{i*} < 0) \quad (i=1, 2, 3, 4)$$

For the temperatures of the working fluid passing through the regenerator ends, T_{2*} and T_{3*} ,

$$T_{2*} = T_{23} \quad (w_{2*} < 0), \quad T_{3*} = T_{34} \quad (w_{3*} \geq 0)$$

As to for the temperatures of the working fluids in the regenerator T_{23} and T_{34} ,

$$T_{i,i+1} = \langle T_{mi} \rangle + \Delta T_{mi} + \Delta T_{mgi} \quad (i=2, 3)$$

where $\langle T_{mi} \rangle$, ΔT_{mi} , ΔT_{mgi} represent the mean temper-

ature of the matrix at the regenerator ends, the temperature change of the matrix, and the temperature difference between the matrix surface and the working fluid, respectively. ΔT_{mi} is given by the following experimental correlation⁽¹¹⁾ and was modified based on experimental data of a series of this work.

$$\Delta T_{mi} = w_{i*} c_v (T_{w2} - T_{w3}) / (2.5 n m_m c_m) \quad (6)$$

where,

c_m : Matrix specific heat capacity
 m_m : Matrix mass
 n : Engine speed.

The energy equation applied to the regenerator gives ΔT_{mgi} as follows.

$$c_{mi} m_{mi} (dT_{mi}/dt) = A_{mi} \alpha_{mi} (T_i - T_{mi}) = A_{mi} \alpha_{mi} \Delta T_{mgi} \quad (7)$$

In the case of decreasing the calculation step sufficiently, $\Delta t \rightarrow 0$, the temperature difference between T_{mi} at $t - \Delta t$ and at t is supposed to be dT_{mi} .

$$\Delta T_{mgi} = c_{mi} m_{mi} (dT_{mi}/dt) / (A_{mi} \alpha_{mi}) \quad (8)$$

$\langle T_{mi} \rangle$ is required to obtain an instantaneous average value of T_{mi} , can be calculated from the average of T_{mi} during the former cycle. For the initial value of $\langle T_{m2} \rangle$ and $\langle T_{m3} \rangle$, the heater-wall and cooler-wall temperatures are used, respectively.

To solve this model, Eqs. (1)-(5) were non-dimensionalized and rearranged to ordinary differential equations corresponding to four energy equations and four momentum equations. For the purpose of obtaining stable results, the Runge-Kutta-Gill method is adopted and is reckoned with a calculation interval of 0.5 degree. As the initial values of the calculation, results of the Schmidt-cycle model (isothermal cylinder model)⁽¹²⁾ are used.

For the friction-loss factors and heat transfer coefficients which are used in the fundamental equations, the following equations were established based on the experimental results. Generally, in a circular tube, the inner-tube diameter is used as the hydraulic diameter, D_H , and the circle diameter which has the same area of the wetted surface area in a double-tube or complicated-shape path is used. For D_H of the matrix, the diameter of the circle corresponding to the distance around an open area of the screen mesh, or the diameter of the wire is used.

(1) Friction factor in single and double tube^{(13),(14)}
 Laminar ($Re \leq 2100$) :

$$f = 16/Re$$

Transient ($2100 < Re \leq 4000$) :

$$f = f_{2100} + (f_{4000} - f_{2100})(Re - 2100)/1900$$

Turbulent ($Re > 4000$) :

$$f = [1 + (D_H/l)^{0.5}] (0.0014 + 0.025 Re^{-0.32}) \quad (9)$$

where Re represents Reynolds number, and f_{2100} and f_{4000} are the friction factors at $Re=2100$ and 4000, respectively.

(2) Friction factor for screen mesh of matrix⁽¹⁵⁾

Creeping flow ($0.1 < Re_d \leq 500$):

$$f = 2n_m / \{1/2 - e - \ln[Re(1 - \eta_v)/4]\}$$

Turbulent ($Re_d > 500$):

$$f = n_m(1 - \eta_o)^2 \eta_v / [8(1 - \eta_v)] / [Re(1 - \eta_v)/4] \quad (10)$$

where Re_d represents Reynolds number with a wire diameter of the screen mesh. n_m , p , and d are number, pitch, and diameter of the screen mesh, respectively. For the porosity, $\eta_v = (1 - \pi p d / 4)$, $\eta_o = (1 - p d) / 2$.

(3) Friction factor of contraction and expansion in flow⁽¹⁶⁾

Contraction: ($Re \leq 3000$): $f = 1 - 0.4A$

$$(Re > 3000): f = 0.5 - 0.4A \quad (11)$$

Expansion: ($Re \leq 3000$): $f = 1 - 2.6A + 1.005A^2$

$$(Re > 3000): f = 1 - 2.0928A + 0.996A^2 \quad (12)$$

where, A is an aspect of the flow area, A_1/A_2 , ($A_1 < A_2$).

(4) Heat transfer coefficient in circular tube⁽¹⁷⁾

Laminar ($Re \leq 2100$):

$$St = (aD_H/\lambda) / (RePr) = 1.86Re^{-2/3}(D_H/l)^{1/3}Pr^{-2/3}$$

Transient ($2100 < Re \leq 4000$):

$$St = St_{2100} + (St_{4000} - St_{2100})(Re - 2100)/1900$$

Turbulent ($Re > 4000$):

$$St = 0.036Re^{-2}(D_H/l)^{1/18}Pr^{-2/3} \quad (13)$$

where Pr , λ represent Prandtl number and thermal conductivity of the working fluid.

(5) Heat transfer coefficient in double tube (from the actual-engine data).

Laminar ($Re \leq 2100$): $Nu = aD_H/\lambda = 4.43$

Transient and Turbulent ($Re > 2100$):

$$Nu = 0.069Re^{0.66} \quad (14)$$

(6) Heat transfer coefficient for screen mesh⁽¹⁸⁾

$$St = \phi Re_a^m Pr^{-1} \quad (15)$$

where,

$$m = 0.43\eta_v + 0.15,$$

$$\phi = \eta_v/2 : (\eta_v < 0.39),$$

$$\phi = 1.54 - 6.36\eta_v + 7.56\eta_v^2 : (0.39 \leq \eta_v < 1)$$

and the porosity is given in Eq.(10) above.

From the above equations, each work or heat quantity related to the cycle is calculated. The powers by the expansion and compression pistons, and heats which flow from/to the five control volumes are calculated from the following equations.

$$W_j = n \oint P_j (dV_j/dt) dt \quad (j=1, 5) \quad (16)$$

$$Q_j = n \oint A_{wj} a_j (T_{wj} - T_j) dt \quad (j=1, 2, 4, 5) \quad (17)$$

where W_1 and W_5 are defined as the expansion power, W_e , and compression power, W_c , respectively. The definition of the indicated power is $W_i = W_e + W_c$. The reheat loss, Q_{rloss} is defined as,

$$Q_{rloss} = nC_p \oint w_{2*} T_{2*} dt = nC_p \oint w_{3*} T_{3*} dt \quad (18)$$

The effective heat which enters the engine through the heater is defined as $Q_{eff} = W_e + Q_{rloss}$.

2.2 Mechanical analysis module

From the above thermodynamic analysis module, the pressure waveform in the working space will be predicted. Using these data, the forces which load the engine mechanism at each calculation step will be calculated and the mechanical friction losses will be analyzed. The fundamental equations about mechanical losses are given as

$$Q_{fric} = \mu F |u| \quad (19)$$

$$Q_{bear} = \kappa F |u| c_d / (2r) \quad (20)$$

where,

c_d : Resistance coefficient

F : Force

Q_{bear} : Bearing loss

Q_{fric} : Friction loss

r : Bearing radius

u : Velocity

κ : Calculated coefficient from the journal theory⁽¹⁹⁾

μ : Friction factor.

Equation (19) is applied for ball bearings with grease. Also, Eq.(20) is applied for oil-required journal-type bearings. For piston rings, rod seal rings, and mechanical seals, Eq.(19) can be adopted⁽²⁰⁾. The pumping loss in the backspace of each piston, W_{pump} was correlated from the experimental data as a function of the mean pressure and engine speed.

The output power is calculated from

$$W_s = W_i - (\sum n \oint Q_{fric} dt + \sum n \oint Q_{bear} dt + W_{pump}) \\ = W_i - W_{mech} \quad (21)$$

In order to calculate the total thermal efficiency, an experimental correlation for the heating system including a preheater and a combustor was obtained from the experimental data. The simple correlation of Q_{in} has a function of the mean pressure, heater-wall temperature, engine speed to represent the actual system with an enough accuracy.

$$\eta = W_s / Q_{in} \quad (22)$$

3. Stirling Engines Used for Evaluation

The applicability of SETMA had been evaluated by the experimental results on the engine and cooling mode by using a 3 kW two-piston type Stirling engine, NS03T⁽²¹⁾ and a 10 W displacer type Stirling engine, SD01⁽²²⁾. As shown in Fig. 2, the NS03T engine is a two-piston type engine for a gas-fired residential heat pump. The two cylinders arrange in 'V' formation at an angle of 60 degrees. Crank-pins of the crankshaft are offset with a 40 degree angle to obtain an optimum

Table 1 Specifications of NS03T engine (1987 model)

Engine type	Two-piston type
Working fluid	Helium
Mean engine press.	6 MPa (max.)
Heater wall temp.	1093 K (max.)
Engine speed	500-1500 rpm
Bore x Stroke (Exp)	82 x 36 mm
(Comp)	82 x 32 mm
Piston phase angle	100°
Heater	Double tube x 24 OD: 9.0, ID 7.5, SP: 6-4, L: 115 mm
Regenerator	Screen matrix Stainless steel #150 OD: 110, ID: 85, L: 70 mm
Cooler	Annular tube x 48 OD: 6.0, ID: 4.5, L: 105 mm

Table 2 Specifications of SD01 engine

Engine type	Displacer type
Working fluid	Air / Helium
Mean engine press.	0.5 MPa (max.)
Heater wall temp.	973 K (max.)
Engine speed	500-3000 rpm
Bore x Stroke	45 x 13 mm
Piston phase angle	60°
Heater	Thin tube x 35 OD: 2.5, ID 1.5, L: 70 mm
Regenerator	Wool/Tube matrix Stainless steel OD: 50, ID: 46, L: 13 mm
Cooler	Thin tube x 55 OD: 2.0, ID: 1.0, L: 40 mm

Fig. 2 NS03T engine (1987 model)

Fig. 3 SD01 engine

phase difference of 100 degree between the expansion and compression pistons. The engine specifications are shown in Table 1. Using helium as the working fluid, the engine has 34.0% maximum thermal efficiency and 4.1 kW maximum output power.

The SD01 engine (Fig. 3) is a 10 W single-acting displacer type. It is a coaxial displacer type integrated in a cylinder including a unique cylindrical cam mechanism. The specifications of the SD01 engine are shown in Table 2.

Because the cyclical process followed by the Stirling cooler on the thermodynamic state of the working fluid follows a path exactly opposite that of the engine, SETMA is able to estimate the performance of the coolers without any modification.

4. Analysis Results for Engines

The reliability of an analysis model may be confirmed by comparing with experimental data of actual machines. This is the most important process for the modification of the design of the machines. At the same time, by comparing with the performance of the machines which were developed based on a detailed design process with the analysis model, it becomes possible to find some problems in the model. As a result, the model becomes more useful and reliable.

Using experimental data of the 1984 NS03T engine, the difference between the case of adopting the friction-loss factors and heat transfer coefficients for the heat exchanger system which were derived with every calculation step and the case of adopting mean values of each coefficient during a cycle was examined. The calculated data in the case of using coefficients derived at every calculation interval agreed with the experimental data better than the case of using the averaged coefficients. However, the former model needed three times the calculation time of the later. The model with the averaged coefficients is more suitable for the analysis.

(a) Expansion space

(b) Compression space

 $T_{w2}=972\text{ K}, T_{w4}=293\text{ K}, n=1000\text{ rpm}$

- Cal. 1 Using instantaneous friction-loss factors and heat transfer coefficients at each calculation step
 Cal. 2 Using averaged and optimized friction-loss factors and heat transfer coefficients and careful scanning of the dimensions of the working space

Fig. 4 P - V diagrams of 1984 NS03T engine

Fig. 5 Effect of inner diameter on indicated work

The analysis and experimental results of the 1984 NS03T were compared on P - V diagrams. In Fig. 4, analysis results are shown which use the coefficients derived at every calculation step and another result which uses the averaged and optimized coefficients. Furthermore, the latter model scanned the dimensions in the working space carefully. Generally, additional experimental information is needed to optimize such coefficients for a precise mathematical model. As shown in Fig. 4, the combination of optimization of the friction-loss factors and heat transfer coefficients and the careful scanning of the dimensions improves the accuracy of the calculations.

The effect of both the core diameter of the double tube for the cooler and the engine speed on the indicated work of the 1994 engine was investigated. Figure 5 shows the effect of the core diameter on the indicated work, when the inner diameter of the outer tube is 13 mm. The indicated work increases with an increase of the core diameter. The value of the core diameter corresponding to the maximum lies between 11.6 mm and 12.4 mm. By increasing the core diameter more than 12.4 mm, the indicated work decreases significantly. As the core diameter increases, the heat transfer coefficient increases and the dead volume decreases. In turn, the compression ratio decreases.

Figure 6 shows the comparison between the calculated and experimental results of the 1987 NS03T engine. Very good agreement is shown. However, the behavior of the calculated reheat loss disagrees with the experimental data slightly. It seems that there is an unknown problem with Eq.(6). In Fig. 7, the behavior of the engine output power and thermal

Fig. 6 Heat and power of NS03T engine

(a) Output power

(b) Total thermal efficiency
 $T_{w2}=1013\text{ K}, T_{w4}=298\text{ K}$
 Fig. 7 Performance of NS03T engine

efficiency with variation of the engine speed and the pressure is shown. There is a discrepancy in the reheat loss, especially, at higher engine speeds. The comparisons establish that the accuracy is sufficient to be used for the purpose of industrial applications, within $\pm 5\%$ uncertainty. Concerning the SD 01 engine, SETMA predicted the engine performance of 4 W output power at 900 rpm and 0.1 MPa engine pressure and the performance of the maximum engine speed.

The cooling characteristics of the NS03T engine and the SD01 engine were also evaluated. There is good agreement between the experimental and calculated results of the NS03T machine on the cooling modes with helium, as shown in Fig. 8. The data depicted in Fig. 9 show the cooling capacity curves of the SD01 machine with a 0.25 MPa mean pressure of helium at a 283 K cooling wall temperature and a 287 K radiator wall temperature. The performance predicted with the SETMA agrees with the testing

Fig. 8 Performance of NS03T in cooling mode

Fig. 9 Cooling performance of SD01

Fig. 10 Compression-space pressure of SD01 in cooling mode

results within $\pm 5\%$ uncertainty.

Figure 10 shows the pressure curve of the SD01 engine in the cooling mode. There is good agreement between the analysis and the measured results. The discrepancy at the high pressure period was caused by the large pressure loss which occurred due to an overlap position of the piston and the manifold of the heat exchanger.

Fig. 11 Effect of equation of state on engine power

5. Equation of State of the Working Fluid

From the viewpoint of a simple calculation, analysis models generally apply the ideal-gas equation of state. Since the coefficient of Eq. (4) is only the gas constant, the available range will be limited. On the other hand, numerous equations of state for real gases have been proposed. Two equations which have fewer terms are the Soave-Redlich-Kwong (SRK) equation⁽²³⁾ and the Peng-Robinson (PR) equation⁽²⁴⁾, both of which modified the attractive-force term of the van der Waals equation. A comparison was made between a reliable database⁽²⁵⁾ for helium and air and the ideal-gas equation of state, SRK and PR equations. No large differences were found at the atmospheric pressure of 0.1 MPa. However, at increasing pressures and at decreasing temperatures, the deviations become large. In the case of helium at 10 MPa and 400 K, the deviation of the ideal-gas equation of state reaches up to -7%. The SRK equation represents the thermodynamic information within $\pm 1\%$, slightly better than the PR equation. The both equations can be represented with the same function.

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)+c(v-b)} \quad (23)$$

where v represents specific volume, m^3/kg . For the SRK equation, the coefficients of a , b , c are

$$a = 0.42747 \frac{R^2 T_c^2}{P_c} \alpha, \quad b = 0.08664 \frac{RT_c}{P_c}, \quad c = 0 \quad (24)$$

where P_c and T_c represent the critical temperature (K) and pressure (Pa). The coefficient, α which depends on the temperature, is given by the reduced temperature $T_r = T/T_c$ and acentric factor, ω .

$$\alpha = [1 + \kappa(1 - T_r^{1/2})]^2, \quad \kappa = 0.480 + 1.574\omega - 0.176\omega^2 \quad (25)$$

Figure 11 shows the deviation between the calculation results using the ideal-gas equation of state and the SRK equation for helium. The engine specifications were for the NS03T engine. Above 6 MPa, the difference becomes larger than the analysis

Fig. 12 Effect of equation of state on cooling power

uncertainty of $\pm 5\%$.

In the case of the cooler in which the temperature of the working fluid is relative low, large deviations are observed, Fig. 12, especially in the case of cooler charged higher pressure about 10 MPa. W_c shows larger deviation than W_e . So, the sum of these work terms which corresponds to the indicated work, W_i , shows a complicated behavior. It becomes clear that the equation of state of the real fluid should be applied to analyze when using the SETMA analysis on higher-pressure-charged Stirling machines.

6. Conclusions

SETMA which is combined with a five-divided working space and detailed mechanical models, was developed to facilitate the design of Stirling machines and to improve their performance. From the comparison between the SETMA calculation and the experimental results on the two engines in the engine and cooling modes, it was confirmed that SETMA is reliable within $\pm 5\%$ analysis uncertainty and that the SETMA has wide applicability. Also, the following issues were obtained as new information.

(1) By optimizing the friction-loss factors and heat transfer coefficients in the working space and careful scanning of the detailed dimensions of the working space for the modeling, the calculation accuracy is improved.

(2) From the consideration of the reliability on the thermodynamic properties of the working fluid referred by SETMA, it was clarified that a reliable equation of the state of the real fluid is more suitable than the ideal-gas equation of state, especially for higher-pressure-charged Stirling machines.

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