Management of a Stronger Wall Jet by an Embedded Streamwise Vortex*

(Reynolds Stresses Distributions and Evaluation of the Vorticity Transport Equation)

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An experimental study was carried out on the Reynolds stress distributions and production terms involved in the vorticity transport equation in a three-dimensional turbulence field of a stronger wall jet managed by an embedded streamwise vortex. Even in the strong three-dimensional flow field the mean velocity gradients $\partial U/\partial y$ and $\partial U/\partial z$ are of primary importance to the quantitative feature of the Reynolds shear stresses $-\overline{uv}$ and $-\overline{uw}$, such as the maximal and minimal in their cross-stream distribution at a local streamwise position, respectively. A hypothesis of spanwise wandering of the introduced streamwise vortex was presented, to understand the large quantitative discrepancy in the stress-strain relationship between the Reynolds stress $-\overline{vw}$ and the rate of strain S_{yz} . The non-isotropic and non-uniform distributions of Reynolds stresses play a dominant role on the decay of the streamwise vortex. In a comparison with some results obtained in turbulent boundary layers disturbed by streamwise vortices, structural differences between the wall jet and the boundary layer reveal in the non-isotropy of the Reynolds shear stresses, $-\overline{uv}$ and $-\overline{uw}$.

Key Words: Stronger Wall Jet, Management, Streamwise Vortex, Turbulence, Vorticity Transport Equation

1. Introduction

In the previous paper⁽¹⁾ we demonstrated the management of a stronger wall jet by introducing a steady streamwise vortex generated with a half-delta wing and investigated the evolution of the streamwise vortex and its influence on the development of the mean flow field. The experiment shows that the streamwise vortex indeed increases momentum flux just as that observed in boundary layers, that is, the vortex generator is an effective device even for the management of a wall jet. Otherwise, the streamwise vorticity decays much faster than in the turbulent boundary layers with a much greater growth rate of vortex radius.

Reynolds shear stress measurements in three-dimensional turbulent flows disturbed by streamwise

vortices have been carried out mostly in boundary layers⁽²⁾⁻⁽⁴⁾. Takagi and Sato⁽⁵⁾ gave an interesting mechanism for turbulent production inside the streamwise vortex core. Qualitative discussions based on the vorticity transport equation, especially, vorticity production, usually classified into Prandtl's first and second kinds of secondary flow, were given by Matsumoto⁽⁶⁾ and Osaka-Gabriela⁽⁷⁾. Osaka and Gabriela⁽⁷⁾ pointed out that abrupt changes of vortex paths near the solid wall could be related with the reorientation phenomenon of vorticity vectors.

Detailed measurements of the mean velocity and turbulence in self-preserving wall jets are performed by Irwin⁽⁸⁾ and Osaka et al.^{(9),(10)}. Development of the mean velocity and turbulence fields in self-preserving stronger and weaker wall jets were investigated experimentally by adjusting the velocity ratio of $U_m/U_e = 1.2 - 4^{(9),(10)}$. Osaka et al.^{(11),(12)} studied the turbulent structure and relaxation process of a wall jet disturbed by a circular cylinder. However, to predict the behavior of a streamwise vortex in a turbulent wall jet as well as the structure of the three-dimensional turbulence, nonuniform distribution of the

* Received 10th September, 1998. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 64, No. 617, B (1998), p. 100-106 (Received April 21, 1997)

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components of Reynolds shear stress tensors producing nonconservative forces for rotating motions must be measured by experimentally.

In the present study the interaction between turbulent flow field and the streamwise vortex is examined in terms of production involved in the vorticity transport equation. We should note the characteristics of the present stronger wall jet: turbulence governed by large-scale spanwise eddy in the outer layer with $\partial U/\partial y < 0$ is subjected to adverse pressure gradient and keeps a relatively high level of turbulence intensity. Building an experimental database of such complex turbulent flows will contribute to the improvement of the turbulence model for three-dimensional flow fields as well as the optimization of the boundary layer control technique with vortex generators or film-cooling techniques for turbine blades etc.

2. Experimental Apparatus and Technique

Figure 1 shows the schematic of the flow field, coordinate system and nomenclature. An experiment was carried out in a low turbulence wind tunnel in the Fluid Mechanics division of Yamaguchi University. A plane jet exits from a 10×400 -mm² nozzle (aspect ratio $AR=40$) along the bottom wall of a test section. The free stream velocity was adjusted by the roof and a mesh screen at the end of the test section, so that the velocity ratio U_m/U_e remained at a constant value of 4 throughout this experiment. A 1 mm thin stainless plate divided the free stream and the jet before the flow entering the test section.

A half-delta wing that is 7 mm in height and 50 mm in cord length was employed to generate a streamwise steady vortex. The wing was located on the wall with an angle of attack $\alpha=15^\circ$ at $x_0=620$ mm where a self-preserving development was established in the non-disturbed flow⁽⁹⁾. The height of the vortex generator $h=7$ mm was chosen to be equal to the

Fig. 1 Schematic of flow field, nomenclature, and coordinate system

inner layer thickness of the nondisturbed wall jet. The streamwise generated vortex in clockwise rotation (positive streamwise vorticity $\Omega_x = \partial W/\partial y - \partial V/\partial z > 0$) had the circulation of $\Gamma_0/(U_m b_m) = 1.08$ which was obtained from the experimental results at $x/h = 0.5$. The nozzle exit Reynolds number $U_j S_i/\nu$ was set at 1.2×10^4 and the free-stream turbulence level was 0.3%.

Measurements of the three components of the mean velocity and six Reynolds stresses were carried out with constant-temperature anemometers and a single slanted hot-wire probe. A tungsten filament, 3 μ m in diameter and 0.6 mm in length, was welded to the tips of a pair of prongs with a slanted angle of 60° , so that the experimental uncertainty in the present strongly three-dimensional flow field is reduced a minimal value. The sensor length Reynolds numbers lu_τ/ν ranging from 20 to 30 have a reasonable spatial resolution for turbulence measurements⁽¹⁶⁾. At each position, the main and secondary velocities and six Reynolds stress components were obtained from 24 measuring states at different angles of the hot-wire, with the least-square method⁽¹⁷⁾. Second-order turbulence terms for the mean velocity components and all the third order terms for the Reynolds stresses were neglected in the response equation⁽¹⁸⁾. Experimental uncertainties estimated at the closest position to the wall $y=1.7$ mm, $x/h=3$ were 2%, 7%, and 33% for mean velocity, secondary velocities, and Reynolds stresses, respectively. Discrepancies in the measured mean velocity and Reynolds shear stress between the single slanted wire method and crossed wire technique were typically 5.4% and 13%, respectively, over the spanwise region of $z/h = -20$ to 20 at $x/h=3$.

3. Results and Discussion

3.1 Reynolds stresses

Figure 2 shows the contours of the Reynolds stresses and related quantities in the cross-stream section. The results were obtained at $x/h=3$, where the increase in momentum flux due to the streamwise vortex reaches a maximum and vortex rebounding⁽⁷⁾ is observed. The outer layer velocity scale U_0 and the height of the vortex generator h are used for normalized expression. For the sake of reference to the extent occupied by the streamwise vortex in the cross-stream section, the vortex center is marked in all the contour maps.

The normal Reynolds stress $\overline{u^2}/U_0^2$ has relatively large values around the vortex center. The peak value of $\overline{u^2}/U_0^2 = 0.45$ is located at a slightly upward position from the vortex center ($z/h=0.1$, $y/h=0.5$). Because the turbulence production term $-\overline{uw} \partial U/\partial z$ gives a larger contribution to the $\overline{u^2}$ component, the

Fig. 2 Distribution of the Reynolds stresses and related quantities at $x/h=3$

peak position of $\overline{u^2}/U_0^2$ does not coincide with the peak of $-\overline{uv}\partial U/\partial y$ which is usually a dominant production term in two-dimensional flows; the maximum value of $-\overline{uw}\partial U/\partial z$ is about 1.5 times that of $-\overline{uv}\partial U/\partial y$. In the region of $y/h=1-3$ at $z/h=-1$, where the secondary flow is directed upwards, the contour lines are slightly deformed probably due to decrease of the production term $-\overline{uv}\partial U/\partial y$ and convection by the secondary velocity component $V\partial \overline{u^2}/\partial y$. Far from the wall in the outer layer, the turbulent intensity $\overline{u^2}/U_0^2$ is slightly decreased, corresponding to a decrease in the mean velocity gradient

$|\partial U/\partial y|$ under the influence of the streamwise vortex. The same trend can also be recognized in the other two turbulent intensities $\overline{v^2}/U_0^2$ and $\overline{w^2}/U_0^2$ in the outer layer. It follows that the deformation of the turbulent structure in the outer layer is not serious.

The Reynolds normal stress component $\overline{v^2}/U_0^2$ takes larger values above the vortex center. The peak value at $z/h=-0.2$ and $y/h=0.8$ is $\overline{v^2}/U_0^2=0.042$ which is comparable in magnitude to the streamwise component $\overline{u^2}/U_0^2$. Positive large turbulence production rates in terms of $-\overline{vw}\partial V/\partial z$ ($z/h=0$ and $y/h=1$) and $-\overline{v^2}\partial V/\partial y$ ($z/h=-0.2$ and $y/h=1$) are responsible

for the peak value of the $\overline{v^2}/U_0^2$ component. These production terms for the $\overline{v^2}/U_0^2$ component are of the same order of magnitude of $-\overline{uv} \partial U/\partial y$ and $-\overline{uw} \partial U/\partial z$ for streamwise component $\overline{u^2}/U_0^2$ in the disturbed flow.

The spanwise component of the Reynolds normal stress $\overline{w^2}/U_0^2$ has relatively smaller values around the vortex center, however, it increases above the vortex center. The peak value $\overline{w^2}/U_0^2=0.040$ which is slightly smaller than that of the streamwise component $\overline{u^2}/U_0^2$ is located at $z/h=0.3$ and $y/h=1$ at which the corresponding production terms $-\overline{vw} \partial W/\partial y$ and $-\overline{w^2} \partial W/\partial z$ take large positive peaks. In particular, $-\overline{w^2} \partial W/\partial z$ term being about twice the $-\overline{vw} \partial W/\partial y$ term at $z/h=0.3$ and $y/h=1$ primarily contributes to the production of the spanwise component $\overline{w^2}/U_0^2$. The two major production terms $-\overline{vw} \partial W/\partial y$ and $-\overline{w^2} \partial W/\partial z$ have positive and negative peaks at $z/h=-0.3$ and $y/h=0.8$, respectively, however, these peaks cancel each other. Interestingly, the spanwise turbulence component $\overline{w^2}/U_0^2$ is slightly reduced in magnitude inside the extent occupied by the streamwise vortex and in its positive spanwise positions. In this spanwise extent, lateral stretching $\partial W/\partial z > 0$ suppresses the magnitude of the turbulence intensity $\overline{w^2}/U_0^2$ by negative values of the corresponding production term $-\overline{w^2} \partial W/\partial z$.

These experiments show in fact that, around the vortex center, the three components of the Reynolds normal stresses are influenced in magnitude by the introduction of the streamwise vortex; the streamwise and transverse components $\overline{u^2}/U_0^2$ and $\overline{v^2}/U_0^2$ are increased, but the spanwise component $\overline{w^2}/U_0^2$ is reduced. Consequently, the magnitude of these components are in the unusual order of $\overline{u^2}/U_0^2 > \overline{v^2}/U_0^2 > \overline{w^2}/U_0^2$ (in a usual undisturbed two-dimensional flow field $\overline{u^2}/U_0^2 > \overline{v^2}/U_0^2 > \overline{w^2}/U_0^2$). However, a qualitative understanding of these deformations can be obtained simply in turbulence production terms.

Next, we will examine the Reynolds shear stress components and these production terms. The Reynolds shear stress $-\overline{uv}/U_0^2$ is negative around the vortex center but positive above the vortex center. The leading term $\overline{v^2} \partial U/\partial y$, which contributes to producing $-\overline{uv}$, has negative and positive values around these positions. Note that the term $\overline{v^2} \partial U/\partial y$ is one order greater than the other three terms $\overline{uv} \partial V/\partial y$, $\overline{uw} \partial V/\partial z$ and $\overline{vw} \partial U/\partial z$. The interesting fact follows that even in such three-dimensional complex turbulent flows the term $\overline{v^2} \partial V/\partial y$ maintains a dominant role in producing the Reynolds shear stress $-\overline{uv}$. Momentum transport due to transverse velocity component V induced by the streamwise vortex forms an asymmetrical distortion in the mean velocity profile

and also Reynolds shear stress distribution in the spanwise direction. Asymmetry of the Reynolds shear stress distribution in the present wall jet is much weaker than that observed in the outer layer of the turbulent boundary layers in which the streamwise vortex survives for a much longer streamwise extent⁽²⁾. In another experiment in which the streamwise vortex decays faster by a factor of 2 in a three-dimensional boundary layer, this asymmetry of the Reynolds shear stress $-\overline{uv}$ was observed along the transverse direction. These asymmetries in the Reynolds shear stress distribution suggest that there exists a mechanism to maintain the streamwise vorticity due to the nonuniform shear stress distribution, resulting from deformation that will be examined in the next section.

The other Reynolds shear stress component $-\overline{uw}/U_0^2$ yields positive and negative peaks aligned in the spanwise coordinate. The leading production term $\overline{w^2} \partial U/\partial z$, for which the velocity gradient $\partial U/\partial z$ is responsible, has positive and negative peaks corresponding to these Reynolds shear stress peaks. In the experimental results for turbulent boundary layers in which the streamwise vortex survives for a much longer streamwise extent, the Reynolds shear stress measurements show positive and negative peaks of the $-\overline{uw}$ component and then strong asymmetric distribution in the transverse direction around the vortex center. It is assumed that these asymmetrical distributions of the Reynolds shear stress components $-\overline{uv}$ and $-\overline{uw}$ roughly depending on the velocity gradients $\partial U/\partial y$ and $\partial U/\partial z$, respectively, create different circumstances for maintaining the streamwise vortex. In the next section, we will examine the streamwise vorticity budget in evaluating the production terms involved in the vorticity transport equation.

The Reynolds shear stress component $-\overline{vw}/U_0^2$ is negative around the vortex center, but positive above the vortex center, and negative at the height of $y/h=1.2$. The corresponding rate of strain $S_{yz}=\partial W/\partial y + \partial V/\partial z$ quantitatively has quite a different distribution: positive values are much larger than the negative ones in the contour map of S_{yz} . This serious discrepancy between the shear stress and the rate of strain can be quantitatively explained in the following hypothesis based on an assumed spanwise unsteady vortex oscillation. In the previous study, vortex wandering velocity in the spanwise direction roughly agreed with the induced velocity due to the mirror image against the solid wall in the calculation using measured circulation and Biot-Savart law. The circulation of streamwise vortex is supposed to fluctuate due to background unsteadiness, for instance, the

passing of large-scale coherent spanwise vortex, and, of course, the vortex wandering velocity fluctuates. This signifies that the streamwise vortex oscillates in the spanwise direction around the time-averaged path. When we think of the correlation between the fluctuating spanwise velocity and the streamwise vorticity fluctuation $\overline{w\omega_x}$, it most probably has positive values in the positive spanwise positions ($\partial\Omega_x/\partial z > 0$) around the time averaged vortex center z_c . Otherwise, in the negative spanwise positions around the vortex center, the negative correlation $\overline{w\omega_x} < 0$ is expected. The component $\overline{w\omega_x}$ is an outer product of the velocity and vorticity vectors and produces a body force⁽²¹⁾. With the aid of an analysis near the wall, based on the present experimental results, this body force can be balanced by the Reynolds shear stress force $\overline{vw} \approx \partial(-vw)/\partial z$. Consequently, the Reynolds shear stress component $-vw$ tends to have negative values inside the vortex core due to the spanwise oscillation. If this hypothesis is accepted, the Reynolds shear stress component $-vw$ is affected by the spanwise vorticity gradient $\partial\Omega_x/\partial z$ rather than the corresponding rate of strain S_{yz} . The negative value of the $-vw$ component strengthens the positive contribution of turbulence production $-\overline{vw}\partial V/\partial z$ for $\overline{v^2}$ and otherwise negative contribution of turbulence production $-\overline{vw}\partial W/\partial y$ for $\overline{w^2}$ around the vortex center where $\overline{v^2} > \overline{w^2}$ is observed. It is useful to refer the experimental results obtained in an interaction process between a boundary layer and an artificially generated spanwise oscillated streamwise vortex by Westphal and Mehta⁽²²⁾, because an unusual result for $\overline{v^2} > \overline{w^2}$ was observed. In their experiment, the height of the vortex generator was of the order of the boundary layer thickness, vortex strength was $\Gamma/(U_0 h) = 0.27$, and the oscillating frequency was chosen to be 1 Hz ($fh^2/\Gamma = 0.003$). If the passing frequency of a large-scale unsteady spanwise vortex is a trigger for the assumed quasi-periodic spanwise oscillation, the non-dimensionalized frequency fh^2/Γ is about 0.02 in the present wall jet⁽²³⁾.

The turbulent kinetic energy $\overline{q^2}/U_0^2$ takes the maximum value of 0.11 above the vortex center ($z/h=0$, $y/h=0.8$) where the Reynolds shear stress $-\overline{uv}/U_0^2$ has a positive peak value. In a boundary layer interacting with a relatively weak streamwise vortex, the smaller maximum value $\overline{q^2}/U_0^2 = 0.024$ was observed in a vortex core region. On the other hand, the much larger maximum value $\overline{q^2}/U_0^2 = 0.14$ was measured in an interaction with a strong streamwise vortex.

3.2 Evaluation of the streamwise vorticity transport equation

According to Townsend's book⁽²⁴⁾, the transport

equation of a streamwise vorticity for a steady flow can be written as follows,

$$\begin{aligned} & \left(U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} + W \frac{\partial}{\partial z} \right) \Omega_x \\ &= \underbrace{\Omega_x \frac{\partial U}{\partial x}}_{P1_1} + \underbrace{\Omega_y \frac{\partial U}{\partial y}}_{P1_2} + \underbrace{\Omega_z \frac{\partial U}{\partial z}}_{P1_2} + \underbrace{\frac{\partial}{\partial x} \left(\frac{\partial \overline{uv}}{\partial z} - \frac{\partial \overline{uw}}{\partial y} \right)}_{P2_1} \\ &+ \underbrace{\frac{\partial^2}{\partial y \partial z} (\overline{v^2} - \overline{w^2})}_{P2_2} + \underbrace{\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) (-vw)}_{P2_3} \\ &+ \underbrace{\nu \nabla^2 \Omega_x}_{VD} \end{aligned} \quad (1)$$

where $\Omega_x = \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z}$, $\Omega_y = \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}$, $\Omega_z = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Hereafter the letters shown under each term will denote the terms on the right hand side of Eq.(1). The PS term contributes to vorticity production by streamwise stretching, $P1_1$ and $P1_2$ indicate productions by interaction between the vorticity and velocity gradient, $P2_1 - P2_3$ indicate productions due to non-uniform distribution of non-isotropic Reynolds shear stress, and VD means the viscous diffusion of vorticity⁽²⁴⁾. Prandtl's first and second kind of secondary flows are supposed to be produced by $P1_1 + P1_2$ and $P2_1 - P2_3$, respectively⁽²⁵⁾. Concerning the much faster decay of streamwise vorticity in the stronger wall jet as compared with the boundary layers, we will evaluate the production terms in the transport equation. At the vortex center of $x/h=3$, the sum of the evaluated production terms differs from the sum of the evaluated left-hand side convection terms typically by 35%. The viscous diffusion term is two orders smaller than the leading production term within the measuring range.

In Fig. 3, cross-stream distributions of the production term at $x/h=3$ are normalized with the velocity scale U_0 and the height of the vortex generator h . The broken lines indicate streamwise vorticity contour lines, which have a 10% value of the maximum vorticity in the cross-stream section. The stretching term PS has a positive contribution to maintain the introduced streamwise vorticity under acceleration $\partial U/\partial x > 0$ accompanied by the recovering velocity deficit inside the vortex core region. The skew-induced terms $P1_1$ and $P1_2$ have negative and positive vorticity productions at $z/h=0.25$ and $y/h=0.4$, respectively. Both the productions are raised around the intersection of the two vortex sheets, $\Omega_z = -\partial U/\partial y > 0$ and $\Omega_y = -\partial U/\partial z > 0$, consequently, these peaks cancel each other. It is seen that the curvatures of the mean streamlines $\partial V/\partial x$ and $\partial W/\partial x$ are negligibly small and that the sum of the skew-induced terms

Fig. 3 Production terms in the vorticity transport equation at $x/h=3$

$P_{11} + P_{12}$ is approximately equal to $\partial U/\partial z \cdot \partial U/\partial y - \partial U/\partial y \cdot \partial U/\partial z$. The magnitude of the sum $P_{11} + P_{12}$, which is positive for $z > z_c$ and negative for $z < z_c$, is about $0.02 U_0^2/h^2$.

The first term of the stress-induced production term P_{11} takes positive ($z/h=0.25$, $y/h=0.75$) and negative ($z/h=-0.25$, $y/h=0.75$) peaks above the vortex center. The positive peak that is mostly due to $\partial/\partial x (\partial uv/\partial z)$ is greater than the negative peak in magnitude and contributes to maintain the introduced streamwise vortex. Note that, for incompressible flows, the first term P_{21} can be derived from the skew-induced production terms by Reynolds decomposition and time-averaging, whereas the other two stress-induced terms P_{22} and P_{23} are derived from the convection terms.

In the second stress-induced term P_{22} , the negative values are higher than the positive ones and their magnitudes are about twice the value of the stretching term PS. The large negative vorticity production around the vortex center decays the introduced streamwise vortex. The third stress-induced production term P_{23} contains a noticeable negative peak that reaches 2.4 times the second stress-induced term. Obviously, the strong negative vorticity production contributes to the faster decay of the streamwise vortex.

The magnitude of each production term along the vortex center is given at three streamwise locations where a streamwise vortex is recognized in Fig. 4. The stretching term PS produces positive vorticity at

Fig. 4 Evolution of the production terms at the vortex center

$x/h=3$, however, its contribution vanishes at $x/h=10$ and 30. The skew-induced production terms P_{11} and P_{12} , which cancel each other around the vortex center, have the largest magnitude at $x/h=3$ and 10. The first term in stress-induced production P_{21} produces positive vorticity, however, its magnitude is the smallest and vanishes at $x/h=10$ and 30. The second term of stress induced production P_{22} gives a negative contribution to the introduced positive vorticity and the magnitude of negative production is the largest at $x/h=10$. The last term of stress-induced production P_{23} produces a relatively large negative vorticity over $x/h=3-30$. From the evaluation of all the production terms, the last two terms of stress-induced production, P_{22} and P_{23} play a dominant role on the evolution of the introduced streamwise vortex and

Fig. 5 Comparison of the non-isotropy $(\overline{v^2} - \overline{w^2})/U_{ref}^2$ at the vortex center

contribute to the decay of the positive vorticity by producing negative vorticity.

The comparison between the present experiment for the wall jet and the results of the turbulent boundary layers shows remarkable differences in the growth rate and decay of the streamwise vortex. We should obtain more detailed comparisons of the mechanisms that result in the differences in the evolution process of the streamwise vorticity. Unfortunately, no experiments were carried out in which the evaluation of all the production terms was made completely. In Fig. 5 a comparison is shown for only the available quantity, non-isotropy of the normal Reynolds stress components $(\overline{v^2} - \overline{w^2})$ along the center of a positive vortex. Here, the free stream velocity is taken as the reference velocity scale U_{ref} for the boundary layers. In the present wall jet, the degree of non-isotropy in $(\overline{v^2} - \overline{w^2})/U_{ref}^2$ takes relatively large positive values due to the increase of normal Reynolds stress $\overline{v^2}$ at the vortex center within the immediate downstream positions behind the vortex generator $x/h < 3$. The positive value of $(\overline{v^2} - \overline{w^2})/U_{ref}^2$ becomes negative and recovers, as the streamwise vortex decays over the downstream of $x/h > 3$. Whereas the non-isotropy in $(\overline{v^2} - \overline{w^2})/U_{ref}^2$ is very small in the boundary layers and gradually increases in the downstream direction over $x/h < 30$. We should obtain comparisons for the other stresses, for instance, the cross-stream distribution of Reynolds shear stress $-\overline{uv}$ and $-\overline{uw}$ components, depending roughly on the corresponding mean velocity gradients $\partial U/\partial y$ and $\partial U/\partial z$, respectively. However, this result of the non-isotropy of the normal Reynolds stresses reveals a large structural difference in the disturbed turbulence field as well as the evolution of streamwise vorticity between boundary layers and wall jets.

4. Conclusions

Reynolds stress measurements and evaluation of

production terms in the vorticity transport equation in a stronger wall jet managed by introducing a steady streamwise vortex provides the following conclusions:

(1) The three components of Reynolds normal stress $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$ take maximal or minimal peaks around the vortex center. The maximal and minimal values can be qualitatively explained by the turbulent production terms involved in the equations for each component.

(2) Even in the three-dimensional turbulent flow strongly disturbed by the streamwise vortex, the term $\overline{v^2} \partial U/\partial y$ plays a dominant role for producing the Reynolds shear stress $-\overline{uv}$ as observed in the two-dimensional flow. It is possible that the maximal and minimal values of $-\overline{uv}$ and $-\overline{uw}$ components can be explained by the corresponding values of the mean velocity gradients $\partial U/\partial y$ and $\partial U/\partial z$.

(3) A large difference is clearly recognized in a stress-strain relationship between the Reynolds shear stress $-\overline{vw}$ and the corresponding rate of strain $S_{yz} = \partial V/\partial z + \partial W/\partial y$ in the streamwise vortex region: $-\overline{vw}$ tends to negative, while S_{yz} tends to be positive. A hypothesis for the spanwise wandering of the generated streamwise vortex is presented, to qualitatively understand this phenomenon.

(4) Both the second and third stress-induced vorticity production terms, P_{22} and P_{23} , due to nonuniform distribution of non-isotropic Reynolds stresses play a dominant role in the decay of the introduced streamwise vortex by producing negative vorticity.

(5) The comparison of results in the turbulent boundary layers reveals a structural difference with a large degree of the non-isotropic Reynolds stress in the present wall jet. A difference in the spatial distribution of the Reynolds shear stresses $-\overline{uv}$ and $-\overline{uw}$ involved in the first stress-induced production term is observed.

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