LDA Flow Rate Measurements Applied for Analysis of Transient Injection Characteristics*

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In the present study an LDA-based technique was applied to analyze details of a transient pipe-flow induced upstream by a fuel injector. Due to highly accurate measurements of the instantaneous time series of velocity, fuel flow rate and pressure gradient were achieved. The pipe flow has been driven under an injection pressure of 0.6 MPa at high injection frequencies of 25 and 50 Hz. The influence of the outflow orifice diameter, varied in range from 100 to 500 μm , on the transient pipe flow dynamics has also been described. The results show that transient behavior of rapid oscillating pipe flow is remarkably and systematically dependent on injection frequency and duration as well as on the orifice diameter.

Key Words: Fuel Injection, Transient Flow, Instantaneous Flow Rate, Laser-Doppler Anemometer

1. Introduction

In the field of automotive engineering, much effort is being made on direct fuel injection research and development^{(1),(2)}, especially on the applicability to Otto and Diesel type engines, that could perform optimal fuel consumption and reduced environmental pollution in the near future. On account of this concept it seems very important to achieve optimal characteristics of spraying and equally its stability⁽³⁾⁻⁽⁵⁾. The exact initial conditions of fuel injection, in particular based on instantaneous fuel flow rate are one of the challenges faced today to forward practical application of these fuel injectors, as well as basic modeling of spray formation.

The theoretical background and an experimental

LDA-based instrument have recently been developed⁽⁶⁾ for accurate measurements of instantaneous flow rate in a periodically oscillating pipe flow. The experimental results showed that temporal characteristics of the pipe flow driven by fuel injector are significantly dependent on the ratio of the injection period and injection duration, especially in a case when the cyclic period is shorter than the viscous time constant. This method also permits reconstruction of the time-dependent velocity field in capillary pipe flow that can not be represented by parabolic radial distribution characterizing steady pipe flow. By introducing a high temporal resolution for measurements of the pipe flow the method charges simultaneous information with respect to velocity field, axial pressure gradient and flow rate. This information obtained through LDA-based measurements in the fuel injection pipeline significantly helps to understand time-varying dynamics of the spray developing afterwards downstream from the injector.

Motivated by this idea we have considered a basic objective for this present study with respect to measurement and analysis of instantaneous pipe flow

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characteristics as velocity, flow rates and pressure gradient with a focus on transient injection. In paper Section 2, theoretical background to the method is briefly reviewed in order to outline objective treatment procedures needed to construct the time-dependent series of flow rate and pressure gradient from LDA-measurements of the centerline velocity. The experimental setup used for actual measurements is described in Section 3. Main results and their discussion are framed in Section 4. Conclusions are stated in Section 5.

List of symbols

- a : sonic wave propagation speed
- c_j : complex Fourier coefficient
- $C.C.$: complex variable conjugate
- $d_p, (R)$: inner diameter (radius) of pipe
- f : operational injection frequency
- I : complex unity
- J_0, J_1 : Bessel's function of zero-th and first order
- L, l : pipe length and length of LDA control measurement volume
- $m, \dot{m}$: mass and mass flow rate
- $\partial P/\partial z$: pressure gradient
- p_0, p_n : amplitude components of pressure
- r : radial position
- Re : Reynolds number
- T : time
- T : cycling period of injection
- T_a : Taylor number
- U : axial velocity
- $\dot{V}$: volumetric flow rate
- μ, ν : molecular and kinematic viscosity
- ρ : fluid density
- τ : injection time
- ω : angular velocity

2. Theoretical Background

The method⁽⁶⁾ was basically developed to measure instantaneous flow rate in a capillary type fast oscillating pipe flow through the time-series of centerline velocity. It is based on three equations written with respect to non-stationary laminar pipe flow. Commonly, if a time series of one time-dependent value is measured, the computations described below permits exact analytical solution for each of three instantaneous values that are velocity, pressure gradient and volumetric flow rate.

The pressure gradient is superposed by a Fourier expansion to fit any arbitrary periodic flow:

$$\frac{\partial P}{\partial z} = -\rho \left[p_0 + \sum_{n=1}^{\infty} (p_n e^{in\omega t} + C.C.) \right] \quad (1)$$

where here and later the complex conjugate $C.C.$ presents corresponding complex argument of a given

value.

Taking into account the linearity of the Navier-Stokes momentum equation on the pressure gradient term and using a superposition of the Lambossy solution⁽⁷⁾ for each induced particular harmonics the exact solution for velocity field can be rewritten in the form of

$$U(r, t) = \frac{R^2 p_0}{4\nu} \left[1 - \left(\frac{r}{R} \right)^2 \right] + \sum_{n=1}^{\infty} \left\{ \frac{p_n}{n\omega} \cdot i e^{in\omega t} \left[\frac{J_0 \left(i^{3/2} T a_n \frac{r}{R} \right)}{J_0 \left(i^{3/2} T a_n \right)} - 1 \right] + C.C. \right\}, \quad (2)$$

where the Taylor number $T a_n = R \sqrt{\omega n / \nu}$, on one hand, defines partial velocity profile with respect to particular oscillations indexed as to n , and, on the other hand, presents the normalized ratio of dynamic and viscous time constant, $T_\mu = R^2 / 4\nu$, being about 250 ms in the present experiments. If the particular oscillation period $T_n = 2\pi / \omega n$ is longer than T_μ , the corresponding velocity profile will be fully developed quasi-parabolic. Otherwise, the particular profile will present a flat-flow with a strong shear at the wall.

An integration treatment of velocity over a circular cross section yields the volumetric flow rate

$$\dot{V}(t) = \frac{\pi R^2}{2} \left(\frac{R^2 p_0}{4\nu} + \sum_{n=1}^{\infty} \left\{ \frac{p_n}{n\omega} i e^{in\omega t} \left[\frac{4 i^{1/2} J_1(i^{3/2} T a_n)}{T a_n J_0(i^{3/2} T a_n)} - 2 \right] + C.C. \right\} \right). \quad (3)$$

Now, for the reconstruction of introduced Eqs. (1), (2) and (3) one needs to deduce harmonics coefficients of p_0 to p_n from the measurements of a time-varying value that can be represented by a time series of velocity or pressure gradient.

Depending on the control measurement point settled into pipe flow and temporal resolution needed to detect pipe flow dynamics, measurement technique and treatment procedure must be reasonably applied. One effective way to simplify the reconstruction treatment is based on a center-line time-varying velocity measurement. In this case the modified expression for axis velocity is deduced from Eq. (2):

$$U(r=0, t) = \frac{R^2 p_0}{4\nu} + \sum_{n=1}^{\infty} \left\{ \frac{p_n}{n\omega} i e^{in\omega t} \left[\frac{1}{J_0(i^{3/2} T a_n)} - 1 \right] + C.C. \right\}. \quad (4)$$

Another way is that the velocities time series based on the time-resolved LDA-measurements referenced to a number of encoded bins N_{exp} within a cyclic period, can be transformed into the Fourier expansion

$$U(r=0, t) = \frac{C_0}{2} + \sum_{n=1}^{N_{exp}} (c_n e^{in\omega t} + C.C.) \quad (5)$$

that finally permits computation of the quantities of unknown coefficients

$$p_0 = \frac{2c_0\nu}{R^2} \quad \text{and} \quad p_n = \frac{c_n i n \omega}{1 - \frac{1}{J_0(i^{3/2} T a_n)}}. \quad (6)$$

The governing Eqs. (1), (2), (3) and (4), (5), (6) are the base to reconstruct velocity field and flow rate in any arbitrary oscillating pipe flow. To get a final solution for the present equations system, it is necessary to input time-series of measured values at a certain set of the encoded time span number N_{exp} within the oscillation period, the basic cyclic frequency $\omega = 2\pi/T = 2\pi f$, the inner pipe diameter $2R$ and the kinematic viscosity of flowing fluid ν .

A measurement point must be settled in a pipe flow section where fully developed laminar flow is expected and validated through the measurements. In practice, this point must be settled with respect to a position located far away from both pipe ends, entrance inflow section and oscillating injector section, more than 100 times of inner pipe diameter ($2R$). Influence of injector characteristics such as orifice size, injection duration and injection pressure on flow rate and pipe flow field can be evaluated only after analysis of instantaneous and integral values of volumetric or mass rates.

As the results will show, the capillary time-dependent injection pipe flow includes short-time impact phases when the injector valve is charged to be open or close. A fast transient regime occurs at these moments, and a high temporal resolution is required to reconstruct the transient flow dynamics. LDA flow metering technique meets the basic requirement that has been indicated in the previous study⁽⁶⁾. In the present investigation this method was used to identify transient flow properties depending on orifice diameter at different high oscillation frequencies of 25 and 50 Hz.

3. Experiments

The experimental setup was shown in Fig. 1 and the specification of the LDA was described in Table 1. The same test rig⁽⁶⁾ developed in the previous paper was used in the present experiment in order to obtain accurate LDA-measurements of the centerline velocity time series. The pipe flow was driven under injection pressure $p=0.6$ MPa and the oscillation time setup was driven by a time-phasing control unit connected to a serial fuel injector.

The control unit was accurately able to vary injection frequency f and injection duration $\tau/T = \tau f$ within intervals of 0.5 - 83 Hz and 0.01 - 1.00 respectively. A glass test pipe section with an inner diameter of $d_p = 2.0$ mm and a measurement section length of $L = 200$ mm was installed along the pipe-portion of the test rig. The study concentrated on investigating

the high injection frequencies, $f = 25$ and 50 Hz, and the orifice size was flexibly changed by using optical pin-holes with diameter $d = 100, 200$ and $500 \mu\text{m}$, referred to hence forth as " $d100$ ", " $d200$ " and " $d500$ ", connected in-line with the injector. Tap water was used as a fuel model. Thus, in the present experiments, the injected spray was not subjected to a combustion chamber. The injected fluid was collected through a flexible pipe to a mass balance container as shown in Fig. 1 to obtain mean mass rate.

To observe the details of short-time transitions all measurements have been monitored with a time-phase encoder at a temporal resolution of 1 000 bins/cycle. Accurate statistics for each bin-phase was achieved when more than 30 000 cycles were measured and phase-sorted within the oscillation period. The accuracy of the LDA flow rate measurements has been simultaneously controlled by mass balance. The fluid mass (m) injected into the container and subsequently collected on the electronic mass meter was integrated during a long time (t) at the LDA-measurements. This permits one to compute the average mean mass rate $\dot{m} = dm/dt$. On the other hand, the first term of the LDA flow rate Eq. (3), referred to as " p_0 term" is also a mean volumetric flow rate $\dot{V}_{LDA} = dV/dt$. This

Table 1 Specifications of the LDA

Power of the He-Ne gas laser	10 mW
Laser wavelength	0.6328 μm
Focal length of transmitting lens	110 mm
Beam separation at the lens	50 mm
Full crossing angle of the beams	25 deg.
Effective shift frequency	400 kHz
Measuring volume length	204 μm
Measuring volume diameter	47 μm

Fig. 1 Pressurized injection test rig and LDA flow rate meter

Fig. 2 Comparison of mass flow rates measured by mass-balance and LDA in oscillating pipe flow at injection frequency 25 and 50 Hz

term can be obtained after applying a Fourier transform to the velocity time series that corresponds to “ c_0 term” in Eq. (5).

Using the mass balance measurements $\dot{m}_{\text{mass balance}}$ the relative accuracy of the present LDA-measurements $\dot{V}_{\text{LDA}}$ was evaluated through comparison of these two integral mean values:

$$\delta = \frac{\dot{V}_{\text{LDA}}\rho - \dot{m}_{\text{mass balance}}}{\dot{m}_{\text{mass balance}}} \quad (7)$$

where ρ is fluid density.

The result of comparing the mass balance and LDA flow rate measurements are shown in Fig. 2. Relative accuracy accorded to Eq. (7) lies within a width of 2%. Disagreement between the measurements is based on two basic reasons. The first is a length l (0.204 mm) of the LDA-control measurement volume (CMV) inside of capillary pipe with limited inner diameter $2R$ (2.0 mm). In the present experiments, after collimating of the intersecting laser beams, the ratio of $l/(2R)$ became 0.1 (0.204 mm/2.0 mm respectively) which for a steady flow corresponds to 1% ideal accuracy of centerline velocity measurement. The second factor is vibration of the glass pipe test section connected to the oscillating injector. Vibration shows virtual flow velocity due to the dynamic displacement of the CMV with respect to a fixed measuring point on the pipe centerline.

4. Results and Discussion

Measurements have been systematically carried out for different injection setups in order to consider what factor among the operating values is most influential on transition behavior of injected flow rate.

The minimum injection frequency was 25 Hz ($T=40$ ms) and the maximum duration of injection rate, τ/T , was 0.5, which corresponds to the absolute longest injection duration of 10 ms. Because all timing scales were much smaller than the viscous time constant, 250 ms, a dynamic pipe flow was expected to be transient and not fully developed. The shortest injection duration, 1 ms, corresponds to the injection frequency 50 Hz and an injection duration rate $\tau/T=0.1$, was limited by practical operation of the applied injector.

4.1 Factors of injection duration rate and orifice size

The time series of centerline velocity measured at a frequency of 50 Hz with different orifice diameters, and instantaneous flow rate reconstructed from the centerline velocity are sampled in Fig. 3. At the flow rate curves in larger orifice size, $d=500$, it was observed that a phase-shift according to injection durations. A curve at the maximum peak of all velocity and flow rate series becomes triangular while injection duration rate $\tau/T=0.1$. By decreasing orifice size, “ $d200$ ”, the transient slopes are separated both in time and amplitudes, and in the case of the smallest orifice size, “ $d100$ ”, the series corresponding to different injection duration, have non-similar transitions.

In both cases, “ $d100$ ” and “ $d200$ ”, a rising transient slope, from start to peak decreases with respect to the injection duration. Transient behavior of velocity and flow rate time-series is considerably dependent on pipe flow configuration, particularly on outflow orifice size. Results in Fig. 3 indicate that injection duration rate is also influential on instantaneous flow rate.

Varying orifice size at the same measurement condition, the velocity and flow rate behavior indicates similarity in time phasing and transition. This fact is illustrated in Fig. 4 with respect to the first 10 ms of injection period at $f=50$ Hz. The data was obtained under constant set of $\tau/T=0.1$. It is clearly seen that all transient time phases are settled at the same positions independently on the orifice diameter.

Therefore, two values among others, injection time and orifice size can be basically considered as operational factors dealt with transitions on the fuel flow rate. The plots of Fig. 5 correspond to the transient instantaneous flow rate within the second stage at the operation frequencies of 25 and 50 Hz. The

Fig. 3 Velocity (upper) and flow rate (lower) time series for variable injection time $0.1T$, $0.2T$ and $0.5T$ at $T=20$ ms

Fig. 4 Instantaneous velocity and flow rate for different orifice diameter $d=100$, 200 and 500 at $f=50$ Hz and $\tau/T=0.1$

Fig. 5 Initial transition of reconstructed flow rate at frequency 25 and 50 Hz

shortest injection duration rate, $0.1T$, causes the fastest transition and this effect becomes remarkable at the high injection frequency, 50 Hz. Certain transient injection can be adjusted by varying oscillating frequency and injection duration rate (τ/T) as well as by setting different orifice size.

4.2 Transient stages

Identical time intervals are defined by a pressure wave propagation along the pipe line (fuel transport

line) and the time phases at which injector valve is charged open or closed. On the basis of the results framed in Fig. 3 and Fig. 4, depending on a practical target, one can subdivide all injection cycles in a number of transient stages. To simplify further discussion, we have considered the four transient intervals.

The first stage, a pressure propagation interval, is started immediately after charging the injector valve to be open ($t=0$). Under initial high-pressure (p) condition, its waves are propagated along the pipe line (L) with a sound of the fluid (a). The fluid mass is excited and forced to move to injector exit. The pressure wave propagation takes a time $t_{prop}=2L/a$ that is 1.5 ms in present measurements. The same time interval will be presented in each further dynamic stages as the flowing time-delay ($t_d=t_{prop}$).

The second stage starts after full excitation of fluid mass, a transition dynamics causes the flow shown in Fig. 4 as "opened". It lasts until the valve, driven by the timing control signal, closes which is referred to as "max" in Fig. 4. The injection time, $\tau/T=0.1$ or $\tau=2$ ms, and due to added time of the first stage (1.5 ms), the maximum velocity and flow rate is settled at the time-phase of 3.5 ms. Positive temporal derivatives of velocity and flow rate characterize injection transitions. Their magnitude deals with orifice size as shown in lower half of Fig. 4. For $d200$ and $d100$ the peak values of flow rate and centerline velocity are proportional to orifice diameter.

The third stage, from maximum peak to a "zigzag" oscillation settled at the time of 5.8 ms ("closed"), is a transition which responds to the valve closing. The total duration of this phase (2.3 ms) is longer than the time delay itself (1.5 ms) because it also consumes a particular time to penetrate the valve into dynamic nozzle flow. "Zigzag" is a major characteristic of ending of the third stage that can be clearly seen in time-series of Fig. 3. and Fig. 4. For the velocity time series it lies on or slightly above zero line, for the flow rate series the "zigzag" characteristic presents itself in a negative zone due to a reversed velocity field discussed below.

The fourth stage is a series of damping oscillations after a "zigzag" point following the "closed" valve shown in Fig. 3. Opening or closing of the injector valve causes a delta type impact to pressure distribution along the pipe. Time response to this delta impact associates pressure sonic wave propagation in pipe flow. Closing valve, the second quasi-delta impact causes pipe flow and generates remaining oscillations. These transient oscillations are clearly observed at high frequency because cyclic fundamental time, $T=1/f$, is less than a pipe flow viscous time

Table 2 Mean velocity near the pipe wall

r mm	d100	d200	d500
0.947	0.110 m/s	0.220	0.230
1.000	0.0	0.0	0.0

Fig. 6 Reconstructed radial velocity profiles for time phases at maximum peak and closed valve

constant, $T_\mu=R^2/4\nu$.

4.3 Transient velocity field

In the previous study, development of the velocity field in pipe flow within an injection period was discussed in detail⁽⁶⁾. Presently, in Fig. 6, the radial velocity profiles for two time phases referenced in Fig. 4 to "max", t_m , and "closed", t_c respectively, are illustrated for different orifice diameters. During injection the transient flow field is clearly dependent on orifice size. The velocity profiles are gradually changed and show the same pattern of a parabolic profile of a steady pipe flow. Since the whole injection time of the second stage, 2 ms, is much less than the viscous time constant at the transient pipe flow, 250 ms, the flow profiles at the "max" are not fully developed. The velocity shear derivatives $\partial U/\partial r$ at the wall ($r=R=1$ mm) is also proportional to the nozzle diameter in accordance to computed results by using Eq. (2) and as tabulated in Table 2. At the end of the third stage, a point referred to as a "zigzag" in the time series, all velocity profiles are reversed. Because of this, the

Fig. 7 Reconstructed flow rate and pressure gradient time series

instantaneous flow rate becomes negative as indicated in Fig. 3 and Fig. 4.

4.4 Pressure gradient

In engineering practice the most suitable fuel flow rate indicators are based on pressure measurements⁽⁸⁾. To evaluate some characteristics of the pressure flow rate measurements, the pressure gradient time series was reconstructed on the basis of Eq. (1) from the centerline velocity measurements. The results obtained at $f=50$ and 25 Hz, $p=0.6$ MPa and $\tau=0.1 T$, are shown in Fig. 7. To compare the series in the same absolute time scale one-half and whole period respectively to $f=25$ and 50 Hz are represented here.

As was described above, since the first stage, the oscillation of pressure increases due to sonic wave propagation and excitation of fluid mass. At the moment of "opened" valve, at 1.5 ms, a pressure shock, short-time oscillation, indicates the start of a flow transition. In maximum peak, the pressure gradient changes a sign and the pipe flow is decelerated. After the instance of "closed" point the second pressure shock eventually settles to a "zigzag" point of the flow rate time series.

If it is available to employ a pair of good differential pressure indicators located separately

along a high-pressure pipeline, one can easily measure the instantaneous flow rate instead of the present LDA-based instrument. However, until now there is no such advanced indicator, since the LDA flow rate meter is effective.

5. Conclusions

Previous investigation by the authors has been performed with respect to the basic solution obtained for precision measurements of instantaneous flow rate and its experimental facility verification. Presently, the method was particularly applied to study transient injection properties that are dependent on the injection duration rate, τ/T , and the outflow orifice size. Based on the results presented we reached the following conclusions:

1. At high injection frequency, short injection duration rate, τ/T , influences transitions of velocity, flow rate and pressure gradient with respects to (i) variable injection frequency and (ii) orifice size settled downstream from the injector.

2. The cyclic injection period can be subdivided in a number of transient stages. The main reference points in the measured and reconstructed time-series are dealt with in the time-phases when the injector valve is charged to either open or close. At these phases a delta-type pressure impact causes pipe flow.

3. At the maximum peak in the time series of velocity and flow rate, the magnitude and velocity shear derivatives at the pipe wall are proportional to the outflow orifice diameter that is for diameter smaller than $200 \mu\text{m}$.

As a result, the present technique can be used for precise measurements of instantaneous flow rate with respect to advanced fuel injectors in order to analyze the basic injection characteristics aiming at their improvement and optimal operation.

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References

- (1) Harada, J., et. al., Development of Gasoline Direct Injection Engine, SAE Paper 970540, (1997).
- (2) Iwamoto, Y., et. al., Development of Gasoline Direct Injection Engine, SAE Paper 970541, (1997).
- (3) Liu, Z. and Reitz, R.D., Modeling Fuel Spray Impingement and Heat Transfer between Spray

- and Wall in Direct Injection Diesel Engines, Numerical Heat Transfer, Part A, (1995), p. 515-529.
- (4) Long, W., Ohtsuka, H. and Obokata, T., Characterization of Conical Spray Flow for Diesel Engine by Means of Laser Doppler Methods, JSME Int. J., Ser. B, Vol. 39, No. 3 (1996), p. 554-561.
 - (5) Ikeda, Y., et. al., Spray Formation and Dispersion of Size-Classified Fuel Droplet of Air-Assist Injector, Proceeding of the 8-th Int. Symp. on Appl. of LDA Techniques to Fluid Mech., Vol. 1 (1996), 13.6.1-13.6.8
 - (6) Durst, F., Ismailov, M. and Trimis, D., Measurements of Instantaneous Flow Rates in Periodically Operating Injection Systems, Experiments in Fluids, Vol. 20 (1996), p. 178-188.
 - (7) Lambossy, P., Oscillations Forcees d'un Liquide Incompressible et Visqueux dans un Tube Rigide et Horizontal, Calcul de la Force Frottement, Helv. Phys. Acta, Vol. 25 (1952), p. 371-386.
 - (8) Bosch, W., The Fuel Rate Indicator: A New Measuring Instrument for Display of the Characteristics of Individual Injection, SAE Paper 660749, (1966).
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