Direct Numerical Simulation of Turbulent Heat Transport at High Prandtl Numbers*

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Turbulent scalar transport phenomena at high Prandtl numbers up to $Pr=100$ were examined through direct numerical simulations of forced isotropic turbulence with a constant mean temperature gradient. The main storage required for computation was significantly reduced by employing a different number of numerical grids for each of the velocity and temperature fields, which contained markedly different microscales. At high Prandtl numbers, the pressure temperature-gradient correlation was found dominant, instead of the molecular dissipation, as a sink term in the turbulent heat flux budget. It was also found that low-wave-number components of the velocity fluctuation were solely responsible for the cascade of the temperature fluctuation irrespective of the Prandtl number.

Key Words: Turbulence, Heat Transfer, Computational Fluid Dynamics, Direct Numerical Simulation, Isotropic Turbulence, Prandtl Number, Turbulent Heat Flux

1. Introduction

Turbulent scalar transport phenomena such as heat transfer and mass diffusion are of great technological interest. In many engineering predictive codes, so-called turbulence closure models are being used, and the success of numerical prediction hinges on the accuracy of these turbulence models. Since turbulence models are generally established by modeling each of the fundamental processes including the production, diffusion and dissipation of turbulent statistics, detailed knowledge of these turbulence processes is indispensable for further improvement of turbulent models. With the recent advancement of high performance computers, quantitative evaluation of fundamental turbulent processes has become possible by exploiting a powerful numerical method called direct numerical simulation (DNS, hereafter), which solves the Navier-Stokes equation without any turbulent models.

Among DNSs involving scalar transport, Shirani

et al.⁽¹⁾, Kerr⁽²⁾ and Ruetsch & Maxey⁽³⁾ carried out DNSs of homogeneous isotropic turbulence without mean velocity gradient. Shirani et al.⁽¹⁾ performed simulations at $Pr=0.2-0.5$ and found that the Prandtl number markedly affects the integral scale, microscale and time scale of the scalar field. Kerr⁽²⁾ investigated the relationship between the high-order correlation functions and the turbulent structure, and found that the vorticity, deformation and scalar gradient are closely related to each other. Ruetsch & Maxey⁽³⁾ studied small-scale structures of turbulent vorticity and scalar fluctuation, and found that the scalar fluctuation is effectively dissipated by continuous deformation of the flow and that the area of high scalar dissipation has a sheet-like structure. On the other hand, Kim & Moin⁽⁴⁾, Kasagi et al.⁽⁵⁾ and Kasagi & Otsubo⁽⁶⁾ carried out DNSs of scalar transport in channel flow. Kim & Moin⁽⁴⁾ calculated the joint probability density function for velocity and temperature at $Pr=0.1, 0.71$ and 2.0 , and found that the scalar flux near the wall can be modeled by the same treatment applied to the Reynolds stress. Kasagi et al.⁽⁵⁾ studied the variance of temperature fluctuation, its dissipation rate and the budget of turbulent heat flux at $Pr=0.7$; they found that the fundamental processes of both heat and momentum transport show similar characteristics. Kasagi & Otsubo⁽⁶⁾ further

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carried out a DNS of low Prandtl number flow ($Pr = 0.025$) and found that the turbulent heat flux perpendicular to the wall becomes relatively small at low Prandtl number. They also reported that molecular dissipation, instead of the temperature pressure-gradient correlation, is dominant as a sink term in the turbulent heat flux budget at $Pr = 0.025$.

Thus, many investigations have been undertaken for turbulent scalar transport at $Pr < 1$, whilst few DNSs have been tried for a Prandtl number well beyond unity. Hence, little knowledge on the structure and statistics of high Prandtl number scalar turbulence has been established except for some extrapolative knowledge deduced by a dimensional analysis. Based on these backgrounds, we presently carry out DNS of high Prandtl number scalar turbulence; the Prandtl number is varied from 0.7 through 100 and its effect is examined. The objective flow field is homogeneous isotropic turbulence with a constant mean temperature gradient imposed in one direction. In order to keep the flow field stationary, numerical forcing is applied.

A general requirement for a respectable DNS is that the spatial and temporal resolution should be fine enough to numerically capture a broad range of turbulence scales, i.e., from energy-containing large scales to dissipative small scales. This means, for calculation of the velocity field, the number of computational grid points should be increased in proportion to $Re^{9/4}$. For the associated scalar field, however the number of grid points required becomes far more than that for the velocity field when Pr increases beyond 1. This fact is the major difficulty for DNS of high Pr scalar turbulence. Therefore, a different number of grid points is employed for each of the velocity and temperature fields in order to reduce the total computational load of required memory and CPU time. Thus, the second objective of this study is to explore a possibility of future DNSs of high Prandtl number turbulence.

2. Symbols

- E : Three-dimensional energy spectrum
 E_θ : Three-dimensional spectrum of temperature fluctuation
 k : Turbulent kinetic energy, $= \overline{u_i u_i} / 2$
 k_θ : Temperature variance, $= \overline{\theta^2}$
 L_b : Size of computational domain
 p : Pressure fluctuation or three-dimensional wave number, $= (p_1^2 + p_2^2 + p_3^2)^{1/2}$
 Pr : Prandtl number, $= \nu / \alpha$
 $P_{\theta\theta}$: Production rate of turbulent heat flux of $\overline{\nu\theta}$,
 $= -\overline{\nu^2} \frac{\partial \Theta}{\partial y}$

- q : Three-dimensional wave number, $= (q_1^2 + q_2^2 + q_3^2)^{1/2}$
 R : Time scale ratio, $= (k_\theta \epsilon) / (\epsilon_\theta k)$
 Re_t : Turbulent Reynolds number, $= k^2 / \nu \epsilon$
 S_θ : Transfer function of $\overline{\theta^2}$, Eq. (13)
 S_{ij} : Strain rate tensor, $= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$
 t : Time
 T_θ : Transfer function of $\overline{\theta^2}$, Eq. (15)
 u, v, w : x -, y - and z -components of velocity fluctuation
 u_i : i -th component of velocity fluctuation
 $\vec{u}$: Velocity vector
 V_θ : Transfer function of $\overline{\theta^2}$, Eq. (16)
 x, y, z : Cartesian coordinate system
 x_i : Cartesian coordinate system ($i = 1, 2, 3$)
 α : Thermal diffusivity
 ϵ : Dissipation rate of turbulent kinetic energy,
 $= \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}$
 ϵ_θ : Dissipation rate of temperature variance, $=$
 $2\alpha \frac{\partial \theta}{\partial x_j} \frac{\partial \theta}{\partial x_j}$
 $\epsilon_{\nu\theta}$: Dissipation rate of turbulent heat flux of $\overline{\nu\theta}$,
 $= (\alpha + \nu) \frac{\partial v}{\partial x_j} \frac{\partial \theta}{\partial x_j}$
 $\phi_{\nu\theta}$: Pressure temperature-gradient correlation,
 $= \overline{p \frac{\partial \theta}{\partial y}}$
 $\vec{\phi}$: Vector potential
 χ_i : Wave number in i -th direction
 χ : Three-dimensional wave number, $= (\chi_1^2 + \chi_2^2 + \chi_3^2)^{1/2}$
 $\vec{\chi}$: Wave number vector
 ν : Kinematic viscosity
 Θ, θ : Mean and fluctuating temperature
 $\vec{\omega}$: Vorticity vector
 Ω_{ij} : Rotation tensor $= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$

Superscripts

- $-$: Ensemble average
 $\wedge$: Fourier component

3. Computational Method

The computational domain and coordinate system are shown in Fig. 1. The domain is a cube which has a side length of 2π . The periodic boundary condition is used in all three directions and a constant mean temperature gradient is imposed in the y -direction.

A set of governing equations solved in the present simulation is shown below:

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{u} \times \vec{\omega}) - \nu \nabla^2 \vec{\omega} \quad (1)$$

$$\nabla \times \vec{\phi} = \vec{u} \quad (2)$$

$$\nabla^2 \vec{\phi} = -\vec{\omega} \quad (3)$$

Fig. 1 Computational domain and coordinate system

$$\frac{\partial \theta}{\partial t} + \vec{u} \cdot (\nabla \theta) = -v \frac{\partial \theta}{\partial y} - \alpha \nabla^2 \theta \quad (4)$$

These are the vorticity equation derived by taking the rotation of an incompressible Navier-Stokes equation, the continuity equation, the Poisson equation of vector potential derived from the definition of vorticity, and the energy equation, respectively. Neglecting the effect of buoyancy, temperature fluctuation is regarded as a passive scalar. The Fourier-spectral method is used for spatial discretization, whereas the Adams-Bashforth and Crank-Nicolson methods are employed for time advancement of the advection and dissipation terms, respectively. A forcing method used by Chen et al.⁽⁷⁾ is adopted in order to maintain the stationary flow field; otherwise it would simply decay without energy input. In this method, low wave number components of velocity fluctuations are multiplied with a constant larger than unity at each time step so that their energy level should be kept constant.

Computational conditions are shown in Table 1. Simulations were carried out at two Reynolds numbers, i.e., $Re_t=30$ (Case A) and $Re_t=85$ (Case B). In Case A, which needed a moderate number of grid points for the velocity field, the Prandtl number was changed over a wider range of $Pr=0.7-100$. In Case B, which demands more grid points, however, a smaller range of $Pr=0.7-20$ is covered. Note that, as shown in Table 1, different grid points have been used for velocity and temperature fields to save required memory and computational time, although it is a common practice to use the same grid points for both fields. For example, in Case A-4 of $Pr=100$, the number of grids is increased to 128^3 only for the temperature field, whilst that for the velocity field is kept the same; this resulted in a memory of 200 MB. However, a main memory of 500 MB would be required if 128^3 grid points were employed for both velocity and temperature fields. Thus, the required memory is reduced to about 2/5 by adopting this treatment. To do this, after the velocity field is renewed at each time step, higher wave number modes of velocity, e.g., wave numbers of 17 to 64 in Case

Table 1 Numerical grids and required memory

		Prandtl Number	Grid Points		Required Memory
			Velocity Field	Temp. Field	
Case A ($Re_t=30$)	A-1	0.7	32^3	32^3	8MB
	A-2	5.0	32^3	64^3	35MB
	A-3	20.0	64^3	64^3	70MB
	A-4	100	32^3	128^3	202MB
Case B ($Re_t=85$)	B-1	0.7	64^3	64^3	70MB
	B-2	20.0	64^3	128^3	242MB

A-4, are added with zero values, and then all the wave number components are transformed into physical space by an inverse fast Fourier transformation. This velocity field is then used for integrating the associated energy equation.

According to Eswaran & Pope⁽⁸⁾, the minimum number of grid points for the velocity field is given as $\chi_{\max} \eta > 1$, where $\eta(=(\nu^3/\varepsilon)^{1/4})$ is the Kolmogorov microscale and $\chi_{\max}$ is the highest wave number in simulation. In the present work, $\chi_{\max} \eta$ ranges over 1.5-2.0 and satisfies the above requirement for the velocity field. It is also known that, in order to obtain reliable statistical data, the ratio between larger and small time scales should be taken into account. For the temperature field, this ratio is increased in proportion to $Pr^{1/2}$. Iida & Kasagi⁽⁹⁾ employed a time step of $\Delta t=0.005$ and the total integration time was $t=5.5$ in their DNS of heat transport in isotropic turbulence at $Pr=0.2-1.5$. The total time steps were 1100. Since simulations at about 100 times larger Prandtl number are carried out in this study, a smaller time step of $\Delta t=0.002$ and longer integration time of $t=20.0$ are employed so that the computation is extended over about $100^{1/2}=10$ times more time steps.

Initial energy spectra are given by the following equations in Cases A and B, respectively:

$$E(\chi)=0.2 \exp(-0.1\chi) \quad (\text{Case A})$$

$$E(\chi)=0.25\chi^2 \exp(-0.8\chi) \quad (\text{Case B})$$

and conditioned to be statistically isotropic.

4. Turbulent Statistics

Temporal evolution of turbulent kinetic energy k and turbulent Reynolds number Re_t are shown in Figs. 2 and 3, respectively. While turbulence would decay in homogeneous isotropic turbulence without mean shear, the flow in this calculation reaches a statistically steady state after $t=10$ owing to numerical forcing. The turbulent Reynolds number is approximately 30 and 85 in Cases A and B, respectively.

The three-dimensional spectra of turbulent energy k and temperature variance k_θ at $t=20.0$ for

Fig. 2 Temporal evolution of turbulent kinetic energy

Fig. 3 Temporal evolution of turbulent Reynolds number

Fig. 4 Spectra of turbulent kinetic energy (k) and of variance of temperature fluctuation (k_θ) in Case A

Cases A and B are shown in Figs. 4 and 5, respectively. Note that these curves are normalized by $2k$ and k_θ . The spectra of k and k_θ are closely similar at $Pr = 0.7$ showing that both velocity and temperature fluctuations have almost the same scale. At higher Prandtl numbers, however, the spectrum of k_θ extends to higher wave numbers, and this fact means that the

Fig. 5 Spectra of turbulent kinetic energy (k) and of variance of temperature fluctuation (k_θ) in Case B

Fig. 6 Time scale ratio for various Prandtl numbers

scale of temperature fluctuation becomes smaller as the Prandtl number increases.

Figure 6 shows the time scale ratio $R [= (k_\theta \epsilon) / (\epsilon_\theta k)]$ for various Prandtl numbers. Previous DNSs of homogeneous decaying turbulence for $Pr = 0.25$ – 2.0 (Shirani et al.⁽¹⁾, Iida & Kasagi⁽⁹⁾, Miyauti & Ishizu⁽¹⁰⁾) show that the time scale ratio grows as the Prandtl number increases, and the present results confirm this tendency at $Pr = 0.7$ – 100 . However, Shirani et al.⁽¹⁾ pointed out that time scale ratio R becomes less dependent on the Prandtl number as the turbulent Reynolds number increases. This is also the case for the present results (see, data of Case B).

The budget of the turbulent heat flux in the direction of the mean temperature gradient (y -direction), $v\theta$, is shown in Fig. 7. The transport equation

Fig. 7 Budget of turbulent heat flux $\overline{v\theta}$ in Case B

of $\overline{v\theta}$ is derived as:

$$\frac{\partial \overline{v\theta}}{\partial t} = \underbrace{-\overline{v^2} \frac{\partial \theta}{\partial y}}_{P_{v\theta}} + \underbrace{p \frac{\partial \theta}{\partial y}}_{\phi_{v\theta}} - (\alpha + \nu) \underbrace{\frac{\partial v}{\partial x_i} \frac{\partial \theta}{\partial x_i}}_{\varepsilon_{v\theta}} \quad (5)$$

It is seen that, as the Prandtl number increases, the pressure temperature-gradient correlation term $\phi_{v\theta}$, instead of the dissipation term $\varepsilon_{v\theta}$, becomes dominant as a sink term. It is evident that this is partly because the molecular diffusion rate diminishes at high Prandtl numbers. As for $\phi_{v\theta}$, although the root-mean-square temperature gradient in the y -direction at $Pr=20.0$ is five times larger than that at $Pr=0.7$, the value of $\phi_{v\theta}$ itself remains almost the same. This fact means that the correlation coefficient between the pressure and the temperature gradient becomes smaller at higher Prandtl numbers.

The microscale of temperature fluctuation η_θ is a scale representing the dissipation rate of temperature fluctuation. It is defined using the dissipation rate ε_θ and its dissipation rate $\varepsilon_{\varepsilon_\theta}$, (Tennekes & Lumley⁽¹¹⁾),

$$\eta_\theta = \left(\frac{\alpha \varepsilon_\theta}{\varepsilon_{\varepsilon_\theta}} \right)^{1/2} \quad (6)$$

where α is thermal conductivity. Temperature fluctuation components of scales smaller than η_θ would be rapidly decayed due to effective heat conduction. Batchelor⁽¹²⁾ estimated the microscale of temperature fluctuation as follows:

$$\eta_\theta \propto \left(\frac{\alpha}{\gamma} \right)^{1/2} = \left(\frac{\alpha^2 \nu}{\varepsilon} \right)^{1/4} = Pr^{-1/2} \eta \quad (7)$$

assuming that, when $Pr \gg 1$, a scalar segment is stretched by flow field deformation as thin as $L_\theta \sim (\alpha/\gamma)^{1/2}$ before finally dissipating by molecular diffusion. Note, in Eq.(7), γ is a typical strain rate and estimated as $\gamma = (\varepsilon/\nu)^{1/2}$.

The microscale of temperature fluctuation η_θ normalized by the Kolmogorov microscale η is shown in Fig. 8. The ratio of η_θ/η is distributed linearly against the Prandtl number on this logarithmic plot. When fitted by the least squares method, the present data gives a correlation, which is proportional to $1.15Pr^{-0.45}$; this is in good agreement with Batchelor's hypothesis, Eq.(7). Thus, it is

Fig. 8 Ratio of microscale of temperature fluctuation and Kolmogorov microscale

reconfirmed that the number of computational grid points required for a temperature field should be proportional to $Re^{9/4} Pr^{3/2}$ when $Pr > 1$.

5. Structure of Temperature Field

Behavior of the temperature field is strongly affected by the structure of the velocity field. Hunt et al.⁽¹³⁾ classified characteristic regions of a turbulent velocity field using the second invariant of deformation tensor $II = (\partial u_i / \partial x_j)(\partial u_j / \partial x_i)$; e.g., the vortex region of $II < 0$ and the stagnation region of $II > 0$. The second invariant II can be decomposed as follows,

$$II = S_{ij}S_{ji} - \Omega_{ij}\Omega_{ji} \quad (8)$$

where S_{ij} and Ω_{ij} are strain rate and rotation tensors, respectively. From Eq.(8), it is clear that II can be interpreted as an index representing the ratio of strength between strain and rotation⁽¹⁴⁾. Instantaneous velocity vectors and a contour plot of the second invariant of deformation tensor at $t=20.0$ in Case B are shown in Figs. 9 and 10, respectively. It is observed that II has a small value in each of the vortex regions where the velocity vectors exhibit a rotational pattern.

The transport equation of square temperature gradient is derived as follows,

$$\frac{\partial}{\partial t} \left(\frac{1}{2} G_i^2 \right) = \underbrace{-\frac{1}{2} S_\theta G_{ji} u_{i,j}}_{P_{c1}} - \underbrace{G_i G_j S_{ij}}_{P_{c2}} + \alpha \left(\frac{\partial G_i}{\partial x_j} \right)^2 - \alpha \frac{\partial^2}{\partial x_i^2} \left(\frac{1}{2} G_i^2 \right) \quad (9)$$

where $G_i = \partial \theta / \partial x_i$. There are two production terms in the above equation; the production caused by the mean temperature gradient P_{c1} , and that generated by the deformation of the velocity field P_{c2} . If these

Fig. 9 Instantaneous velocity vectors in Case B

Fig. 10 Distribution of the second invariant (II) of deformation tensor in Case B: White to black, $II = -5$ to 5

two production terms are compared, it turns out that P_{c1} remains to be about $1/5$ of P_{c2} in Case B-1 ($Pr = 0.7$), but it reduces to $1/100$ in Case B-2 ($Pr = 20.0$). Therefore, the contribution of P_{c1} to the budget of G_i^2 decreases as the Prandtl number increases. On the other hand, P_{c2} is the main production term at high Prandtl numbers. Ruetsch & Maxey⁽³⁾ have also pointed out that a steep temperature gradient is generated in the region of continuous deformation.

Figure 11 shows the distribution of square temperature gradient G_i^2 in Case B-1 ($Pr = 0.7$) at the same instant as in Figs. 9 and 10, e.g., $t = 20.0$. It is found that regions of steep temperature gradient correspond to those of $II > 0$. The distribution of G_i^2 at a higher

Fig. 11 Distribution of G_i^2 in Case B-1 ($Pr = 0.7$): White to black, $G_i^2 = 0$ to 35

Fig. 12 Distribution of G_i^2 in Case B-2 ($Pr = 20.0$): White to black, $G_i^2 = 0$ to 10^3

Prandtl number ($Pr = 20.0$) in Case B-2 is shown in Fig. 12, where the structure of temperature fluctuation becomes much finer. In both Figs. 11 and 12, a vortex associated with a high temperature gradient is observed at the lower right. This engulfment of temperature gradient into a vortical motion is called *wrapping* (Guezennec et al.⁽¹⁵⁾). When $Pr = 20.0$ (Fig. 12), wrapping reaches even the center of the vortex. In this case, the temperature gradient generated around a vortex is exposed to a continuous deformation for a long time, because, when $Pr \gg 1$, the temperature fluctuation has a longer timespan before it is dissipated.

6. Transfer Functions in Fourier Space

In Fourier space, unlike in physical space, turbulent scales can be defined and evaluated quantitatively. Here, so-called transfer functions of temperature variance $\bar{\theta}^2$ are analyzed to investigate the cascade phenomenon of temperature fluctuation in Fourier space.

Assuming decaying homogeneous turbulence which has no production of temperature fluctuation, the energy equation is derived as follows:

$$\frac{\partial \theta}{\partial t} + \frac{\partial(u_i \theta)}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i^2} \quad (10)$$

Transformation of Eq.(10) into Fourier space leads to:

$$\frac{\partial \hat{\theta}(\vec{\chi})}{\partial t} + \alpha |\vec{\chi}|^2 \hat{\theta}(\vec{\chi}) = \underbrace{i \chi_i \sum_{\vec{p} + \vec{q} = \vec{\chi}} \hat{u}_i(\vec{p}) \hat{\theta}(\vec{q})}_{\text{Advection Term}} \quad (11)$$

Taking the real part after multiplying the complex conjugate $2\hat{\theta}^*(\vec{\chi})$ on both sides of Eq.(11), a transfer function of temperature variance is derived:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + 2\alpha \chi^2 \right) (\hat{\theta}(\vec{\chi}) \hat{\theta}^*(\vec{\chi})) \\ &= 2\chi_i \text{Im.} \left[\hat{\theta}^*(\vec{\chi}) \sum_{\vec{p} + \vec{q} = \vec{\chi}} \hat{u}_i(\vec{p}) \hat{\theta}(\vec{q}) \right] \quad (12) \end{aligned}$$

Advection Term

The right hand side of Eq.(12) represents the transfer of $\bar{\theta}^2$ between different wave numbers, $\vec{\chi}$ and $\vec{q}$, caused by nonlinear interaction of velocity and temperature fluctuations, that is, *cascade*. This term is defined as a transfer function with the absolute value of χ as a variable as follows:

$$T_\theta(\chi) = 2 \sum_{\chi \leq |\vec{\chi}| < \chi+1} \chi_i \text{Im.} \left[\hat{\theta}^*(\vec{\chi}) \sum_{\vec{p} + \vec{q} = \vec{\chi}} \hat{u}_i(\vec{p}) \hat{\theta}(\vec{q}) \right] \quad (13)$$

Further information about transfer of $\bar{\theta}^2$ can be obtained if Eq.(13) is regarded as a function of χ , p and q without taking summation over p and q . Thus, a function $T_\theta(\chi|p, q)$ is defined as follows:

$$T_\theta(\chi|p, q) = 2 \sum_{\substack{\chi \leq |\vec{\chi}| < \chi+1 \\ \vec{p} \leq \vec{p} < \vec{p}+1 \\ \vec{q} \leq \vec{q} < \vec{q}+1}} \chi_i \text{Im.} [\hat{\theta}^*(\vec{\chi}) \hat{u}_i(\vec{p}) \hat{\theta}(\vec{q})] \quad (14)$$

where $\vec{p} + \vec{q} = \vec{\chi}$. The function $T_\theta(\chi|p, q)$ is derived by decomposing the transfer function $T_\theta(\chi)$ into each wave number of velocity and temperature components. Taking summation over p or q for $T_\theta(\chi|p, q)$, two related functions are obtained:

$$S_\theta(\chi|q) = \sum_p T_\theta(\chi|p, q) \quad (15)$$

$$V_\theta(\chi|p) = \sum_q T_\theta(\chi|p, q) \quad (16)$$

where $S_\theta(\chi|q)$ and $V_\theta(\chi|p)$ represent the contributions of wave number components of velocity and temperature to the transfer of $\bar{\theta}^2$, respectively.

The transfer function $T_\theta(\chi)$ in Case B-1 ($Pr=0.7$)

Fig. 13 Transfer function of $\bar{\theta}^2$, $T_\theta(\chi)(Pr=0.7)$

Fig. 14 Transfer function of $\bar{\theta}^2$, $S_\theta(k|q)(Pr=0.7)$

Fig. 15 Transfer function of $\bar{\theta}^2$, $V_\theta(k|p)(Pr=0.7)$

is shown in Fig. 13. It is evident that the temperature variance $\bar{\theta}^2$ is transferred from low to high wave numbers, since $T_\theta(\chi)$ is negative at $\chi < 5$ while positive at $\chi > 5$.

The contribution of each wave number of temperature, q , to a particular wave number of $\chi=3, 6, 12$ and 24 is shown in Fig. 14. Focusing on $\chi=6$ as an example, the curve has a negative value at $\chi < 6$, while a positive value at $\chi > 6$. This fact means again that $\bar{\theta}^2$ is transferred from lower to higher wave number parts. In addition, the wave number modes of temperature within $\chi \pm 5$ are found to be mainly responsible for the cascade; namely, the local transfer should be dominant. In the same manner, the contribution of velocity component to the cascade of temperature variance at each wave number χ is shown in Fig. 15, where the wave numbers of $\chi < 10$ only contribute to the transfer. Furthermore, it should be noted that these characteristics at $Pr=0.7$ are also observed in the transfer functions at $Pr=20.0$ (see, Figs. 16 to 18)

Fig. 16 Transfer function of $\bar{\theta}^2$, $T_\theta(\chi)$ ($Pr=20.0$)

Fig. 17 Transfer function of $\bar{\theta}^2$, $S_\theta(\chi/q)$ ($Pr=20.0$)

Fig. 18 Transfer function of $\bar{\theta}^2$, $V_\theta(\chi/p)$ ($Pr=20.0$)

and at $Pr=100$ (Case A-4).

Although turbulent kinetic energy is mainly exchanged locally between two close wave number modes, the transfer is induced by non-local interaction between these two modes and a lower wave number component (Domaradzki & Rogallo⁽¹⁶⁾). Regarding the transfer of scalar, Yeung⁽¹⁷⁾ calculated the transfer function for fluids of $Pr(Sc)=0.25-1.0$, and showed that $\bar{\theta}^2$ is transferred between two close wave number modes of temperature, $\vec{\chi}$ and $\vec{q}$, by interaction with a lower wave number of velocity component $\vec{p}$. In the present study, the transfer functions at higher Prandtl numbers ($Pr=0.7-100$) have been calculated and the results showed similar tendency as those of Yeung⁽¹⁷⁾ irrespective of the Prandtl number.

Finally, as shown in Figs. 15 and 18, higher wave numbers of velocity component have less contribution to the development of the temperature field. Thus, the computational method proposed in this study, which saves the numerical grids for wave number compo-

nents of velocity higher than the Kolmogorov wave number $\chi_{Kol}=(\varepsilon/\nu)^{1/4}$, is found to be appropriate and useful for future DNS of scalar turbulence at high Prandtl numbers.

7. Conclusions

DNSs of homogeneous isotropic turbulence with heat transport at $Re_t=30-85$ and $Pr=0.7-100$ have been carried out, and the following conclusions are obtained.

(1) It is possible to perform DNS at a Prandtl number much larger than unity with moderate increase of computational load by employing approximately $Pr^{3/2}$ times more grid points only for the temperature field than that required for velocity field. Turbulent statistics can be calculated with sufficient accuracy by the present method, because the wave number components of velocity higher than the Kolmogorov wave number have little effect on the development of the temperature field.

(2) The time scale ratio R increases as the Prandtl number is increased; this is in good agreement with the tendency found in previous DNSs at lower Prandtl numbers.

(3) The pressure temperature-gradient correlation, instead of the molecular dissipation, is dominant as a sink term in the budget of turbulent heat flux $\overline{v\theta}$ at high Prandtl numbers.

(4) The microscale of temperature fluctuation η_θ is found to be proportional to $Pr^{-1/2}$ showing good agreement with the prediction of Batchelor⁽¹²⁾.

(5) Wrapping, which is a region of high temperature gradient turning into a vortex, is observed in the present flow field. At high Prandtl numbers, a steep temperature gradient reaches the center of vortex without the temperature fluctuation decaying.

(6) The transfer function of temperature variance $\bar{\theta}^2$ is analyzed in Fourier space. In a manner similar to the cascade of turbulent kinetic energy, two close wave number modes of temperature and a lower wave number mode of velocity are found responsible for the cascade transfer of $\bar{\theta}^2$ irrespective of the Prandtl number.

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