Low-Frequency Unsteadiness in Turbulent Elliptic Wakes*

Yukinobu ABE**, Masaru KIYA***
and Osamu MOCHIZUKI****

Velocity fluctuations in the near wake of an elliptic disk are analyzed by the wavelet transform with Morlet wavelet. The wavelet transform demonstrates two dominant periodic components in the velocity fluctuations. The periodic components, which correspond to an alternate vortex shedding in the minor plane and a meandering motion in the major plane, are found to have a low-frequency unsteadiness. The modulus of the wavelet transform reveals that the low-frequency unsteadiness has a central frequency of approximately one-fifth of the frequency of the corresponding periodic component in each plane. Moreover, the unsteadiness is in phase on both sides of the wake in the same plane but out of phase in the different planes. The unsteadiness is of large spatial extent in that the representative wavelength of the low-frequency unsteadiness is of the order of 30 minor diameters.

Key Words: Fluid Dynamics, Turbulence, Wake, Elliptic Disk, Vortex Shedding, Coherent Structure, Low-Frequency Unsteadiness, Wavelet Transform

1. Introduction

There are a number of asymmetric bodies in engineering applications. A typical example is side-view mirrors of cars. The wake of the mirrors passes along side windows of the cars, producing pressure fluctuations on the windows which may cause noise in the cabin. Structure of turbulent wakes of asymmetric bodies is not clarified yet, at least to the same extent as that of the axisymmetric wakes which have been studied mostly for spheres⁽¹⁾⁻⁽⁵⁾ or circular disks^{(6),(7)}.

Turbulent wake of an elliptic disk, which is a simple shape of asymmetric bodies, was found to have two periodic components of velocity fluctuations if the major diameter is 2 - 3 times the minor diameter; one is in the major plane while the other is in the minor plane^{(8),(9)}. The periodic component in the minor plane

is due to alternate shedding of hairpin-like vortices, while that in the major plane is likely to be associated with a meandering motion of the hairpin-like vortices⁽¹¹⁾. The elliptic wake also experiences an axis switching, which is a cross-over of half widths in the major and minor planes, being similar to that in an elliptic jet^{(10),(11)}.

Amplitude of the periodic velocity fluctuations in the elliptic wake is expected to have a low-frequency modulation which was observed in the turbulent vortex-street wakes of cylindrical bodies⁽¹²⁾⁻⁽¹⁵⁾ and in plane mixing layers⁽¹⁵⁾. The purpose of the present paper is to study this low-frequency unsteadiness in the elliptic wake by means of the wavelet transform with Morlet wavelet. Spatiotemporal characteristics of the low-frequency unsteadiness will be shown to be obtained by the auto and cross correlations of the fluctuation components of modulus of the wavelet transform.

2. Experimental Apparatus and Methods

The flow configuration and definition of main symbols are shown in Fig. 1. The x -axis is taken in the longitudinal direction, the y -axis along the major axis, and the z -axis along the minor axis. The origin is at the centre of the front face of the disk. The time-mean and fluctuating velocity components in the x direction are denoted by U and u . The r. m. s. value

* Received 30th March, 1999. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 64, No. 626, B (1998), p. 3156-3161 (Received 9th March, 1998)

** Division of Mechanical Science, Hokkaido University, N16W8, Kita-ku, Sapporo 060-8628, Japan, E-mail: abe@ring-me.eng.hokudai.ac.jp

*** Division of Mechanical Science, Hokkaido University, Japan. E-mail: kiya@eng.hokudai.ac.jp

**** Division of Mechanical Science, Hokkaido University, Japan. E-mail: osamu@eng.hokudai.ac.jp

Fig. 1 Elliptic disk, coordinate system, major and minor planes and velocity components

of u is denoted by u' . The xy -plane will be referred to as the major plane, while the xz -plane as the minor plane.

Experiments were performed in a through-flow wind tunnel with a 30.0 cm wide, 30.0 cm high and 100.0 cm long working section. The flow is introduced into the working section through a bell entrance of 1 : 9 contraction. A filter of glass wool is attached to the inlet of the calming chamber in which two stages of gauze screens are installed. The free-stream turbulence level is 0.5% at a main-flow velocity U_∞ of 15.0 m/s, which is employed in the present study. The velocity U is uniform within $\pm 0.5\%$ of U_∞ in the cross section at 0.30 m downstream of the inlet of the working section, except for the boundary layers on the tunnel walls. The thickness of the boundary layer at this section, which is turbulent, is approximately 3 cm.

Two elliptic disks of the minor diameter D of 20.0 mm were manufactured by machining from a brass plate of 3.0 mm in thickness. One has the major diameter L of 40.0 mm and the other has $L=60.0$ mm. The disks will be referred to by the aspect ratio $AR(=L/D)$, which is 2.0 for the former and 3.0 for the latter. The edge of the disks are bevelled towards the downstream side by the angle of 45.0° .

The elliptic disk was fixed at 0.30 m downstream of the inlet in the middle of the working section by three steel wires of 0.2 mm diameter. The wires are thin enough for their effects on the wake to be neglected. Moreover the blockage ratio of the disks is 0.70–1.0%, so that their effects on the wake can also be neglected. No sensible oscillation of the disks was observed during the experiment at the main-flow velocity $U_\infty=15.0$ m/s.

Measurements of the velocity components were made by constant-temperature hot-wire anemometers (KANOMAX Model 1010 & Model 1011) using I-wire probes of 5 μ m diameter and 1.0 mm working length, and a split-film probe of 132 μ m (TSI Model

Fig. 2 Morlet wavelet with $k_\phi=6.0$. —, real part; ---, imaginary part.

1287) with the plane of split normal to the longitudinal direction. The split-film probe was used in a region of intermittent reverse flow, while the I-wire probes were employed only in a region where such reverse flow is negligible. The instantaneous velocity components were digitized with a time interval of $\Delta t=0.5$ ms over 4.092 s to obtain the time-mean and fluctuating components and the wavelet transform. Measurements of the velocity were made in the major and minor planes.

The experiments were made at Reynolds number $Re(=U_\infty D/\nu, \nu$: kinematic viscosity) of 2.0×10^4 .

3. Wavelet Transform

The wavelet transform of the velocity fluctuation $u(t)$, where t is the time, is defined by

$$W(a, b) = \frac{1}{\sqrt{a}} \int \phi^* \left(\frac{t-b}{a} \right) u(t) dt \quad (1)$$

where a is scale parameter, b is translation, ϕ is a wavelet, and ϕ^* is the complex conjugate of ϕ . The wavelet employed in this study is Morlet wavelet defined by

$$\phi(T) = \exp(\sqrt{-1} k_\phi T) \exp\left(-\frac{T^2}{2}\right) \quad (2)$$

where $T=(t-b)/a$. The value of constant k_ϕ is chosen as 6.0 by the recommendation of Farge⁽¹⁶⁾. Figure 2 shows the real and imaginary parts of Morlet wavelet. This wavelet has been employed in the study of coherent structures in wakes^{(13),(15)} and jets⁽¹⁷⁾.

Amplitude of the real part of $W(a, b)$, which is denoted by W_r , follows the amplitude of the velocity fluctuation u if the scale a is fixed at the inverse of the vortex-shedding frequency; that is, a high amplitude of u corresponds to a high amplitude of W_r , and vice versa. On the other hand, a high amplitude of the imaginary part of $W(a, b)$, which is denoted by W_i ,

corresponds to a high amplitude of $\partial u/\partial t$ because the imaginary part of $\phi(t)$ is antisymmetric. Thus the time history of modulus $|W| (= (W_R^2 + W_I^2)^{1/2})$ is the envelope of the time history of the real part and the imaginary part. This implies that the fluctuating part of the modulus includes information on the amplitude modulation of the velocity fluctuation.

4. Results and Discussion

Figure 3 shows distributions of the time-mean velocity U and the r.m.s. velocity fluctuation u' in the major and minor planes in the wake of the elliptic disk of aspect ratio $AR=3.0$, which is defined as the major diameter divided by the minor diameter. The velocity fluctuation u has periodic components as shown by its power spectrum in Fig. 4. The spectrum of u in the minor plane has a sharp peak at a frequency of 80 Hz ;

Fig. 3 Distributions of time-mean velocity U and r. m. s. velocity fluctuation u' in the major and minor planes in the wake for $AR=3.0$. $\bullet$, U ; $\circ$, u' .

Fig. 4 Power spectrum of longitudinal velocity fluctuation u at (a) $(x/D, y/D, z/D) = (4.0, 0.0, 2.0)$ in the minor plane and (b) $(x/D, y/D, z/D) = (4.0, 2.0, 0.0)$ in the major plane for $AR=3.0$. Vertical scale is arbitrary linear

this frequency will be denoted by F_{minor} . The spectrum of u in the major plane also has a sharp peak at a lower frequency of 45 Hz ; this frequency will be denoted by F_{major} . In the middle of the major and minor planes, the spectrum had two peaks at the frequencies F_{minor} and F_{major} (Abe et al.^{(8),(9)}). Both spectrum peaks were detected as far as $x/D=20$. The spectrum peaks were also found for the disk of $AR=2.0$ at different frequencies of $F_{\text{minor}}=90$ Hz and $F_{\text{major}}=60$ Hz (Abe et al.^{(8),(9)}).

The periodic components will now be analyzed by the wavelet transform to study the low-frequency unsteadiness. In what follows, results for the disk of $AR=3.0$ will be presented because features of the low-frequency unsteadiness were basically the same for $AR=2.0$ and 3.0 .

Figure 5 shows contours of W_R and $|W|$ in the (a, b) parameter space, which were constructed from u near the edge of the wake in the minor plane. High peaks of W_R appears at the scale $a/\Delta t = 1/(F_{\text{minor}}\Delta t) = 24.0$, which is the period of the vortex shedding. Note that $\Delta t (=0.5$ ms) is the time interval of digitization of u . At the same scale, $|W|$ attains peaks with an interval much longer than that for W_R . This is interpreted as the low-frequency unsteadiness. Similar results are obtained for the velocity fluctuations in the major plane as shown in Fig. 6. In this case the high peaks of W_R appear at the scale $a/\Delta t = 1/(F_{\text{major}}\Delta t) = 41.0$.

Figure 7 shows the simultaneous fluctuating components of modulus in the major and minor planes, $|W|'_{\text{major}}$ and $|W|'_{\text{minor}}$, respectively. The velocity fluctuations were measured at the same positions as in

Fig. 5 Morlet wavelet transform of u at $(x/D, y/D, z/D) = (4.0, 0.0, 2.0)$ in minor plane for $AR=3.0$. (a) Real part W_R (positive values only), (b) modulus $|W|$ (values greater than 30% of maximum $|W|$), and (c) velocity fluctuation u . $\Delta t = 0.5$ ms is the interval of digitization of u

Fig. 6 Morlet wavelet transform of u at $(x/D, y/D, z/D) = (4.0, 2.0, 0.0)$ in major plane for $AR=3.0$. (a) Real part W_R (positive values only), (b) modulus $|W|$ (values greater than 30% of maximum $|W|$), and (c) velocity fluctuation u . $\Delta t = 0.5$ ms is the interval of digitization of u

Fig. 7 Time history of fluctuating part of modulus in the major plane at scale $a/\Delta t = 24.0$, $|W'|_{\text{major}}$, and that in the minor plane at $a/\Delta t = 41.0$, $|W'|_{\text{minor}}$ for $AR = 3.0$. —, $|W'|_{\text{major}}$; ---, $|W'|_{\text{minor}}$. Vertical scale is arbitrary linear. Positions of measurement of u are the same as those in Figs. 5 and 6

Figs. 5 and 6. The representative frequency of the low-frequency unsteadiness can be obtained in terms of the power spectrum of the fluctuating component, which is shown in Fig. 8. The spectrum in the minor plane has a broad peak at a frequency in a range 15–20 Hz, while that in the major plane has a broad peak at a frequency in another range 7–10 Hz. These representative frequencies are of the order of $1/5$ of the corresponding periodic components F_{minor} and F_{major} . In other words, every five shed vortices experience modulation in their strength. This was also the case for the elliptic disk of $AR=2.0$. In passing it is worth mentioning that the representative frequency for the

Fig. 8 Power spectrum of $|W'|$ (a) in the minor plane ($a/\Delta t = 24.0$) and (b) in the major plane ($a/\Delta t = 41.0$) for $AR=3.0$. The vertical axis is arbitrary linear

Fig. 9 Time history of fluctuating component of modulus at positions of maximum u' in minor plane for $AR=3.0$: $|W'|_+$ on side $z>0$ and $|W'|_-$ on side $z<0$ ($a/\Delta t = 24.0$). —, $|W'|_+$; ---, $|W'|_-$. Vertical axis is arbitrary linear

vortex-street wake of a circular cylinder is of the order of $1/25$ of the vortex-shedding frequency at Reynolds numbers of $80-10^4$ (Kiya and Ishikawa⁽¹⁵⁾).

Another aspect of the low-frequency unsteadiness can be obtained in terms of the cross correlation of $|W'|$ constructed from simultaneous velocity fluctuations at two opposite positions in each plane, a sample record of the $|W'|$ being given in Fig. 9. Figure 10 shows the cross correlation of $|W'|_+$ and $|W'|_-$ in the minor plane, $C_z(\tau)$, where τ is the time lag; $|W'|_+$ is $|W'|$ on the side $z>0$ while $|W'|_-$ is that on the opposite side $z<0$. The velocity fluctuations used to construct $|W'|_+$ and $|W'|_-$ are those at positions of maximum u' . The cross correlation takes a significant positive value of approximately 0.35 at $\tau=0$. This is also the case for the cross correlation $C_y(\tau)$ in the major plane. The above results indicate that the low-fre-

Fig. 10 Cross correlation coefficient of modulus $|W|_+$ and $|W|_-$ in major plane $C_y(a/\Delta t=24.0)$ and in minor plane $C_z(a/\Delta t=41.0)$ for $AR=3.0$. —, $C_y(\tau)$; ---, $C_z(\tau)$. τ is time lag

quency unsteadiness in the minor plane, for example, occurs in phase on both sides. The same is true in the major plane. If the high amplitudes of the velocity fluctuation are assumed to correspond to stronger shed vortices or higher amplitude of the meandering motion and thus a greater instantaneous width of the wake, the greater width should appear simultaneously on both sides in each plane. The reverse is true for the low amplitudes of the velocity fluctuation.

The phase relation of the low-frequency unsteadiness in the major and minor planes can be obtained in terms of the cross correlation coefficient of the fluctuating components of modulus $|W|'_{\text{major}}$ and $|W|'_{\text{minor}}$ (say $C_{yz}(\tau)$) based on the simultaneous velocity fluctuations. The modulus $|W|'_{\text{major}}$ and $|W|'_{\text{minor}}$ obtained at the positions of maximum u' were used to calculate the cross correlation. As shown in Figs. 11 and 12, $C_{yz}(\tau)$ attains a significant negative value of -0.4 for $AR=2.0$ and -0.25 for $AR=3.0$ at $\tau=0$, indicating that the unsteadiness in the two planes is out of phase. That is, when the wake is in the phase of enlargement in the major plane, the wake in the minor plane is in the phase of shrinkage, and vice versa. The same results were obtained in a range $x/D=2.0-8.0$.

The average distance between shed vortices is approximately $6.0D$ in the minor plane, so that the representative wavelength of the low-frequency unsteadiness is of the order of $30.0D$ because five consecutive vortices, on average, are included in one period of the unsteadiness. Thus the low-frequency modulation is of large spatial extent.

Low-frequency unsteadiness of the same nature has been observed in separated-and-reattaching flows^{(18),(19)} and in the vortex-street wake of cylindri-

Fig. 11 Cross correlation coefficient of $|W|'_{\text{major}}$ and $|W|'_{\text{minor}}$ at position of maximum u' , $C_{yz}(\tau)$, at $x/D=4.0$ for $AR=3.0$

Fig. 12 Cross correlation coefficient of $|W|'_{\text{major}}$ and $|W|'_{\text{minor}}$ at position of maximum u' , $C_{yz}(\tau)$, at $x/D=4.0$ for $AR=2.0$

cal bodies⁽¹²⁾⁻⁽¹⁴⁾. The unsteadiness in the separation bubbles is explained by a feedback loop. On the other hand, the unsteadiness in the vortex-street wake is interpreted as the large-scale vortex dislocations⁽¹²⁾ or as the beat of two slightly different frequencies of the vortex shedding at nearby spanwise positions. In the elliptic wake it is difficult to assume vortex dislocations or a feedback loop. Moreover, since $F_{\text{minor}}=85$ Hz and $F_{\text{major}}=50$ Hz for $AR=3.0$, the beat frequency should be 35 Hz, which is more than twice the representative frequency of the unsteadiness in the minor plane and five times that in the major plane. Thus the beat frequency is not consistent with the observed frequency of the unsteadiness. The mechanism of the low-frequency unsteadiness in the elliptic wake remains to be clarified in the future.

5. Conclusion

The present paper has described properties of low-frequency unsteadiness in the turbulent wake of

elliptic disks of aspect ratio of 2.0 or 3.0 at Reynolds number of 20 000. Main results may be summarized as follows.

(1) There are two periodic components of velocity fluctuations in the elliptic wake. One is in the minor plane, while the other is in the major plane. The component in the minor plane is associated with the alternate shedding of hairpin-like vortices. A structure responsible for the component in the major plane is likely to be a meandering motion of the wake in the same plane.

(2) The wake experiences a low-frequency unsteadiness which can be visualized as enlargement and shrinkage of the wake on the time scale of approximately five times the periods of the vortex shedding or the meandering motion. This was demonstrated from the wavelet analysis of the longitudinal velocity fluctuations in the minor and major planes, using the fluctuating component of modulus of the Morlet wavelet transform. The representative frequency of the unsteadiness is different in the two planes, being approximately one-fifth of the frequency of the vortex shedding in the minor plane and that of the meandering motion in the major plane.

(3) The unsteadiness is in phase on both sides of the wake in the same plane but out of phase in the different planes. The unsteadiness is of large spatial extent because the representative wavelength of the low-frequency unsteadiness is of the order of 30 minor diameters in the minor plane for $AR=3.0$. This could be compared with the distance between two consecutive vortices in the minor plane, which is approximately 6 minor diameters. The mechanism responsible for the unsteadiness remains to be clarified.

Acknowledgments

The authors express their sincere thanks to Mr. T. Sampo for his help in the construction of the experimental apparatus.

References

- (1) Achenbach, E., Vortex Shedding from Spheres, *J. Fluid Mech.*, Vol. 62 (1974), p. 209-221.
- (2) Taneda, S., Visual Observation of the Flow Past a Sphere at Reynolds Numbers between 104 and 106, *J. Fluid Mech.*, Vol. 85 (1978), p. 187-192.
- (3) Kim, H.J. and Durbin, P.A., Observations of the Frequencies in a Sphere Wake and of Drag Increase by Acoustic Excitation, *Phys. Fluids*, Vol. 31 (1988), p. 3260-3265.
- (4) Cannon, S., Champagne, F. and Glezer, A., Observations of Large-Scale Structures in Wakes Behind Axisymmetric Bodies, *Exp. Fluids*, Vol. 14 (1993), p. 447-450.
- (5) Sakamoto, H. and Haniu, H., A Study of Vortex Shedding from Spheres in a Uniform Flow, *ASME J. Fluids Eng.*, Vol. 112 (1990), p. 386-392.
- (6) Fuchs, H.V., Mercker, E. and Michel, U., Large-Scale Coherent Structures in the Wake of Axisymmetric Bodies, *J. Fluid Mech.*, Vol. 93 (1979), p. 185-207.
- (7) Berger, E., Scholz, D. and Schumm, M., Coherent Vortex Structures in the Wake of a Sphere and a Circular Disk at Rest and under Forced Vibrations, *J. Fluids & Struct.*, Vol. 4 (1990), p. 231-257.
- (8) Abe, Y., Kiya, M. and Mochizuki, O., Turbulence Properties in the Wake of an Elliptic Disk, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 63, No. 609, B (1997), p. 1520-1525.
- (9) Abe, Y., Kiya, M. and Mochizuki, O., Turbulence Structure in Elliptic Wakes, *Proc. JSME Cent. Grand Conf., Int. Conf. Fluid Eng.*, Vol. II (1997), p. 747-752, The Japan Society of Mechanical Engineers, Tokyo.
- (10) Ho, C. M. and Gutmark, E., Vortex Induction and Mass Entrainment in a Small-Aspect-Ratio Elliptic Jet, *J. Fluid Mech.*, Vol. 179 (1987), p. 383-405.
- (11) Hussain, F. and Husain, H. S., Elliptic Jets. Part 1. Characteristics of unexcited and Excited Jets, *J. Fluid Mech.*, Vol. 208 (1989), p. 257-320.
- (12) Williamson, C. H. K., The Natural and Formation of Spot-Like 'Vortex Dislocations' in the Transition of a Wake, *J. Fluid Mech.*, Vol. 243 (1992), p. 393-441.
- (13) Ishikawa, H., Kiya, M. and Mochizuki, O., POD and Wavelet Analyses of Coherent Structures in a Turbulent Mixing Layer, *Proc. JSME Cent. Grand Conf., Int. Conf. Fluid Eng.*, Vol. II (1997), p. 583-588, The Japan Society of Mechanical Engineers, Tokyo.
- (14) Najjar, F.M. and Balachandar, S., Low-Frequency Unsteadiness in the Wake of a Normal Plate, *J. Fluid Mech.*, Vol. 370 (1998), p. 101-147.
- (15) Kiya, M. and Ishikawa, J. 1998 Coherent Structures and Large Low-Frequency Irregularity in Free Turbulent Shear Flows Educed by Wavelet Transform and Proper Orthogonal Decomposition, *Proc. 2nd China-Japan Workshop on Turbulent Flows* (ed. Qing-Ding Wei), (1989), p. 8-22, Beijing University, Beijing.
- (16) Farge, M., Wavelet Transform and Their Applications to Turbulence, *Ann. Rev. Fluid Mech.*, Vol. 24 (1992), p. 395-457.
- (17) Walker, S.H., Gordeyev, S.V. and Thomas, F.O., *Exp. Fluids*, Vol. 23 (1997), p. 229-238.
- (18) Eaton, J.K. and Johnston, J.P., Low-Frequency Unsteadiness of a Reattaching Turbulent Shear Flow, In *Turbulent Shear Flows 3* (eds Bradbury, L. J. S. et al.), (1992), p. 162-170, Springer-Verlag, Berlin.
- (19) Kiya, M., Separation Bubbles, In *Theoretical and Applied Mechanics* (eds Germain, P. et al.), (1989), p. 173-191. Amsterdam, North-Holland.