Study on Criterion of Acoustic Resonant Vibration with Unsteady Fluid Dynamic Force*

Hiroki TANAKA**, Kazuhiro TANAKA**,
Fumio SHIMIZU** and Tohru IJIMA***

Ducts containing a tube bundle undergo acoustic resonant vibration. It is considered that the acoustic vibration is a kind of forced vibration by means of Karman vortex. However, it will be more reasonable to think that the vibration is a kind of mutual excited vibration, because fluid dynamic force excites a sound pressure and the sound pressure also generates the new fluid dynamic force. Equation of motion of sound pressure was derived from the mutual excited mechanism with the unsteady fluid dynamic forces. It was revealed from the equation that the imaginary part of the fluid dynamic force acting on the tube bundle supplies the energy of sound and on the other hand, radiation of sound consumes the energy. By comparing the supplying and consuming energies, it became possible to calculate a boundary where the acoustic resonant vibration will occur. The mechanism of supplying and consuming energies is discussed in this paper.

Key Words: Flow Induced Vibration, Aerodynamic Acoustics, Oscillatory Flow, Fluctuating Pressure, Unsteady Fluid Dynamics

1. Introduction

Loud noise some times occurs in boilers, heat exchangers, etc. when the outer tube gas flow reaches a certain velocity. It is considered that the loud noise is generated when the frequency of vortex shedding from tube bundle corresponds to the acoustic natural frequency of the cavity containing the tube bundles. The vortex shedding frequency is proportional to the gas velocity, so the resonant vibration occurs at a certain flow velocity. Many studies have been conducted to make the problems clear, as reviewed in papers⁽¹⁾⁻⁽³⁾ and textbooks^{(4),(5)}. The papers have stated that acoustic resonant vibration fundamentally occurs when the vortex shedding frequency coincides with the acoustic natural frequency of the cavity.

As many plant designers or operators have experienced,

however, the acoustic resonant vibrations do not always occur even though the frequencies coincide with each other. Conditions for occurrence of acoustic resonant vibration are diagrammatically shown in Fig. 1 and it suggests that some ducts produce strong acoustic vibrations and others do not. It is very important for designers to know at the design stage whether acoustic vibration will occur or not in the plant.

In terms of the point, several studies have been made⁽⁶⁾⁻⁽¹⁴⁾. The studies are mainly concerned the effects of the tube arrangement, Reynolds Number, Strouhal Number, etc. and many matters were made clear. However, the acoustic resonant vibrations are also influenced by the shape of the duct containing the

Fig. 1 Block diagram of acoustic resonant vibration

* Received 24th February, 1999. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 64, No. 626, B (1998), p. 3293-3298 (Received 24th December, 1997)

** Department of Mechanical System Engineering, Kyushu Institute of Technology, 680-4 Kawazu, Iizuka-City, Fukuoka, Japan. E-mail: hiroki@mse.kyutech.ac.jp

*** Muroran Institute of Technology, 27-1 Mizumoto-Machi, Muroran-City, Hokkaido, Japan

tube bundles and so the effects of the shape of the duct, the position of tube bundle, number of tubes, etc. must be taken into account to predict the occurrence of acoustic vibration.

A hypothesis was introduced that a supplying energy is necessary to maintain the acoustic resonant vibration and the supplying energy must be equal to or larger than the consuming energy. As shown in Fig. 2, it is considered that the flow around the tubes supplies the acoustic energy by the aerodynamic forces and the propagating sound though the duct walls or open holes consumes the energy. The supplying energy must be larger than the consuming energy, so it is necessary to get the amounts of supplying and consuming energy. In this paper, the mechanism of acoustic resonant vibration is described on the viewpoint of energy.

Nomenclature

- F : external force acting on a unit mass of gas
 p_t, p_l : pitch ratio of tubes
 T : kinetic energy
 U : steady uniform flow
 V : potential energy
 $V_r = 2\pi U/d\omega$: reduced velocity
 W : work due to fluid dynamic force
 a : area
 c : sound velocity
 C_m, C_d, C_k : mass, damping, stiffness coefficient of fluid dynamic force
 C_R, C_I : real and imaginary part of fluid dynamic force
 d : reference length (diameter of tube)
 f : vector of fluid force
 i : imaginary part
 $k = \omega/c$: acoustic reduced frequency
 l : reference length of cavity
 $l_a = l/d$: ratio of reference length
 p : sound pressure
 $p^* = p/\rho c^2$: pressure coefficient
 u : velocity of fluid particles induced by sound
 x : displacement of fluid particles
 ζ : damping ratio of sound pressure
 ρ : density of fluid
 ϕ : mode of sound pressure

Fig. 2 Supplying energy and scattering energy

ω : acoustic angular frequency

∇ : Hamiltonian

p_f : pressure induced by fluid dynamic force

v : volume

$s_t = \omega d/2\pi U$: mechanical Strouhal number or acoustic Strouhal number

ϕ : phase difference of fluid force

2. Mutual Excited Vibration of Sound Pressure and Fluid Dynamic Force

A sound pressure produces a fluid particle movement and the movement makes an oscillating flow. The oscillating flow gives a strong influence on the fluid dynamic force acting on a tube bundle^{(15)–(19)}. The fact means that the fluid dynamic forces produced by the oscillating flow in sound pressure are more or less different from the forces that are produced by a steady flow in a silent duct. So it is necessary to make clear the fluid dynamic forces which are affected by the oscillating flow. On the other hand, the sound pressure is maintained by the fluid forces, so the acoustic resonant vibration is considered to be a kind of mutual excited vibration of fluid dynamic force and sound pressure. Mechanism of the vibration is shown in Fig. 3 as a block diagram. The study⁽¹⁹⁾ suggested a possibility of the mutual excited vibration but the details have not been made clear. On the point of mutual excited vibration, many problems are left unsolved. Especially, in terms of the energy, studies are almost not conducted.

3. Vibration of Fluid Particles Induced by Sound Pressure

Sound is a vibration of pressure and so the sound induces a fluid particle movement. Velocity potential is introduced to express the velocity of fluid particle movement as followings;

$$u = \nabla \phi \quad (1)$$

Unsteady Bernoulli's equation shows that the unsteady pressure is described by the differential of the velocity potential with time as follows;

$$p = -\rho \frac{\partial \phi}{\partial t} \quad (2)$$

Subtracting the velocity potential ϕ from Eq. (1) and

Fig. 3 Block diagram of mutual-excited vibration

(2), the following equation is obtained;

$$\dot{\mathbf{u}} = -\frac{1}{\rho} \nabla(p) \quad (3)$$

The equation corresponds to the Euler's equation.

It can be assumed that acoustic vibration is sinusoidal and so the acceleration term is expressed as follows;

$$\dot{\mathbf{u}} = i\omega \mathbf{u} \quad (4)$$

Then the movement velocity of the fluid particles is expressed as follows;

$$\mathbf{u} = \frac{-1}{i\omega\rho} \nabla p \quad (5)$$

Displacement of the particles is expressed as follows;

$$\mathbf{x} = \frac{\mathbf{u}}{i\omega} = \frac{\nabla p}{\rho\omega^2} \quad (6)$$

As the simplest case, an acoustic resonant vibration between two parallel walls is shown in Fig. 4. Displacement of the fluid particles at the wall must be zero because the particles can not move on the wall. However, fluid particles can move easily in the center so the movement is expressed by a function having the maximum amplitude in the center and zero at the walls as shown in Fig. 4. Gradient of the pressure is proportional to the velocity and so is zero at the wall. This means that the amplitude of the pressure must be maximum at the walls because the gradient is zero. With respect to time, displacement of the fluid particles is in phase with the pressure and displacement velocity is 90 degrees in advance and acceleration is 180 degrees in advance of the pressure.

The angle of the oscillating flow is expressed as follows;

$$\alpha = \frac{u}{U} = \frac{\nabla p}{\omega\rho U} \quad (7)$$

The movement of fluid particles is zero at the walls and maximum in the middle. As shown by Eq.(7), the displacement of fluid particles is in proportion to the pressure gradient.

4. Fluid Dynamic Force and Sound Source

It is considered that the acoustic resonant vibra-

Fig. 4 Sound pressure and fluid displacement mode between two walls

tion is produced or maintained by the fluid particle movement that is induced by the Karman vortex shedding from the tube bundle. So if we examine the sound source, it is necessary to get the unsteady movement of each fluid particle among the tubes in detail. Several researchers are trying to calculate and some of them have succeeded in simple cases. However, the calculation is so complicate and needs so large computational capacity that it may not be able to apply the method to practical problems conveniently.

The unsteady momentum equations are used in this paper to make simple. Length of the cylinder is assumed to be unit and the flow is two dimensions. Control volume is introduced which include a single cylinder. Boundary of the control volume is expressed by s and inner area by a . The fluid dynamic force on a cylinder is expressed as follows; ⁽²⁰⁾

$$\mathbf{f} = \oint p d\mathbf{s} + \rho \oint \mathbf{u}(\mathbf{u} \cdot d\mathbf{s}) + \rho \frac{d}{dt} \iint_a \mathbf{u} da \quad (8)$$

If wavelength of the acoustic vibration is very large as compared to the tube diameter, the flow pattern around a tube must be the same as that of neighboring tubes. So the second terms of Eq.(8) should be zero. Fluid dynamic force is produced by a steady flow and also by a vibrating flow. The combined flow of uniform and vibrating flow is expressed as follows;

$$\mathbf{U} = \mathbf{U}_0 + \mathbf{u}e^{i\omega t} = \mathbf{U}_0 - \frac{\nabla p}{i\rho\omega} e^{i\omega t} \quad (9)$$

The second term shows the velocity induced by the sound pressure.

Let's distinguish the pressures by adding the suffix "s" as the pressure induced by the sound and suffix "f" as the pressure produced by fluid flow. Then the Eq.(8) can be expressed as follows;

$$\mathbf{f} = \oint (p_s + p_f) d\mathbf{s} + \rho \frac{d}{dt} \iint_a (\mathbf{u}_s + \mathbf{u}_f) da \quad (10)$$

As Eq.(3) shows, the pressure gradient of sound must be equal to the inertia force and so the following equation is obtained;

$$\oint p_s d\mathbf{s} + \rho \frac{d}{dt} \iint_a \mathbf{u}_s da = 0 \quad (11)$$

Then the net force acting on a tube due to fluid flow can be expressed as follows;

$$\mathbf{f} = \oint p_f d\mathbf{s} + \rho \frac{d}{dt} \iint_a \mathbf{u}_f da \quad (12)$$

Blevins⁽⁴⁾ has introduced the equation of acoustic vibration using external forces acting as body forces. Now, consider a small area, then equation of motion is described as follows;

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla p = \mathbf{F} \quad (13)$$

Next, equation of continuity is shown as follows;

$$\frac{\partial}{\partial t}(\rho) + \nabla(\rho \mathbf{u}) = 0 \quad (14)$$

Let's differentiate Eq.(13) with differential operator and Eq.(14) with time. Eliminate the same terms from these equations, then the following equation is obtained;

$$\frac{\partial^2}{\partial t^2}(\rho) - \nabla^2 p = -\nabla \mathbf{F} \quad (15)$$

The equation of sound speed is introduced;

$$\frac{\partial p}{\partial \rho} = c^2 \quad (16)$$

Using the above equation, ρ in Eq.(15) can be eliminated as follows;

$$\frac{\partial^2}{\partial t^2}(p) - c^2 \nabla^2(p) = -c^2 \nabla \mathbf{F} \quad (17)$$

Now, consider a small area including a single cylinder, body force generated by fluid dynamic force is equal to the fluid dynamic force on the cylinder.

$$\mathbf{F} dx dy = \mathbf{f} \quad (18)$$

Unsteady momentum equation (12) shows that the fluid dynamic force on a cylinder consists of pressure gradient and momentum change. However, as the momentum change is induced by the pressure gradient generated by sound, the fluid dynamic force can originally be expressed by pressure gradient only. Then, the following equation is obtained;

$$\mathbf{F} dx dy = \mathbf{f} = -(\nabla p_r) dx dy \quad (19)$$

It can be considered that the fluid dynamic force on tubes excites the sound pressure by means of pressure gradient that is generated by fluid flow.

5. Equation for Resonant Vibration

A cavity has its own acoustic natural frequencies and modes. If the frequency of acoustic vibration is in resonance with the natural frequency, the mode of sound pressure becomes stationary because the waves traveling forward and backward are synchronized with each other. The equation of sound pressure in resonant condition has already been deduced by Blevins⁽⁴⁾ so process of the deduction is briefly described. The sound pressure in a resonant condition can separately be expressed by time and space, as follows;

$$p_i(t, x, y) = p_i(t) \psi_i(x, y) \quad (20)$$

The vibrating velocity of fluid particles in i -mode is expressed as follows;

$$\begin{aligned} \mathbf{u}_i(t, x, y) &= \frac{-1}{i\omega_i \rho} \nabla p_i(t, x, y) \\ &= \frac{-1}{i\omega_i \rho} p_i(t) \nabla \psi_i(x, y) \end{aligned} \quad (21)$$

The energy of sound consists of the dynamic energy and the potential energy. The dynamic energy is expressed as follows;

$$\begin{aligned} T &= \frac{\rho}{2} \int \mathbf{u}^2 dv \\ &= \frac{1}{2} \frac{1}{\rho \omega^2} \int (\nabla p)^2 dv \end{aligned}$$

$$= \frac{1}{2} \frac{p(t)^2}{\rho \omega^2} \int (\nabla \psi(x, y, z))^2 dv \quad (22)$$

Introducing the bulk modulus K , the potential energy is expressed as follows;

$$V = \frac{1}{2} \frac{1}{K} \int p^2 dv = \frac{1}{2} \frac{p(t)^2}{\rho c^2} \int \psi(x, y, z)^2 dv \quad (23)$$

where,

$$c = \sqrt{\frac{K}{\rho}} \quad (24)$$

Next, the virtual work due to fluid dynamic force is considered. The displacement of fluid particles in the virtual work is expressed as follows;

$$\delta \mathbf{x} = \frac{1}{i\omega} \delta \mathbf{u} = \frac{1}{\rho \omega^2} \delta \nabla p = \frac{1}{\rho \omega^2} \nabla \psi(x, y) \delta p(t) \quad (25)$$

The virtual work due to the body force that is generated by the fluid dynamic force is;

$$\delta W = \int (\mathbf{F} \cdot \delta \mathbf{x}) dv = \delta p(t) \frac{1}{\rho \omega^2} \int (\mathbf{F} \cdot \nabla \psi) dv \quad (26)$$

Now, sinusoidal vibration is assumed. Multiplying T , V , W by $\rho \omega^2$, we can get the energy equation as follows;

$$\begin{aligned} L &= \rho \omega^2 (V - T - \delta W) = \frac{1}{2c^2} \dot{p}(t)^2 \int (\psi)^2 dv \\ &\quad - \frac{1}{2} p(t)^2 \int (\nabla \psi)^2 dv - \delta p(t) \int (\mathbf{F} \cdot \nabla \psi) dv \end{aligned} \quad (27)$$

Lagrange's equation is expressed as follows;

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{p}} \right) - \frac{\partial L}{\partial p} + \frac{\partial (L)}{\partial (\delta p)} = 0 \quad (28)$$

Then,

$$\begin{aligned} \ddot{p}(t) \frac{1}{c^2} \int (\psi)^2 dv + p(t) \int (\nabla \psi)^2 dv \\ = \int (\mathbf{F} \cdot \nabla \psi) dv = - \int (\nabla p_r \cdot \nabla \psi) dv \end{aligned} \quad (29)$$

The natural frequency of the cavity is expressed as follows;

$$\omega_0^2 = c^2 \int (\nabla \psi)^2 dv / \int (\psi)^2 dv \quad (30)$$

Replacing the term in Eq.(29) by Eq.(30), we get the following equation;

$$\ddot{p}(t) + \omega_0^2 p(t) = -c^2 \int \nabla p_r \cdot \nabla \psi dv / \int \psi^2 dv \quad (31)$$

However, the equation does not have a damping term. It is difficult to know what the damping consists of. So the general expression of damping force is used in the equation. Then, the equation becomes as follows;

$$\begin{aligned} \ddot{p}(t) + 2\zeta \omega_0 \dot{p}(t) + \omega_0^2 p(t) \\ = -c^2 \int \nabla p_r \cdot \nabla \psi dv / \int \psi^2 dv \end{aligned} \quad (32)$$

The equation expresses the sound pressure in resonant condition with the body force induced by the fluid dynamic force⁽⁴⁾. However, the equation does not have a feedback loop, so it is regarded as a forced vibration.

6. Fluid Dynamic Force in an Oscillating Flow

Making the Navier-Stokes' equation to non-

dimensional form, the Reynolds Number and Strouhal number can be induced and it is shown that the unsteady phenomena are obeyed by the Strouhal Number. However, the Strouhal number is generally used to express the vortex shedding frequency, so reduced velocity which is reciprocal of Strouhal number is introduced to express the unsteady flow conditions.

The amplitude of vibration of fluid particles is shown by Eq.(6). Unsteady fluid dynamic force consists of an inertia term, a damping term and a stiffness term so the force can be expressed as a complex number as follows;

$$\begin{aligned} f &= \rho d^2 c_m \ddot{x} + \rho d U c_d \dot{x} + \frac{1}{2} \rho U^2 c_k x \\ &= \frac{1}{2} \rho U^2 x \left\{ -2 \left(\frac{d\omega}{U} \right)^2 c_m + 2i \left(\frac{d\omega}{U} \right) c_d + c_k \right\} \\ &= \frac{1}{2} \rho U^2 x \left(-\frac{8\pi^2}{V_r^2} c_m + \frac{4i\pi}{V_r} c_d + c_k \right) \end{aligned} \quad (33)$$

where c_m, c_d, c_k are the fluid dynamic force coefficients. It is desirable that the coefficients have constant values, but they are generally variable with reduced velocity for its non-linear characteristics.

The inertia terms and stiffness terms are both in real part and so they have the same effect on the sound pressure. Gathering the real parts, the fluid force can be expressed as follows;

$$f = \frac{1}{2} \rho U^2 x (c_R(V_r) + i c_I(V_r)) \quad (34)$$

As can be seen from Eq.(34), the fluid dynamic force has real and imaginary parts and is the functions of reduced velocity and amplitude of fluid particle movement.

7. Acoustic Vibration and Fluid Force

7.1 Non-dimensional form of acoustic vibration

Let's assume the sound vibration is sinusoidal. The pressure is expressed using the non-dimensional form as follows;

$$p = p^* \rho c^2$$

From Eq.(32), following equation is obtained;

$$\begin{aligned} \rho c^2 (-\omega^2 + 2i\zeta\omega\omega_0 + \omega_0^2) p^* \\ = -c^2 \iint_a \nabla p_f \cdot \nabla \phi da / \iint_a \phi^2 da \end{aligned} \quad (35)$$

Fluid dynamic force is taken into account. Substituting Eq.(34) into Eq.(19), we get the following equation;

$$\nabla p_f = \frac{-f}{dx dy} = \frac{-1}{2} \frac{\rho U^2 x (c_R + i c_I)}{dx dy} \quad (36)$$

It is assumed that the pressure is a sinusoidal function of time. Dimensionless angular frequency is defined as follows;

$$k = \frac{l\omega}{c} \quad (37)$$

Introducing the dimensionless pressure, dimensionless

angular frequency, and substituting Eq.(36) into Eq.(35) the vibration can be described as follows;

$$\begin{aligned} (-k^2 + 2i\zeta k k_0 + k_0^2) p^* \\ = \frac{1}{2} \frac{l^2}{c^2} \iint \frac{U^2 x (c_R + i c_I)}{dx dy} \cdot \nabla \phi dv / \iint \phi^2 dv \end{aligned} \quad (38)$$

Here, new dimensionless quantities are introduced;

$$\nabla = \nabla^* / l$$

$$v = l^3 v^*$$

$$x = x^* d, \quad y = y^* d$$

dx, dy represent distance between the tubes, so that;

$$dx = p_i d$$

$$dy = p_i d$$

Substituting these values into Eq.(38), following equation is obtained;

$$\begin{aligned} (-k^2 + 2i\zeta k k_0 + k_0^2) p^* \\ = \frac{1}{2} \frac{V_r^2 k^2}{(2\pi)^2 l_d} \frac{\iint_a \left\{ \frac{x^* (c_R + i c_I) \nabla^* \phi}{p_i p_i} \right\} da^*}{\iint_a \phi^2 da^*} \end{aligned} \quad (39)$$

This is the dimensionless equation of sound pressure induced by fluid dynamic force. If the tube arrangement and fluid dynamic force coefficients are constant on all tube bundles, then the Eq.(39) can be rewritten as follows;

$$\begin{aligned} (-k^2 + 2i\zeta k k_0 + k_0^2) p^* \\ = \frac{1}{2} \frac{V_r^2 k^2}{(2\pi)^2 l_d} \frac{(c_R + i c_I)}{p_i p_i} \frac{\iint_a (x^* \nabla^* \phi) da^*}{\iint_a \phi^2 da^*} \end{aligned} \quad (40)$$

From Eq.(6), the dimensionless amplitude is described as follows;

$$x^* = \frac{x}{d} = \frac{\nabla p}{\rho \omega^2 d} = \frac{l p^* \nabla^* \phi}{d k^2} \quad (41)$$

Substituting the equation into Eq.(40) can be rewritten as follows;

$$\begin{aligned} (-k^2 + 2i\zeta k k_0 + k_0^2) p^* \\ = \frac{1}{2} \frac{V_r^2 p^* (c_R + i c_I)}{(2\pi)^2 p_i p_i} \frac{\iint (\nabla^* \phi)^2 dv^*}{\iint \phi^2 dv^*} \end{aligned} \quad (42)$$

The Eq.(42) has p^* on both sides, so the equation must be reduced to the following equation;

$$\begin{aligned} (-k^2 + 2i\zeta k k_0 + k_0^2) \\ - \frac{1}{2} \frac{V_r^2 (c_R + i c_I)}{(2\pi)^2 p_i p_i} \frac{\iint (\nabla^* \phi)^2 dv^*}{\iint \phi^2 dv^*} = 0 \end{aligned} \quad (43)$$

This is the characteristic equation of the acoustic resonant vibration induced by fluid dynamic force.

7.2 Solution of characteristic equation

We introduced a sinusoidal function of time in the above section. However, the real pressure does not always vibrate with steady amplitude and so pressure is expressed in this section as follows;

$$p(t) = \rho c^2 p^* e^{i\omega t} \quad (44)$$

where

$$\lambda^* = \frac{\lambda l}{c} \quad (45)$$

Then, the characteristic equation can be expressed as follows ;

$$\lambda^{*2} + \zeta k_0 \lambda^* + k_0^2 - f_R - i f_I = 0 \quad (46)$$

where f_R, f_I are real and imaginary parts of fluid dynamic force. Eigenvalue of the Eq.(46) is described as follows ;

$$\lambda^* = \frac{-\zeta k_0 + \text{Re}(a) \pm \text{Im}(a)}{2} \quad (47)$$

where

$$a = \sqrt{\zeta^2 k_0^2 - 4(k_0^2 - f_R - i f_I)} = k_0 r (\cos \theta + i \sin \theta) \quad (48)$$

$$r = \sqrt{\left(\zeta^2 - 4 + 4 \frac{f_R}{k_0^2}\right)^2 + \left(4 \frac{f_I}{k_0^2}\right)^2} \quad (49)$$

$$\theta = \frac{1}{2} \tan^{-1} \left\{ \left(4 \frac{f_I}{k_0^2} \right) / \left(\zeta^2 - 4 + 4 \frac{f_R}{k_0^2} \right) \right\} \quad (50)$$

However, the acoustic resonant vibration occurs almost at the acoustic natural frequency. In this case, the values are approximately expressed as follows ;

$$r = 2 \quad (51)$$

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{f_I}{k_0^2} / -1 \right) = \left(\frac{\pi}{2} - \frac{f_I}{2 k_0^2} \right)$$

Substituting them into Eq.(47) ;

$$\text{Re}(a) = \frac{f_I}{k_0} \quad \text{Im}(a) = 2 k_0$$

Then, the eigenvalue is ;

$$\lambda^* = \lambda_R \pm i \lambda_I = \frac{1}{2} \left(-\zeta k_0 + \frac{f_I}{k_0} \right) \pm i k_0 \quad (52)$$

If the real part of the eigenvalue is positive, the amplitude of sound pressure increases with time and it is expectable to generate a loud noise. So that, the condition where acoustic vibration will occur can be described as follows ;

$$\lambda_R = \frac{1}{2} (-2 \zeta k_0 + f_I / k_0)$$

$$= \frac{1}{2} \left(-2 \zeta k_0 + \frac{1}{2} \frac{V_r^2 c_I}{(2\pi)^2 k_0 p_i p_t} \frac{\oint (\nabla^* \phi)^2 dv^*}{\oint \phi^2 dv^*} \right) \geq 0 \quad (53)$$

After that,

$$\zeta \geq \zeta_e = \frac{1}{4} \frac{V_r^2 c_I}{(2\pi)^2 k_0^2 p_i p_t} \frac{\oint (\nabla^* \phi)^2 dv^*}{\oint \phi^2 dv^*} \quad (54)$$

The equation means that the consuming energy is larger than the supplying energy by fluid force. This is a new and another criterion whether vibration occurs or not.

8. Discussion

Rewrite the Eq.(54), and multiplying the both sides by terms $\rho c^2 p^*$, the following equation is obtained ;

$$\rho c^2 p^* k_0^2 \zeta \oint \phi^2 dv^* \geq \zeta_e$$

$$= \frac{1}{4} \frac{\rho c^2 p^* V_r^2 c_I}{(2\pi)^2 p_i p_t} \oint (\nabla^* \phi)^2 dv^* \quad (55)$$

Left hand side of the equation shows the damping force that consumes the energy of sound and right hand side shows the fluid dynamic force that supplies the energy.

Following items are necessary to apply the criterion. They are as acoustic vibration ;

Natural frequency : k

Damping ratio : ζ

Mode of sound pressure : ϕ

as fluid dynamic force ;

Imaginary part of fluid dynamic coefficient : $c_I (V_r)$

where, the coefficient must be the function of reduced velocity.

The natural frequency and pressure mode can be calculated by finite element method or boundary element method. However, it might not be easy to get the damping ratio. So it was tried to get the damping ratio from a frequency response curve of the sound pressure⁽²⁰⁾. The frequency response curve can be obtained with the boundary element method by putting the various frequency of sound source into the equation. The damping ratio can be calculated from the frequency response curve by the least error method.

The fluid dynamic force acting on a tube in the oscillating flow induced by sound pressure is also necessary to apply the criterion. However, it is not easy to get the forces numerically, so experiment was conducted⁽²¹⁾. Some samples of application of the criterion were shown in the paper⁽²¹⁾.

9. Concluding Remarks

The phenomena of acoustic vibration have complicated and difficult problems, however if we limit the subject on resonant vibration, the vibration has its own natural frequency and vibration mode, so the problems can be reduced to the one-dimensional vibration through the Lagrange's equation⁽⁴⁾. The one-dimensional sound equation however had been limited to the forced vibration and so this could not explain the fact that some plants generate a loud noise and others do not on the resonant condition. As far as the acoustic resonant vibration is concerned, it is more reasonable to consider that the acoustic vibration is the mutual excited vibration through unsteady fluid dynamic forces. In this paper, one-dimensional simple equation of mutual excited acoustic vibration was deduced.

Mechanism of the fluid dynamic force acting on the sound pressure is very difficult to investigate and may changes with case by case. However, if the

diameter of tube is very small as compared to the wavelength of sound pressure, the flow pattern around each tube becomes the same. The fluid dynamic force on a tube can be considered to correspond to the exciting force of sound pressure by the rule of action and reaction force. The fluid dynamic forces vary with the vibrating flow induced by the sound pressure, so the mutual excited vibration is made up.

If the unsteady fluid dynamic force in vibrating flow is obtained, the supplying energy by fluid force can be calculated and if the damping ratio of cavity is obtained, the consuming energy can be calculated. Comparing the supplying and consuming energies, we can judge whether a loud noise will be generated or not. A few samples of applying the criterion were shown in Ref.(22) and they showed a reasonable result. However the method must be applied to many other cases to make sure the accuracy of the judgment.

References

- (1) Paidoussis, M.P., A Review of Flow-Induced Vibrations in Reactor and Reactor Components, Nuclear Engineering and Design, Vol. 74, No. 1 (1982), p. 31-60.
- (2) Blevins, R.D., Review of Sound Induced by Vortex Shedding from Cylinders, Journal of Sound and Vibration, Vol. 92, No. 4 (1984), p. 455-470.
- (3) Weaver, D.S. and Fitzpatrick, J.A., A Review of Flow Induced Vibrations in Heat Exchangers, International Conference on Flow Induced Vibrations, Bowness-on-Windermere, England, May, 1987, p. 1-17.
- (4) Blevins, R.D., Flow-Induced Vibration, (1977), Van Nostrand Reinhold Company.
- (5) Chen, S.S., Flow-Induced Vibration of Circular Cylindrical Structures, (1987), Hemisphere Publishing Corporation.
- (6) Chen, Y.N., Flow-Induced Vibration and Noise in Tube-Bank Heat Exchangers Due to Von Karman Streets, ASME Journal of Engineering for Industry, Feb., (1968), p. 134-146.
- (7) Fitzpatrick, J.A., The Prediction of Flow-Induced Noise in Heat Exchanger Tube Arrays, Journal of Sound and Vibration, Vol. 99, No. 3 (1985), p. 425-435.
- (8) Fitzpatrick, J.A., A Design Guide Proposal for Avoidance of Acoustic Resonance in In-Line Heat Exchangers, ASME Journal of Vibration, Acoustics, Stresses, and Reliability in Design, Vol. 108 (1986), p. 296-300.
- (9) Blevins, R.D. and Blessler, M.M., Acoustic Resonance in Heat Exchanger Tube Bundles-Part 1: Physical Nature of the Phenomenon, Part 2: Prediction and Suppression of Resonance, ASME Symposium on Flow Induced Vibrations, Edit Au-Yang, Chen, PVP-122 (1987), p. 19-34.
- (10) Ziada, S., Oengoen, A. and Buhlmann, E.T., On Acoustical Resonance in Tube Arrays Part 1: Experiments, Journal of Fluids and Structures, Vol. 3 (1989), p. 293-314.
- (11) Ziada, S., Oengoen, A. and Buhlmann, E.T., On Acoustic Resonance in Tube Arrays Part 2: Damping Criteria, Journal of Fluids and Structures, Vol. 3 (1989), p. 315-324.
- (12) Eisinger, F.L., Sullivan, R.E. and Francis, J.T., A Review of Acoustic Vibration Criteria Compared to In-Service Experience with Steam Generator In-line Tube Buncs, ASME Journal of PVP, Vol. 116 (1994), p. 17-23.
- (13) Weaver, D.S., Vortex Shedding and Acoustic Resonance in Heat Exchanger Tube Arrays, Technology for the '90s. A Decade of Progress the ASME Pressure Vessels and Piping Division, Chapter 6, (1993), p. 777-810.
- (14) Eisinger, F.L., Francis, J.T. and Sullivan, R.E., Prediction of Acoustic Vibration in Steam Generator and Heat Exchanger Tube Banks, ASME Journal of Pressure Vessel Technology, Vol. 118 (1996), p. 221-236.
- (15) Blevins, R., The Effect of Sound on Vortex Shedding from Cylinders, Journal of Fluid Mechanics, Vol. 161 (1985), p. 217-237.
- (16) Archbald, D.S., Self-Excitation of an Acoustic Resonance by Vortex Shedding, Journal of Sound and Vibration, Vol. 28, No. 1 (1975), p. 81-103.
- (17) Oengoren, A. and Ziada, S., Vorticity Shedding and Acoustic Resonance in an In-Line Tube Bundle Part II-Acoustic Resonance, ASME, PVP-Vol. 206, Flow-Induced Vibration and Wear, (1991), p. 135-145.
- (18) Okamoto, S., Hirose, T. and Adachi, T., Effect of Sound Pressure on Vortex Shedding from a Single Cylinder (Acoustic Disturbance in Cylinder Axis Direction), Trans. Jpn. Soc. Mech. Eng., (in Japanese), (1980), p. 813-820.
- (19) Tanaka, H., Imayama, Z., Koga, K. and Katayama, K., An Experimental Study on the Acoustic Instabilities of Tube Arrays in Cross Flow, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 55, No. 509 (1989), p. 120-125.
- (20) Imai, I., Fluid Dynamics, (in Japanese), Vol. 1, (1973), p. 404, Shokabo.
- (21) Yan, Z., Tanaka, H., Matsuzoe, J. and Tanaka, K., Estimation of Damping Coefficient of Acoustic Vibration, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 62, No. 604 (1996), p. 4142-4147.
- (22) Yan, Z., Tanaka, H., Matsuzoe, J. and Tanaka, K., Criterion for Judging Acoustic Vibration, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 63, No. 612 (1997), p. 2719-2724.