Stability of a Magnetically Levitated Diamagnetic Fluid Column

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The stability of a magnetically levitated nonmagnetic liquid column in a magnetic fluid and a nonuniform magnetic field is studied theoretically and experimentally. The angular velocity equation of the interfacial wave is derived by assuming that (1) the fluids are inviscid, (2) the undisturbed interface between the levitated fluid and the surrounding fluid is an infinitely long cylindrical surface, (3) the amplitudes of the disturbances are very small, and (4) the magnetic susceptibility of each fluid is much smaller than 1. The stable conditions of different modes are discussed. The experimental study is performed with two very long magnetic north poles set face to face to form the nonuniform magnetic field, using water as the levitated fluid and a diluted oil-base magnetic fluid as the surrounding fluid.

Key Words: Stability, Magnetic Fluid, Wave, Fluid Dynamics

1. Introduction

Remote positioning and control of objects by magnetic field are of great interest in scientific research and engineering. In material processing, drug transport in blood vessels and separation of mixed matter, and in some other fields, the levitation or positioning of magnetic and nonmagnetic solid objects have been studied extensively. However, less attention has been paid to the study of the positioning of nonmagnetic fluid by using a magnetic field, which is very important in creating a containerless flow system without contact with the solid wall.

It is known that a diamagnetic material, even water, can be levitated in a very strong magnetic field\(^2\). By placing two north poles face to face, a nonuniform magnetic field, whose strength is the weakest at the center, and gradually becoming stronger in the outward direction, is obtained. When the field is strong enough, the diamagnetic fluid can stay stably in place around the center of the magnetic field. In a previous paper, we discussed the levitation of a diamagnetic droplet\(^2\).

If the magnetic poles are infinitely long, the center of the magnetic field is a line, which means the fluid forms a column as if in a virtual pipe. Because of this phenomenon, the magnetic field is called a magnetic pipe in our paper. In order to produce the magnetic pipe in air or other gas, the very strong magnet is required. However, with the help of the magnetic fluid as a surrounding fluid, the gravity of the levitated non-magnetic liquid can be balanced by the buoyancy and the magnetic force even in a far weaker magnetic field, and the magnetic pipe is easily simulated.

As the magnetic pipe is a virtual one, formed not by the solid wall but by the magnetic field, the levitated liquid column can be deformed or displaced from the equilibrium position. In the levitation, the stability of the levitated column is of great importance. The present study is on the oscillation of a thin diamagnetic liquid column magnetically levitated in a magnetic fluid under different kinds of small amplitude disturbances. A theoretical analysis is given for

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an arbitrary magnetic pipe. An experimental study was conducted using a very thin oil-base magnetic fluid, water and a pair of permanent magnets.

2. Mathematical Analysis

2.1 Governing equations

Figure 1 shows the analytical model. The diameter of the column is 2r0. The magnetic field in each fluid is

\[ \vec{H} = H_x \hat{i} + H_y \hat{j}, \]

where \( \vec{H} \) is the strength of the magnetic field, \( H_x \) and \( H_y \) are the \( x \)- and \( y \)-components, respectively, and \( \hat{i} \) and \( \hat{j} \) are unit vectors in the \( x \)- and \( y \)-directions, respectively. The wavelength, angular velocity and amplitude of the disturbances are \( \lambda \), \( \omega \) and \( A_0 \), respectively. The densities of the fluid column and the surrounding fluid are \( \rho_1 \) and \( \rho_0 \), respectively. The permeabilities of the two fluids are \( \mu_1 \) and \( \mu_2 \), respectively, and the permeability of the free space is \( \mu_0 \). The magnetic susceptibilities of the two fluids are \( \chi_1 \) and \( \chi_2 \), respectively.

Assuming that the fluids are inviscid, gravitational force is absent and the initial velocity is zero, a velocity potential of the disturbed fluids exists. Letting \( \phi_a \) represent the velocity potentials, where \( a = 1 \) indicates inner fluid 1 and \( a = 2 \) outer fluid 2, the equation of motion and the equation of continuity of each fluid are

\[ \rho_a \frac{D \phi_a}{Dt} = -\nabla (p + p_a), \]

\[ \nabla^2 \phi_a = 0, \]

where \( \rho_a \) is the density of each fluid, \( p_a \) is the magnetic pressure and \( p \) is the thermodynamic pressure,

\[ p = p(\rho, T), \]

\[ p_a = \rho_a \int_0^\infty \left( \frac{\partial M}{\partial \rho} \right)_{H_0} dH, \]

where \( T \) is the temperature, \( M \) is the magnetization and \( \rho = 1/\rho \) is the specific volume. The velocity boundary conditions at the static boundary and the interface between the two fluids are

\[ \left[ \frac{\partial \phi_a}{\partial x_i} \right]_{r=r_i} = 0, \]

\[ \left[ \frac{\partial \phi_1}{\partial x_i} \right]_{r=r_1} = \frac{\partial \phi_1}{\partial x_i} |_{r=r_1}, \]

where \( x_i (i=1, 2, 3) \) represents the directions of the coordinate system. The pressure condition at the interface is

\[ p_s^1 + p_{a1} = p_s^2 + p_{a2}, \]

where

\[ p_s = p + p_a + p_m, \]

\[ p_m = \frac{\mu M H}{\mu_0}, \]

\[ \frac{1}{2} \mu_s^2 \]

where \( R_1 \) and \( R_2 \) are the main radii of curvature of the interface and \( \sigma \) is the surface tension coefficient.

The boundary conditions of the magnetic field at the column surface are

\[ \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0, \quad \hat{n} \times (\vec{B}_1 - \vec{B}_2) = 0, \]

where \( \hat{n} \) is the unit vector normal to the surface and \( \vec{B} \) is the magnetic induction field vector.

Assuming that the amplitude of the oscillation is very small, the initial velocity is zero, and the disturbed velocity and pressure are very small, the nonlinear part of the acceleration in Eq. (2) is a higher order term compared to the unsteady term \( \partial \nabla \phi_a / \partial t \). Omitting the higher order term and letting \( \Pi_s = p + p_m \), Eq. (2) is changed to

\[ \rho_a \frac{\partial \phi_a}{\partial t} = -\Pi_s. \]

In a cylindrical coordinate system, assuming \( \phi_a \) in the form

\[ \phi_a = A_1(r)E, \quad E = e^{i\omega t} e^{i(\omega t - k_x x + k_y y)}, \]

the following solution of Eq. (3) is obtained:

\[ \phi_a = [C_1 I_\nu (k_x r) + C_2 K_\nu (k_x r)] E, \]

where \( I_\nu (k_x r) \) is the modified Bessel function of the first kind of order \( \nu \), and \( K_\nu (k_x r) \) is the modified Bessel function of the second kind of order \( \nu \). \( C_1 \) and \( C_2 \) are constants which should be determined by the boundary conditions. Factor \( \nu \) determines the oscillation type of the liquid column.

2.2 Motion of the interface

It is assumed that the shape of the interface between fluids 1 and 2 is represented by \( r = r_0 + \eta(\theta, z, t) \), and in the course of the motion of the interface, there is no fluid penetration through the interface from one fluid to the other. Then the flow velocity in the \( r \)-direction at the interface is

\[ V_r \approx \frac{\partial \eta}{\partial t}. \]

Thus we have
\[
\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial \gamma}.
\]  
(17)

For inner fluid 1, the velocity potential can be obtained from Eq. (15), as

\[
\phi_1 = C_I I_k(k_2 r_0) E.
\]

Substituting the above formula into Eq. (17), we obtain

\[
\eta = \frac{C_I I_k(k_2 r_0)}{i \omega} E
\]

\[
= \eta f E,
\]  
(19)

where \( \eta = \frac{C_I I_k(k_2 r_0)}{i \omega} \), thus

\[
C^I = \frac{I_{dk}(k_2 r_0) k_2}{I_k(k_2 r_0) k_2},
\]

\[
\rho \frac{\partial \phi}{\partial t} = - \rho \omega^2 \eta I_k(k_2 r_0) E.
\]

For outer fluid 2, the velocity potential is obtained from Eq. (15) as

\[
\phi_2 = [C_I I_k(k_2 r_0) + C_L K_{k_2}(k_2 r_0)] E.
\]

Substituting Eq. (22) into Eq. (6) in the \( r \)-direction, at the solid pipe wall of radius \( R \), which forms the outer boundary of outer fluid 2,

\[
C^I I_k(k_2 R) + C_L K_{k_2}(k_2 R) = 0.
\]  
(23)

On the surface of the levitated column,

\[
C^I I_k(k_2 R) + C_L K_{k_2}(k_2 R) = \frac{i \omega \eta}{k_2}.
\]

The solution of Eqs. (23) and (24) gives \( C^I \) and \( C^L \) which are substituted into Eq. (22) to get \( \phi_2 \). Thus

\[
\rho \frac{\partial \phi_2}{\partial t} = - \left( C_{I1} I_{dk}(k_2 r_0) + C_{I2} K_{k_2}(k_2 r_0) \right) \frac{\rho \omega^2 \eta}{k_2} E,
\]

(25)

where

\[
C_{I1} = \frac{K_{k_2}(k_2 R) I_{dk}(k_2 r_0)}{K_{k_2}(k_2 R) I_{dk}(k_2 r_0) - K_{k_2}(k_2 R) I(k_2 r_0)},
\]

\[
C_{I2} = \frac{I_{dk}(k_2 R) K_{k_2}(k_2 r_0)}{I_{dk}(k_2 R) K_{k_2}(k_2 r_0) - I(k_2 r_0) K_{k_2}(k_2 r_0)}.
\]  
(26, 27)

### 2.3 Capillary pressure

Before the column is perturbed, radii of curvature of the interface are \( R_1 = r_0 \) and \( R_2 = \infty \). After the shape of the interface is changed,

\[
\frac{1}{R_1} + \frac{1}{R_2} \approx \frac{1}{r_0} - \left( \frac{\partial^2 \eta}{\partial \gamma^2} \right) \frac{\partial \eta}{\partial \gamma}.
\]

(28)

Thus the capillary pressure difference in the course of oscillation can be written as

\[
\Delta p_c = \sigma \left[ \frac{1}{r_0} - \frac{1}{r_0} \left( \frac{\partial^2 \eta}{\partial \gamma^2} \right) \frac{\partial \eta}{\partial \gamma} \right] - \frac{\sigma}{r_0} \approx k_2 \sigma + \frac{1}{r_0^2} \sigma.
\]

(29)

### 2.4 Magnetic pressure

It is assumed that the initial magnetic field is \( H_0 \), and after the interface is disturbed, it becomes \( H \), the absolute value \( H \) of which can be represented by

\[
H = \left( H_0 + \frac{\partial H}{\partial \gamma} \right) \eta + h,
\]

(30)

where \( H_0 \) is the absolute value of \( H_0 \), and \( h \) is a small value produced by the shape change of the interface. Recalling the definition of susceptibility,

\[
M = \chi H,
\]

(31)

where \( M \) is the magnetization and, for convenience, letting

\[
k_r = \frac{\partial H}{H_0 \partial \gamma} \eta,
\]

(32)

then, the magnetic pressures \( p_m \) and \( p_m \) are obtained from Eqs. (10), (11), (30), (31) and (32) by integration

\[
p_m = \mu_0 \int_{r_0}^{r_0} \int_{r_0}^{r_0} \frac{\partial^2 \eta}{\partial \gamma^2} \frac{\partial H}{\partial \gamma} \eta + \frac{\partial H}{\partial \gamma} \eta + H_0 \eta H_0 \eta,
\]

(33)

where \( h_r \) represents the \( r \)-component of \( h \) and \( \tilde{M} = \frac{1}{H_0 \gamma} \partial \eta \).  

#### 2.5 Frequency

Taking into account Eq. (13), Eq. (7) is changed to the form

\[
- \rho \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \gamma^2} \eta + \frac{\partial^2 \phi}{\partial \gamma^2} \eta = -\rho \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \gamma^2} \eta + \frac{\partial^2 \phi}{\partial \gamma^2} \eta - \frac{\partial H}{\partial \gamma} \eta - H_0 \eta H_0 \eta,
\]

(35)

where subscript 1 indicates fluid 1, subscript 2 indicates fluid 2, and \( \Pi_\omega \) is the undisturbed \( \Pi \). Substituting Eqs. (33) and (34) into Eq. (35), the following relation is obtained:

\[
- \rho \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \gamma^2} \eta + \frac{\partial^2 \phi}{\partial \gamma^2} \eta = -\rho \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \gamma^2} \eta + \frac{\partial^2 \phi}{\partial \gamma^2} \eta - \frac{\partial H}{\partial \gamma} \eta - H_0 \eta H_0 \eta,
\]

(36)

If the magnetic field is uniform \( (k_r = 0) \), \( H_0 \) is the dominant term. This case, however, is not discussed here.

In the magnetic field shown in Fig. 1, the magnetic field is a nonuniform one with a large gradient. For small-amplitude oscillation, \( h \) is a smaller order value compared with \( k_r \). Bearing in mind that \( \chi \ll 1 \), that we are dealing with a small-amplitude problem and that in the undisturbed position the pressures are balanced, Eq. (36) is simplified to

\[
- \rho \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \gamma^2} \eta + \frac{\partial^2 \phi}{\partial \gamma^2} \eta = -\rho \frac{\partial \phi}{\partial t} - \frac{\partial H}{\partial \gamma} \eta.
\]

(37)

Substituting Eqs. (21), (25) and (29) into Eq. (37), we obtain

\[
- \rho \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \gamma^2} \eta + \frac{\partial^2 \phi}{\partial \gamma^2} \eta = \left( C_{I1} I_{dk}(k_2 r_0) + C_{I2} K_{k_2}(k_2 r_0) \right) \frac{\rho \omega^2 \eta}{k_2} E
\]

\[
+ \rho \omega^2 \eta \frac{\partial^2 \phi}{\partial \gamma^2} \frac{\partial^2 \phi}{\partial \gamma^2} \eta = -\frac{\partial H}{\partial \gamma} \eta - H_0 \eta H_0 \eta.
\]

(38)
The magnetic field produced by \( n \) pairs of magnets, as shown in Fig. 1, is approximated as
\[
H_a = H_0 \left( \frac{r}{r_0} \right)^{2n-1} \cos \left( \frac{\theta}{4n} \right)
\]
\[
H_b = H_0 \left( \frac{r}{r_0} \right)^{2n-1} \sin \left( \frac{\theta}{4n} \right)
\]  
(39)

or
\[
H = \left( \frac{r}{r_0} \right)^{2n-1} H_0,
\]  
(40)

where \( H_0 \) is the magnetic flux density at a fixed radius of \( r = r_0 \). In the experimental study mentioned later, one pair of magnets was used and the magnetic field was \( H = (r/r_0)H_0 \). More generally, setting the magnetic field as
\[
H = f(r)H_0,
\]  
(41)

the angular speed takes the form
\[
\omega^2 = - \frac{k^4 + \nu^2 f'(r_0)f(r_0)N_{so} + \nu^2 - 1}{\left( 1 - \nu \right) f'(r_0) f(k^*)/f(k^*)} k^* \]  
(42)

where \( k^* = k_0 r_0 \) is the dimensionless wave number, \( N_{so} = \mu_0 (\varphi_2 - \varphi_1) H_0^2 r_0 / \sigma \) is the magnetic Bond number which describes the strength of the levitation force, \( \varphi \) is the density ratio of the two fluids and \( \omega^2 = \varphi_1 r_0^2 \omega^2 / \sigma \) is the square of the dimensionless angular velocity.

If the right-hand side of Eq. (42) is negative, the angular frequency \( \omega \) has two pure imaginary roots. In this case, it is seen from Eqs. (14) and (15) that the amplitude of the disturbance grows exponentially with time and the interface is unstable. Otherwise, the amplitude of the disturbance does not change with time, which means the liquid column can be stably levitated, and the disturbance propagates with a finite speed of \( C^* = \omega^* / k^* \).

2.6 Analytical results

(1) Mode of oscillation

Equation (19) describes the shape of the disturbed interface. When \( \nu = 0 \), which is called mode 0 in the present study, \( \eta \) does not change in the \( \theta \)-direction. This means the cross section of the column remains circular during oscillation, which is the case when the disturbance is an axial one. When \( \nu = 1 \), which is called mode 1, \( \eta \) changes in the \( \theta \)-direction in the \( \cos \theta \) pattern. This makes the center of the cross section deviate from the undisturbed position and the amount of deviation changes with time. This is the case when the disturbance is in an \( x \)- or a \( y \)-directional wavy pattern or their combination in the \( \theta \)-direction, which is called a rotational pattern.

When \( \nu = 2 \) (mode 2), the cross section becomes elliptical. For higher modes, the cross section is triangular, square and so on.

(2) Stability condition

\[ f'(r_0)N_{so} \geq \frac{1 - k^* - \nu^2}{\nu^2 f'(r_0)}. \]  
(44)

Because the right-hand side of formula (44) can be positive, in order to satisfy Eq. (44) at any value of \( k^* \) and \( \nu \), the left-hand side must be positive. Thus, when the strength of the magnetic field decreases in \( r \)-direction or \( f'(r_0) < 0 \), the column is stable only when \( N_{so} < 0 \), which means that the susceptibility of inner fluid 1 must be larger than that of the outer fluid. A case similar to this is mentioned in Ref. (3). Because there is no magnetic source or magnetic divergence, a field with \( f'(r) < 0 \) does not exist. In the present paper, only \( f'(r) > 0 \), or the strength of the magnetic field increases in \( r \)-direction, is discussed.

(3) Effect of the magnetic Bond number

In Eq. (42), the right-hand part is always real, and \( \omega^* \) can only be real or pure imaginary, which means the stable condition of the column is
\[ \omega^* \geq 0 \]  
(43)

or

\[ f'(r_0)N_{so} \geq \frac{1 - k^* - \nu^2}{\nu^2 f'(r_0)}. \]  
(44)
Fig. 4 Dispersion relationship of different modes: $N_{B0}=5, \nu=1$

It can be seen from this formula that if the column is stable when $\nu=0$, it is also stable when $\nu>0$. Thus, in order to maintain a stable levitated column, only mode 0 need be studied. Figures 2 and 3 show two of the stable ranges of different types of oscillations when $\nu=1, \nu=0$ and $\nu=1$.

As shown in the figures, when the magnetic Bond number $N_{B0}>1$, the column is always stable. The increases in the strength of the magnetic field or the gradient of the magnetic field stabilizes the levitated liquid column.

When $1>N_{B0}>0$, mode 0 is unstable for the range of $k^*<(2\pi r_0)/(\sqrt{1-\nu^2})(r_0)^2N_{B0}$, but mode 1 is always stable. For the modes 0 and 1 to be stable, the susceptibility of the inner fluid should be less than that of the outer fluid, but for the stability of higher modes, this is not necessarily the case.

(4) Wave velocity

Figure 2 also shows that in the stable range of mode 0, the angular frequency increases with an increase in magnetic Bond number $N_{B0}$. The increase in magnetic Bond number $N_{B0}$ suggests an increase in magnetic pressure which stabilizes the liquid column. Because the wave velocity $C^*=\omega^*/k^*$, the increase in angular frequency also results in the increase in wave velocity. The same is the case for mode 1, as shown in Fig. 3. However, mode 1 is more stable than mode 0, and $\omega^*$ is larger. The minimum $\omega^*$ exists at $k^*=0$, but is not 0, unlike the case of mode 0.

Similar trends are seen at higher modes, as shown in Fig. 4.

(5) Effect of density ratio

Equation (42) shows that with the increase in $x$, $\omega^*$ decreases and the wave velocity decreases. When $x=0$, Eq. (42) describes the case where the outer fluid is gas. In this case, if $N_{B0}=0$, Eq. (42) describes the stability of a liquid jet in air with uniform velocity. When $x\to\infty$, $\omega^*=0$. This is the case when the levitated fluid is gas.

(6) Effect of outer boundary

When the outer boundary of the outer fluid is infinitely far or $R\to\infty$, $C_1=0$ and $C_2=-1$, and the $\omega^*$ and the wave velocity increases. In the analysis, when $R>(3-5)r_0$, the error produced by setting $C_1=0$ and $C_2=1$ is so small that it can be disregarded.

3. Experimental Study

3.1 Disturbances and fluids

It is known from Eq. (44) that $k^*$, $N_{B0}$ and $\nu$ control the stability of the column when the magnetic field and the radius of the column are known. Equation (42) gives the angular speed of the disturbances. In order to examine this result, three groups of experiments were performed. Table 1 shows the levitated fluids, surrounding fluids and types of disturbances in the experiments. Experiment 1 is designed to test mode 0 and experiments 2 and 3, mode 1.

The axial disturbance is produced using a reciprocating piston system, the $y$-directional one, a swinging lever system and the angular one, a rotating eccentric disk.

Magnetic fluids are kerosene-diluted oil-base ones and their physical properties are given in Table 2. The interface tension was measured at the contact surface with the water. The temperature of water is $11^\circ\text{C}$.

Figure 5 shows the magnetization curve of the undiluted magnetic fluid.

3.2 Experimental steps

Figure 6 shows the schematic of the experimental devices. Two magnetic north poles are placed face to
Table 2 Properties of the diluted magnetic fluid

<table>
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<tr>
<th>Volume concentration (%)</th>
<th>Density $\rho_2$ ($\times 10^2$ kg/m$^3$)</th>
<th>Surface tension $\sigma$ ($\times 10^{-2}$ N/m)</th>
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<td>1.0</td>
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<td>3.08</td>
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</tr>
<tr>
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<td>0.790</td>
<td>2.59</td>
</tr>
</tbody>
</table>

Fig. 5 Magnetization of magnetic fluid

Fig. 6 Schematic of the experimental system

Fig. 7 Magnetic field in the middle cross section

face to form a nonuniform magnetic field. The size of the magnet is 200 mm $\times$ 20 mm $\times$ 20 mm. The magnetic flux density on the surface of the magnet is $0.4[T]$. The gap between the two magnets is 24.8 mm. Figure 7 shows the measured magnetic field in the middle cross section of the column. The length of the levitated column is 97 mm. The coordinate system is shown in Fig. 6.

The rotational speed of the shaft ranges from 400 rpm to 700 rpm. An acrylic box filled with magnetic fluid is placed between the two magnets. The size of the acrylic box is 200 mm $\times$ 24 mm $\times$ 24 mm whose cross section is equivalent to a circle with radius $R$. Water is injected into the central part of the magnetic fluid through a small hole in the sidewall of the box. The volume of water is 10, 14 and 18 ml, and the diameter 2% of water column is 5.31 $\sim$ 5.73, 6.28 $\sim$ 6.78 and 7.12 $\sim$ 7.69 mm, respectively. No air is allowed to exist in the box during the experiment.

A high-power electric light is used to generate a beam strong enough to penetrate the diluted magnetic fluid and water so that a clear image can be taken.

A motor and a shaft are used to produce the disturbances. At one end of the shaft, a round concavity (B in Fig. 6) in the surface is attached to contact the levitated water. The centerline of the concavity is deviated eccentrically from the axis of the shaft to produce the rotational disturbance. A frequency control device is used to regulate the rotational speed of the motor to change the frequency of the disturbance.

The axial and $y$-directional disturbances are produced in a similar way, by moving the concavity in the $x$- and $y$-directions using the reciprocating piston system and the swinging lever system.

In the experiment, the levitated column must be shorter than the magnets but longer than the interfacial wavelength.

The interfacial wave pattern caused by the disturbance is recorded by a CCD video camera and is input into a computer through a video card. The wavelength of the interfacial wave is therefore measured using graphic software. The cycle length of the
interfacial wave is obtained by measuring the rotation speed of the motor.

3.3 Experimental results and discussion

In the analysis, the stability condition of the liquid column and the angular velocity of the wave in the stable region were discussed. However, experimental definition of the stability condition was difficult, and the wave velocity in the stable condition was measured.

Figure 8 shows a typical set of images for rotational disturbance. The bright inner part is water. The outer black part is the kerosene-diluted oil-base magnetic fluid. The disturbance source is at the right end.

Figures 9-11 show the experimental results and the theoretical results of the angular velocity depend on the wave number. Plots with similar magnetic Bond numbers are in one figure and only one theoretical line is given because the theoretical lines are very close. These figures show that the experimental and theoretical results agree very well. In Fig. 10, the line with triangles shows some deviation from the theoretical line, the reason being that the radius of the corresponding column is 15% larger than that of the line with circles. Because gravity exists in the experiment, a larger radius of the column leads to a greater deviation of the centerline of the liquid column from the centerline of the magnetic field. Thus the shape of the cross section deviates more from the assumption that the cross section is circular. Therefore the error produced by the assumption that the cross section of the levitated column is circular and that the centerline of the liquid column overlaps with the centerline of the magnetic field increases.

4. Conclusion

1. The stable condition of the liquid column in a magnetic pipe is

\[ f'(r_0) N_{so} \geq \frac{1 - k^2\nu^2}{r_0 f(r_0)} \]

The increase in the strength of the magnetic field or the gradient of the magnetic field stabilizes the levitated liquid column.

2. When \( N_{so} \geq 1 \), the column is always stable.

3. When \( \nu \geq 1 \), the column becomes unstable only when \( N_{so} < 0 \), or the magnetic susceptibility of the inner fluid is bigger than that of the outer fluid.
(4) If $N_{so} > 0$, or the magnetic susceptibility of the inner fluid is smaller than that of the outer fluid, only mode 0 need be discussed in the study of the stability of the liquid column in a magnetic pipe. A diamagnetic fluid column levitated in free space is this case.

(5) For mode 0, at $N_{so} < 1$, the column is unstable when the wavelength of the disturbance is shorter than $(2\pi r_0)/\sqrt{1 - r_0f(r_0)/f(r_0)N_{so}}$.

(6) In the stable range, the propagation speed of the disturbance becomes higher with increase in the strength of the magnetic field and the distance of the outer boundary, and with decrease in the density ratio.

References