The Interaction between Upstream and Downstream Heat Transfers on the Burning and Extinction of Dilute Spray Flames*

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The extinction of dilute spray flames under the influence of the external heat transfer existing in both upstream and downstream regions is studied using activation energy asymptotics. A completely prevaporized burning model and a partially prevaporized burning model are identified. The internal heat transfer embedded in the rich and lean spray respectively provides heat loss and heat gain for the system. However, the external heat transfer results in globally external heat loss and external heat gain, respectively, to the spray system with increasing wall temperature. The C-shaped and S-shaped extinction curves are clearly classified and mapped with parameters of the wall temperature, the initial droplet size and the external heat transfer coefficient. Characteristics differing from previous findings regarding upstream heat transfer are discussed. Furthermore, the phenomenon of reproducing an S-shaped extinction curve from the original C-shaped extinction curve is also presented and discussed.

Key Words: Flame Extinction, Dilute Spray, Asymptotics, Heat Transfer

1. Introduction

A homogeneous laminar premixed flame influenced by external heat loss can be adequately described by a C-shaped extinction curve (a double-valued function) in the classical flame-quenching theory⁽¹⁾⁻⁽³⁾. It is well known that a given combustible premixture will have two possible flame speeds under a fixed amount of heat loss: the upper branch representing stable solution; and the lower branch showing unstable solution. The extinction limit, identified by the critical point in connecting the upper and lower branches, indicates that a sufficiently large external heat loss leads to flame extinction.

The participation of fuel spray effects⁽⁴⁾ further produced so-called internal heat loss (or gain) to the system, and thereby resulted in an S-shaped extinction curve (a triple-valued function) on spray flame extinction. Because the fuel spray absorbs heat for

the gasification process, the internal heat transfer embedded in the rich and lean spray respectively provides heat loss and heat gain for the system. It was generally concluded that the S-shaped extinction curve is found for a rich spray, if the spray is thick enough and consists of liquid droplets large enough. On the contrary, the flame propagation flux of a lean spray is increased with increasing liquid fuel loading and decreasing initial droplet size without the occurrence of flame extinction.

Recently, the interaction between external and internal heat transfers on burning and extinction of dilute spray flames has been examined by Liu and Lin⁽⁵⁾. It was revealed that the external heat loss represented by the bulk heat conduction from the system to a tube wall which maintains at the upstream temperature dominates the tendency of flame extinction, and the internal heat loss associated with the gasification process of the disperse phase gives only small modification which depends on the droplet radius and the liquid loading. The small modification can be enlarged by the improvement of upstream prevaporization to reproduce an S-shaped extinction

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curve from the original C-shaped extinction curve.

In order to further understand the influence of upstream vaporization on the flame extinction of dilute sprays, Hou et al.⁽⁶⁾ varied the wall temperature to cause the spray system to globally experience external heat loss, excess enthalpy burning⁽⁷⁾ and external heat gain. As the wall temperature was higher than a certain minimum, it was found that the flame extinction can occur on the condition of completely prevaporized burning (CPB) which was originally identified to be an unstable solution in the absence of the enhancement of upstream prevaporization, and that by varying the initial droplet size, the line of flame flux at extinction has a jump between the C-shaped curve governed by the completely prevaporized burning (CPB) spray and the S-shaped curve controlled by the partially prevaporized burning (PPB) spray.

The external heat transfer considered in the above studies was simply restricted only in the upstream region of the bulk premixed flame. In the excess enthalpy theory of spray deflagration⁽⁸⁾, it is recognized that the external heat transfer occurring in both upstream and downstream regions allows heat recirculation from downstream products to the upstream fresh mixture. Depending on the balance between internal and external heat transfers, dilute spray flames were characterized by a C-shaped extinction curve or a curve bounded by blow-off and flashback. In view of flame positions varied in the region of external heat transfer⁽⁸⁾, the interaction between upstream and downstream heat transfers is important in describing the overall effect of the external heat transfer on the burning and extinction of dilute spray flames.

In this study, we have further formulated an extinction theory for dilute spray flames, allowing the heat transfer mechanism to be composed of the external heat transfer existing in both upstream and downstream regions and the internal heat transfer associated with droplet gasification. The mathematical technique used is the matched asymptotic analysis in the limit of large activation energy. We shall also restrict our analysis to dilute sprays^{(5),(6),(8),(9)} in which the amount of liquid fuel loading in the total fresh mixture is very small and can be expanded in perturbation analysis.

Nomenclature

Dimensional quantities

- B' : frequency factor
- c_L : specific heat of liquid
- c_{PG}' : specific heat of a homogeneous mixture
- E_a' : activation energy

- l_f' : preheat zone thickness, $\lambda'/(c_{PG}'\dot{m}_P')$
- $\bar{M}'$: average molar mass
- $\dot{m}_P'$: flame flux of a homogeneous mixture
- n' : number density
- Q_c' : heat of combustion per unit mass of fuel
- $\bar{R}$: universal gas constant
- r' : droplet radius

Non-dimensional quantities

- A : equation (5)
- h_{LG} : latent heat of vaporization, h_{LG}/Q_c'
- K : external heat transfer coefficient, $K'\lambda'/(c_{PG}'\dot{m}_P'^2)$
- $\dot{m}$: flame propagation flux, $\dot{m}'/\dot{m}_P'$
- T : temperature, $T'/c_{PG}'Q_c'$
- T_a : activation temperature, $E_a'c_{PG}'/(Q_c'\bar{R})$
- $\dot{W}$: equation (6)
- x : transformed coordinate, x'/l_f'
- Y : mass fraction, $Y_F = Y_F'$ and $Y_O = Y_O'/\sigma$
- Z : density ratio, ρ'_G/ρ'

Geek symbols

- α : $\alpha=1$ and $\alpha=0$ for lean and rich sprays
- γ : $Z_{-\infty}=1-\varepsilon\gamma$
- ε : small expansion parameter, T_{∞}/T_a
- σ : stoichiometric ratio
- ϕ : equivalence ratio

Superscripts

- $+$: downstream of the flame
- $'$: dimensional quantities

Subscripts

- b : boiling state
- E : state at extinction
- e : state at which droplet is completely gasified
- f : flame front
- F, O : fuel and oxygen
- G, L : gas and liquid phases
- v : state at which evaporation initiates
- $0, 1$: zeroth and first order solutions
- $-\infty, \infty$: initial and final states

2. Theoretical Model

We adopt a one-dimensional coordinate system in which a planar flame sits at $x=0$; the two phase combustible mixture composed of various concentrations of oxidizer, nitrogen, fuel vapor, and fuel droplets of a certain radius comes from $x=-\infty$; and equilibrium reaction products move away toward $x=+\infty$, as illustrated in Fig. 1. A completely prevaporized mode ($r'_{\infty} \leq r'_c$) and a partially prevaporized mode ($r'_{\infty} > r'_c$) of flame propagation, shown in Fig. 1 (a) and Fig. 1(b) respectively, are identified by a critical initial droplet size (r'_c) for the droplet to achieve complete vaporization at the premixed flame front. Depending on the available oxidizer

Fig. 1 Schematic diagram of (a) completely prevaporized, and (b) partially prevaporized burning sprays

downstream of the flame, the downstream mixture has either droplet burning for lean sprays or droplet vaporization for rich sprays in the partially prevaporized mode, as shown in Fig. 1(b). We assume that the droplet will start to evaporate at, $x = x_v$, only when the gas temperature has reached the boiling point of the liquid. Droplets then ignite upon crossing the flame, and vanish at $x = x_e$ upon complete combustion for lean sprays or complete evaporation for rich sprays.

We further assume that the external heat transfer being $O(\epsilon)$ is proportional to $(T - T_u)$ in the upstream region of x_v to 0 and to $(T - T_d)$ in the downstream region of 0 to x_e , as the external heat transfer simultaneously exists in the upstream and downstream regions. T_u and T_d denote the wall temperature in upstream and downstream regions, respectively; and are set to be equal in the calculation for the sake of simplicity. The external heat transfer results in globally external heat loss and excess enthalpy burning (which is globally adiabatic), respectively, to the spray system with increasing wall temperature⁽⁶⁾. Since the spray is dilute and the external heat transfer is small compared with the heat release of combustion, it is reasonable to assume that the amount of liquid fuel loading is of $O(\epsilon)$ of the total spray mass and that the amount of external heat transfer is of $O(\epsilon)$ in the asymptotic analysis. The small parameter of expansion ϵ is the ratio of thermal energy to activation energy in the combustion process. Finally, we assume that the fuel and oxidizer reaction for the bulk premixed flame is one-step overall, that the fuel

droplets burn in the flame sheet limit, and that conventional constant property simplifications apply. More detailed assumptions and comments were described in an earlier study⁽⁹⁾.

The total number of droplets crossing any plane normal to the x axis per second are set to constant, $n'u' = n'_{-\infty}u'_{-\infty}$, and the overall continuity is given by $\rho'u' = \rho'_{-\infty}u'_{-\infty} = \dot{m}'$, where n' is the number density, ρ' is the overall density of the mixture, $\rho' = \rho'_G + \rho'_S$, and $u'_{-\infty}$ represents the burning velocity of the mixture. Note that $\rho'_S = (4/3)\pi r'^3 n'$. Quantities with and without primes are dimensional and nondimensional, respectively.

We designate the extent of gas-phase heterogeneity by the parameter $Z = \rho'_G/\rho'$ such that $Z = 1$ represents the completely vaporized state. Following the previous formulation^{(9),(10)}, the non-dimensional equations for gas-phase continuity, conservation of fuel, oxidizer and energy are respectively given by

$$Z\dot{m}\frac{dZ}{dx} = \frac{A}{T}(1-Z)_{-\infty}^{2/3}(1-Z)^{1/3}F(T, Y_0), \quad (1)$$

$$\frac{d}{dx}\left(Z\dot{m}Y_F - \frac{d}{dx}Y_F\right) = \dot{W} + f_F\dot{m}\frac{dZ}{dx}, \quad (2)$$

$$\frac{d}{dx}\left(Z\dot{m}Y_O - \frac{d}{dx}Y_O\right) = \dot{W} + f_O\dot{m}\frac{dZ}{dx}, \quad (3)$$

$$\frac{d}{dx}\left(Z\dot{m}T - \frac{d}{dx}T\right) = -\dot{W} + f_T\dot{m}\frac{dZ}{dx} - \epsilon K_u(T - T_u)H_1(x) - \epsilon K_d(T - T_d)H_2(x) \quad (4)$$

where

$$A = 3 \cdot \left(\frac{\lambda'}{r'_{-\infty}\dot{m}'_F}\right)^2 \cdot \left(\frac{P'\bar{M}'}{\bar{R}c_{PG}Q'_c\rho'_L}\right) \quad (5)$$

$$\dot{W} = -\left(\frac{B'\sigma}{\bar{M}'_O}\right)\left(\frac{P'\bar{M}'}{\bar{R}}\right)\left(\frac{\lambda'}{c'_{PG}\dot{m}'_F}\right)Y_OY_F\exp\left(-\frac{T_a}{T}\right) \quad (6)$$

and the function

$$H_1(x) = \begin{cases} 1 & \text{for } x_v \leq x \leq 0 \\ 0 & \text{for } x > 0 \end{cases} \quad (7)$$

$$H_2(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq x_e \\ 0 & \text{for } x > x_e \end{cases} \quad (8)$$

While $x = x'/l'_f$ is the non-dimensional distance expressed in units of the preheat zone thickness, $l'_f = \lambda'/(c'_{PG}\dot{m}'_F)$. In Eqs. (1)–(4), the function $F(T, Y_0)$ and the constant parameters f_F , f_O , and f_T are respectively $\ln[1 + (T - T_b)/h_{LG}]$, 1, 0, and $-h_{LG}$ for the vaporizing droplet and $\ln[1 + (T - T_b - Y_0)/h_{LG}]$, 0, -1 , and $(1 - h_{LG})$ for the burning droplet. K_u and K_d represent the heat transfer coefficient for the external heat transfer in the upstream and downstream regions, respectively. In this study, we assume $K_u = K_d = K$ for simplification. $\dot{m}$ denotes the flame propagation flux normalized by the premixed value, $\dot{m}'_P$, in the homogeneous mixture.

Performing the inner and outer expansions based on the small parameter of ϵ , and following the

detailed matching procedure of the previous study⁽⁹⁾ to match the inner and outer solutions, we therefore reach the final results as follows:

$$\dot{m}^2 = \exp[T_1^+(0)]. \quad (9)$$

Note that the flame propagation flux is exponentially affected by the first-order temperature downstream near the flame. The first-order temperature $T_1^+(0)$ is expressed by the following equation:

$$\begin{aligned} T_1^+(0) = & \left(\frac{1 - e^{-\dot{m}x_v}}{e^{-\dot{m}x_e} - e^{-\dot{m}x_v}} \right) \left[T_{1\infty} e^{-\dot{m}x_e} \right. \\ & + \frac{\gamma}{T_\infty} (\alpha - Y_{i-\infty}) (1 - e^{-\dot{m}x_e}) \\ & + \frac{\dot{m}\gamma}{T_\infty} (T_\infty + h_{LG} - \alpha) \int_0^{x_e} \xi_0^+ e^{-\dot{m}x} dx \\ & - \frac{K(T_\infty - T_d)}{\dot{m}^2 T_\infty} (1 - e^{-\dot{m}x_e} - \dot{m}x_e e^{-\dot{m}x_e}) \Big] \\ & + \left(\frac{1 - e^{-\dot{m}x_e}}{e^{-\dot{m}x_e} - e^{-\dot{m}x_v}} \right) \left\{ \frac{\dot{m}\gamma}{T_\infty} \int_{x_v}^0 [(T_\infty \right. \\ & + h_{LG}) e^{-\dot{m}x_e} + Y_{i-\infty}] \xi_0^- dx \\ & - \frac{KY_{i-\infty}}{\dot{m}^2 T_\infty} (1 - \dot{m}x_v - e^{-\dot{m}x_v}) \\ & \left. + \frac{K(T_\infty - T_u)}{\dot{m}^2 T_\infty} (1 - \dot{m}x_v e^{-\dot{m}x_v} - e^{-\dot{m}x_v}) \right\}. \quad (10) \end{aligned}$$

For the sake of notation compactness, we use $\alpha=1$ and $i=F$ for lean sprays and $\alpha=0$ and $i=O$ for rich sprays. The liquid fuel loading is represented by γ through the expansion of $Z_\infty=1-\varepsilon\gamma$ for dilute sprays⁽⁹⁾.

Setting $x_e=0$ to achieve the condition of completely prevaporized burning sprays, we obtain

$$\dot{m}^2 = \exp[T_{1\infty}] \quad (11)$$

where

$$\begin{aligned} T_{1\infty} = & \frac{\gamma}{T_\infty} \left[\alpha - h_{LG} - (T_\infty - T_b) - \frac{c_L'}{c_{PG}'} (T_b - T_\infty) \right] \\ & - \frac{K}{\dot{m}^2 T_\infty} [(T_\infty - T_b) - (T_\infty - T_u) \dot{m}x_v] \quad (12) \end{aligned}$$

Equation (12) also indicates that there is no contribution to $T_{1\infty}$ coming from the droplet size.

On the basis of the formulated results, Eqs. (9) and (11), sample calculations for *n*-octane (C_8H_{18}) burning in air are now considered to illustrate the influence of external heat transfer existing in both upstream and downstream regions. The relevant physical properties are referred to in an earlier study⁽⁹⁾. Note that for completely prevaporization burning, the value x_e is equal to zero; the external heat transfer therefore exist only in the upstream region. K , T_u , and T_d corresponding to the external heat transfer, and r'_{∞} and γ , indicating the internal heat transfer, are selected as parameters in the calculation. Results for lean and rich sprays are separately discussed in the following sections.

3. Lean Sprays

The possible extinction curves are clearly classified and mapped in Fig. 2 for a lean spray of $\phi_G=0.8$ and $\gamma=0.02$. The corresponding extinction curves are shown in Fig. 3 for the CPB sprays and represented in Figs. 4–6 for the PPB sprays at various wall temperatures, $T_u=T_d=500, 759.72$, and 800 K, respectively. In the region of $T_u < 759.72$ K and $r'_{\infty} \leq 18.32 \mu\text{m}$, flame extinction is characterized only by the C-shaped curve under CPB conditions, as shown in Fig. 2. With increasing wall temperature, the critical value of flame flux at extinction ($\dot{m}_E$) maintains a constant value, however, the external heat transfer coefficient at extinction increases, as indicated in Fig. 3. It is obvious that a sufficiently large external heat loss results in flame extinction. Therefore, a higher T_u and T_d leads to a larger K at extinction because of

Fig. 2 The map of possible burning and extinction curves for a lean spray

Fig. 3 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the upstream wall temperature T_u for a lean spray with completely prevaporized burning

the smaller temperature gradient between the system and the tube wall. The energy balance occurs upstream, i.e., the spray achieves the excess enthalpy condition⁽⁶⁾, as $r'_{\infty} \leq 18.32 \mu\text{m}$ and $T_u = 759.72 \text{ K}$. In this condition, $\dot{m}$ is independent of K and r'_{∞} . As $T_u > 759.72 \text{ K}$ and $r'_{\infty} \leq r'_c$, $\dot{m}$ increases monotonically with increasing K . This is caused by the globally external heat gain in the upstream region. As $T_u < 759.72 \text{ K}$ and $18.32 \mu\text{m} < r'_{\infty} \leq 30.2 \mu\text{m}$ in Fig. 2, there exists the C-shaped extinction curve, composed of a line of PPB spray and a curve of CPB spray with the junction on the upper branch of CPB spray. This extinction characteristics is expressed by the curve of $r'_{\infty} = 25 \mu\text{m}$ at $T_u = T_d = 500 \text{ K}$ in Fig. 4. The junction shifts toward the extinction limit of the CPB spray along the dot-dash line with increasing r'_{∞} , as shown in Fig. 4. However, the flame flux at extinction in such circumstances remains constant, that is, the same as the extinction point of CPB spray. The superscripts denoted by U, L and E in Fig. 2 represent the junctions of PPB-CPB curves occurring at the upper branch, the lower branch, and the extinction condition of CPB extinction curve, respectively.

In the area underneath the BC line in Fig. 2, the extinction curve consists of an upper branch corresponding to the PPB, a lower branch associated with the CPB, and a junction showing flame extinction, which belongs to the unstable branch of the CPB. This phenomenon is illustrated by the extinction curve of $r'_{\infty} = 40 \mu\text{m}$ in Fig. 4. If r'_{∞} increases beyond a critical value, r'^{*}_{∞} , (e.g., $56 \mu\text{m}$ in Fig. 4), crossing line FC in Fig. 2, for the case of $T_u = T_d = 500 \text{ K}$, then the flame extinction is totally controlled by the PPB spray, and the S-shaped extinction curve is reproduced to be inserted into the C-shaped curve. The quenching limit jumps from the unstable lower branch of the CPB to the PPB, if $r'_{\infty} > r'^{*}_{\infty}$, as was also demonstrated in the previous studies^{(5),(6)}.

Fig. 4 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the initial droplet radius r'_{∞} for a lean spray at $T_u = T_d = 500 \text{ K}$

The r'^{*}_{∞} at which there exists a jump from the CPB to PPB along the $\dot{m}_E$ line increases with increasing wall temperature, as shown along the FC line in Fig. 2. Since the spray burning with external heat loss only occurs upstream⁽⁵⁾, a minimum wall temperature (510.5 K) exists to allow a jump from CPB to PPB along the $\dot{m}_E$ line, i.e., an S-shaped extinction curve is reproduced on the basis of the spray having large enough r'_{∞} and sufficiently large T_u ($r'_{\infty} > r'^{*}_{\infty}$ and $T_u > 510.5 \text{ K}$). However, in the present investigation even at very low wall temperature, e.g., $T_u = T_d = 348.6 \text{ K}$, the S-shaped extinction curve still occurs, but only if the r'_{∞} is large enough. Since the external heat loss occurring in the upstream and downstream regions simultaneously is larger than that of the previous study⁽⁵⁾ in which the external heat loss exists only upstream, flame extinction is more sensitive to the internal heat loss which results in the S-shaped extinction curve.

Under the condition of $r'_{\infty} \leq 18.32 \mu\text{m}$ and $T_u = 759.72 \text{ K}$, the spray burning controlled by the CPB achieves energy balance upstream of the premixed flame, i.e., the overall external heat transfer is zero. In this circumstance, $\dot{m}$ is independent of the heat transfer coefficient and the initial droplet radius, as shown in Fig. 5. For the CPB spray and the homogeneous premixture, flame extinction does not occur. However, the flame flux of the former is larger than that of the latter due to the additional internal heat gain for a lean spray. If $18.32 \mu\text{m} < r'_{\infty} \leq 59.5 \mu\text{m}$ at $T_u = 759.72 \text{ K}$ (the line AC in Fig. 2), the flame flux decreases monotonically with increasing initial droplet radius, resulting in no flame extinction, e.g., the characteristic curves of $r'_{\infty} = 30 \mu\text{m}$ and $40 \mu\text{m}$ in Fig. 5. Increasing T_u to 759.72 K results in external heat loss smaller than that of $T_u = 500 \text{ K}$ with the same value of external heat transfer coefficient (K). A

Fig. 5 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the initial droplet radius r'_{∞} for a lean spray at $T_u = T_d = 759.72 \text{ K}$

Fig. 6 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the initial droplet radius r'_∞ for a lean spray at $T_u = T_d = 800$ K

larger K will promote the upstream prevaporization which produces the downstream internal heat gain resulting from burning the secondary gasified fuel. However, the external heat loss existing in both upstream and downstream regions also increases and finally results in a decrease in $\dot{m}$. Moreover, as the r'_∞ is large enough at $T_u = T_d = 759.72$ K in Fig. 5, i.e., $r'_\infty > 59.5$ μm , the S-shaped extinction curve governed by the PPB is obtained. At a higher wall temperature, e.g., $T_u = T_d = 800$ K, as indicated in Fig. 6, the external heat transfer provides external heat gain for the CPB and the homogeneous premixture. The burning intensity, therefore, increases monotonically with increasing heat transfer coefficient (increasing external heat gain). Figure 6 also shows that an initial droplet radius exists to allow for the flame flux of a PPB spray independent of K , i.e., 18.8 μm shown on the AE line in Fig. 2. As $r'_\infty < 18.8$ μm and $r'_\infty > 18.8$ μm , with increasing K , $\dot{m}$ are enhanced and weakened in response to increasing the external heat gain and external heat loss, respectively. As r'_∞ is large enough ($r'_\infty > 60.3$ μm), the S-shaped extinction curve can be found. A lean spray containing large r'_∞ experiences a weaker upstream prevaporization, provides less internal heat gain, and thereby has a reduced burning intensity. Hence, it can be extinguished by smaller $\dot{m}$ and K , as shown in Fig. 6. It is interesting to note that for a very large r'_∞ (200 μm in Fig. 6), $\dot{m}_E$ is nearly equal to $e^{-0.5}$ corresponding to $\dot{m}_E$ of the homogeneous premixture⁽³⁾, because a large droplet spray has a very small amount of secondary gasified fuel associated with a very small quantity of internal heat gain.

4. Rich Sprays

Figure 7 shows the classification of the possible extinction curves for a rich spray of $\phi_c = 2.0$ and $\gamma = 0.3$. The corresponding extinction curves are shown in

Fig. 7 The map of possible burning and extinction curves for a rich spray

Fig. 8 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the upstream wall temperature T_u for a rich spray with completely prevaporized burning

Fig. 8 for the CPB spray and illustrated in Figs. 9–11 for the PPB sprays at various wall temperature values, $T_u = T_d = 500, 736.23$, and 800 K, respectively. The map of the rich spray (Fig. 7) illustrates similar characteristics as that of the lean spray (Fig. 2), except that $\dot{m}_E$ and the corresponding Q_L for the CPB are respectively smaller than $e^{-0.5}$ and e^{-1} which are the flame flux at extinction ($\dot{m}_E$) and the associated heat loss (Q_L) of the homogeneous premixture⁽³⁾, because the rich spray has overall internal heat loss through the droplet gasification process^{(5),(6)}. Figure 8 shows the extinction curves at various values of wall temperature for the CPB spray. As $T_u > 736.23$ K, the overall external heat transfer becomes heat gain, then $\dot{m}$ is strengthened by increasing the heat transfer coefficient accompanied with a larger heat gain. The wall temperature is determined to be 736.23 K for the energy balance upstream, i.e., the globally external heat loss is equal to zero under the specified condition

Fig. 9 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the initial droplet radius r'_{∞} for a rich spray at $T_u = T_d = 500$ K

of the rich spray. As $T_u = 736.23$ K under CPB conditions, the flame flux remains constant and no flame extinction occurs. As $T_u < 736.23$ K, the C-shaped extinction curves with the same extinction limit ($\dot{m}_E$) are obtained, as shown in Fig. 8. Note that in Fig. 8, a smaller K corresponds to a smaller Q_L at a fixed wall temperature, and that a larger wall temperature accompanied by a larger K can arrive at the same amount of external heat loss which results in flame extinction.

The flame extinction is identified under the extinction point of the CPB curve, as $r'_{\infty} < 27.3$ μm (e.g., $r'_{\infty} = 25$ μm) and $T_u = T_d = 500$ K in Fig. 9. As 27.3 $\mu\text{m} < r'_{\infty} \leq 40.2$ μm , the $\dot{m}_E$ line moves along the lower unstable branch of the extinction curve corresponding to the CPB spray. Moreover, as r'_{∞} is larger than $r'^*_{\infty} = 40.2$ μm (line FC on Fig. 7), the extinction point jumps from the C-shaped curve governed by the lower branch of the CPB spray to the S-shaped curve totally controlled by the PPB spray. Note that even at a very low wall temperature, e.g., $T_u = T_d = 348.6$ K, a jump still occurs from the CPB to the PPB along the $\dot{m}_E$ line, but only if r'_{∞} is large enough (by passing through line FC in which $r'_{\infty} = r'^*_{\infty}$, as shown in Fig. 7). This phenomenon is similar to that of the lean spray but differs from the previous findings^{(5),(6)} based on the upstream heat transfer.

As $T_u = 736.23$ K associated with no external heat transfer upstream, flame extinction does not occur for the CPB spray and the homogeneous premixture burning. However, the flame extinction characterized by an S-shaped curve can be achieved, as $r'_{\infty} > 41.8$ μm for the PPB spray, as shown in Fig. 10. Figure 11 illustrates that if $T_u = T_d = 800$ K, the flame fluxes of the homogeneous premixture, the CPB spray, and the PPB spray having $r'_{\infty} < 17.2$ μm are increased monotonically by increasing the heat transfer

Fig. 10 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the initial droplet radius r'_{∞} for a rich spray at $T_u = T_d = 736.23$ K

Fig. 11 The flame flux $\dot{m}$ as functions of the external heat transfer coefficient K and the initial droplet radius r'_{∞} for a rich spray at $T_u = T_d = 800$ K

coefficient (enhancing the external heat gain). If r'_{∞} is just equal to 17.2 μm (on line AE of Fig. 7), $\dot{m}$ is independent of K . Furthermore, if r'_{∞} is larger than 42.4 μm (by crossing line CD in which $r'_{\infty} = r'^*_{\infty}$, as shown in Fig. 7), the S-shaped extinction curve still occurs.

5. Conclusions

The extinction of the dilute spray flame burning in a steady, one-dimensional, low speed, sufficiently off-stoichiometric, two-phase flow, and experiencing the external heat transfer which exists in both upstream and downstream regions, has been investigated in this article. A completely prevaporized burning mode ($r'_{\infty} \leq r'_c$) and a partially prevaporized burning mode ($r'_{\infty} > r'_c$) were identified by a critical initial droplet size (r'_c) for the droplet to achieve complete vaporization at the premixed flame front.

The interaction between external heat transfer (existing in both upstream and downstream) and

internal heat transfer (contributed by the droplet gasification process) on the burning and extinction of the dilute sprays were clearly described in different burning models. The droplet gasification provides internal heat gain and heat loss for the lean and rich spray, respectively. The C-shaped and S-shaped extinction curves were classified and mapped in response to the wall temperature and the initial droplet radius. Results are generally concluded as follows:

1. Flame extinction characterized by a C-shaped curve are mainly controlled by the external heat loss from the spray to a tube wall upstream and downstream; however, the S-shaped extinction curve is strongly influenced by internal heat transfer associated with the droplet gasification process.

2. As the wall temperature is smaller than the critical value at which the globally external heat transfer is zero in upstream, it is seen that the flame extinction characterized by the C-shaped curve may be achieved on the condition of the completely prevaporized burning which is originally identified to be unstable in the absence of intensification of upstream prevaporization, and that by varying the droplet size, the line of flame flux at extinction can jump between the C-shaped curve governed by the completely prevaporized burning and the S-shaped curve controlled by the partially prevaporized burning.

3. As the wall temperature is just equal to or larger than the critical value at which the excess enthalpy condition occurs (the overall external heat transfer is zero) in upstream, the S-shaped extinction curve, totally controlled by the partially prevaporized burning, may be found when the droplet size is large enough, while no flame extinction occurs if the spray has small droplets.

4. As the wall temperature is increased to a high degree, no flame extinction would be expected to occur because the system experiences globally external heat gain. In this circumstance, the burning intensity is decreased (increased) as the initial droplet size

is increased for the lean (rich) spray.

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References

- (1) Spalding, D.B., A Theory of Inflammability Limits and Flame-Quenching, *Proc. Roy. Soc., Lond.*, A 240 (1957), p. 83.
- (2) Joulin, G. and Clavin, P., Analyse Asymptotique des conditions D'extinction des Flammes Laminaires, *Acta Astronaut.*, Vol. 3 (1976), p. 323.
- (3) Buckmaster, J.D. and Ludford, G.S.S., *Theory of Laminar Flames*, (1982), p. 38, Cambridge University Press, Cambridge, England.
- (4) Huang, C.L., Chiu, C.P. and Lin, T.H., The Influence of Liquid Fuel on the Flame Propagation of Dilute Sprays in Non-Conserved Systems, *J. of Chinese Soc. of Mech. Eng.*, Vol. 10, No. 7 (1989), p. 333.
- (5) Liu, C.C. and Lin, T.H., The Interaction between External and Internal Heat Losses on the Flame Extinction of Dilute Sprays, *Combust. Flame*, Vol. 85 (1991), p. 468.
- (6) Hou, S.S., Liu, C.C. and Lin, T.H., The Influence of External Heat Transfer on Flame Extinction of Dilute Sprays, *Int. J. Heat Mass Transfer*, Vol. 36, No. 7 (1993), p. 1867.
- (7) Weinberg, F.J., Combustion Temperature: the future? *Nature*, Vol. 233 (1971), p. 239.
- (8) Hou, S.S. and Lin, T.H., A Theory on Excess Enthalpy Spray Flame, Atomization and Sprays, Vol. 9 (1999), p. 355.
- (9) Lin, T.H., Law, C.K. and Chung, S.H., Theory of Laminar Flame Propagation in Off-Stoichiometric Dilute Sprays, *Int. J. of Heat and Mass Transfer*, Vol. 34, No. 5 (1988), p. 1023.
- (10) Williams, F.A., *Combustion Theory*, 2nd ed., Benjamin/Cummings, (1985), p. 446, Menlo Park, CA.