Numerical Analysis of Turbulence Structures Induced by Bubble Buoyancy*

Yuichi MURAI**, Yoichiro MATSUMOTO***,
Xiang-qun SONG** and Fujio YAMAMOTO

The turbulence structure in a rising bubbly flow driven by bubble buoyancy has been analyzed using the Eulerian-Lagrangian model. A new method of calculating the two-way interaction between the dispersed bubbles and the liquid phase was proposed to accurately simulate the local fluctuations of liquid flow induced by bubble migration. The present two-dimensional simulation revealed that, (1) in the case of the larger bubble, the turbulence structure has the longer wavelength in the vertical direction, and, (2) the wave spectrum of kinetic energy of the liquid phase is sensitive to the average void fraction and the lift force of the bubbles.

Key Words: Bubbly Flow, Multiphase Flow, Finite Differential Method, Computational Fluid Dynamics, Turbulent Flow, Natural Convection

1. Introduction

Bubbly flow is observed in various industrial and environmental processes, such as nuclear furnace circulation systems, chemical plants, and bioreactors. The performance of such a system is governed by complex flow behavior owing to the gas-liquid interaction and the motion characteristics of bubbles. In order to predict and elucidate the bubbly flow behavior in detail, it is first necessary to construct the governing equations which are formulated precisely according to the actual phenomena in the bubbly flow, and second to carry out the simulation based on an adequate numerical procedure. Recently, it has come possible to predict valid flow structures with spatial scales significantly larger than the bubble diameter, which is known in advance of grid-averaged equations, such as the number density model⁽¹⁾, and the two-

fluid model⁽²⁾. For detailed structures of bubbly flows, the investigations on single bubble motions in various types of liquid flows⁽³⁾⁻⁽⁵⁾ have been advanced mainly through direct numerical simulation and experimental measurement based on image processing. The characteristics of bubble motion and the ambient liquid flow structure acquired from these results contribute to more accurate modelling of gas-liquid interfacial terms. Furthermore, very recently, a new calculation method for free surface motion of bubbles has been developed⁽⁶⁾, which enables us to carry out the direct numerical simulations of bubbly flow containing 200 to 600 bubbles in the calculation domain.

In the previous papers^{(7),(8)}, the authors proposed a Eulerian-Lagrangian model for bubbly flows (E-L model) and discussed the numerically predicted detailed flow structures caused by a bubble plume. The merits of the E-L model were confirmed as follows: (1) practical density ratio of gas against liquid can be treated correctly, (2) more than ten thousands of bubbles can be easily analyzed in the computational domain, (3) spatial resolution of the analysis is higher than that of the two-fluid model. In this paper, the following two objectives have been set.

(1) To obtain a phenomenologically invariant mechanism of the structure of turbulence in the liquid

* Received 5th April, 1998. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 64, No. 626, B (1998), p. 3257-3263 (Received 11th September, 1996)

** Department of Mechanical Engineering, Fukui University, 3-9-1 Bunkyo, Fukui 910-8507, Japan. E-mail: murai@fv.mech.fukui-u.ac.jp

*** Department of Mechanical Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

phase induced by many buoyant bubbles.

(2) To establish a new numerical method by which the bubble-induced turbulence can be predicted accurately (in order to achieve the first objective).

2. Revision of Two-Way Coupling in the E-L Model

The turbulence which is induced by individual dispersions in dispersed multiphase flow is distinguished from "true" turbulent flow and is called "pseudo turbulence"⁽⁹⁾. In order to correctly simulate pseudo turbulence using the E-L model, a reliable analytical scheme which can exactly handle the interaction of dispersion to continuous phase is required. In this section, the current problems of the two-way analysis method in the E-L model and the solution method are described.

2.1 Problems of conventional two-way coupling

In the two-way analysis of dispersive two-phase flow, three kinds of interfacial interactions between dispersed phase and continuous phase are considered, that is, mass (or volume fraction), momentum and energy. In the E-L model, the following point should be noted as compared to the Eulerian-Eulerian model⁽¹⁾. The physical quantities of the continuous phase are defined by the grid averaged values whose data are memorized in the Eulerian-type array. In contrast to this, those of the dispersed phase are defined at each point of dispersion. Therefore a difference in the predicted results occurs depending on the data conversion method of the physical variables between the dispersed and continuous phases. When a microscale flow such as turbulence is analyzed, the difference of prediction performance due to the data conversion method must be clarified.

First, to determine the influence of continuous phase flow on dispersion motion, the physical quantities in the continuous phase at each dispersion point can be calculated by Lagrange interpolation using physical quantities obtained from the surrounding grid. In this case, linear interpolation⁽¹⁰⁾, second-order interpolation, or cubic spline interpolation can be applied. The higher-order interpolation will lead to the predicted results with higher order spatial resolution, and are used in many kinds of computations, such as one-way simulation of particle motion in a turbulent flow, visualization of calculated results and measured flow fields by particle path lines. For calculating the influence of the dispersion on the continuous phase, the following two simple methods are frequently employed.

(1) The physical quantity of dispersion is attributed only to one grid volume where the center of gravity of dispersion exists.

(2) The physical quantity of dispersion is attributed to several grid volumes which involve a partial volume of the dispersion.

Method (1) is feasible when each volume of the dispersion is sufficiently small compared to the unit volume of a grid. Method (2) is often employed when the dispersion volume is not negligible. The method (1) was adopted in the author's previous papers^{(7),(8)}, and method (2) was adopted in the particle tracking method⁽¹⁰⁾. However, it is found that a significantly large pulse turbulence affects the predicted result by both the methods. This pulse like turbulence does not exist in the practical bubbly flow. This turbulence is induced by quick changes of intensity of the interaction from gas to liquid when the dispersion migrates across the grid-grid interfaces. In this paper, this impulsive turbulence due to the grid division is named "numerical pseudo turbulence", and a solution to this problem is proposed below.

2.2 Generation mechanism of numerical pseudo turbulence

The origin of the numerical pseudo turbulence is due to the fact that the influence of physical quantity of each dispersion is only treated up to the spatial resolution whose minimum unit is the grid size. For instance, if the above methods (1) and (2) are applied, the liquid fluctuation cannot be calculated when a bubble migrates inside a grid. The reason for this is that the grid averaged volume fraction is constant even though the bubble moves inside the grid. However, a large fluctuation occurs when the bubble penetrates the grid interface. The liquid velocity of numerical pseudo turbulence is easily estimated by the conservation equation of liquid mass:

$$\frac{\partial(1-\alpha)\rho_L}{\partial t} + \text{div}(1-\alpha)\rho_L \mathbf{u}_L = 0. \quad (1)$$

where ρ is the density, α is the volume fraction, and $\mathbf{u}$ is the velocity vector, and the subscript L stands for the liquid phase. In the case that a bubble with volume V_b enters into a grid with volume V_c while time proceeds at time step δt , the external velocity of liquid flow at the grid interface is derived as:

$$u'_L = \frac{(1/\delta t) V_b V_c^{-\frac{2}{3}}}{2M(1-\alpha)} \quad (2)$$

where, M is the spatial dimension of the flow field of interest, and α is the void fraction in a grid before the bubble enters, V_b stands for volume of bubble, and V_c is volume of the grid. Equation (2) indicates the grid-averaged quantity of liquid-flow fluctuation due to the entrance of the bubble. One recognizes from this equation that the over estimated liquid flow is calculated if time step δt decreases, so that a resultant numerical pseudo turbulence appears. In the case that the volume of a bubble is laid across several grids, the

liquid-flow fluctuation can be derived according to the following equation because the transient change of void fraction is given by $(V_c/V_c)(u_c \delta t)(V_c^{1/3})$:

$$u_L = \frac{u_c V_c^{2/3} V_c^{-2/3}}{2M(1-\alpha)} \quad (3)$$

where, u_c is the translational velocity component perpendicular to the grid interface. As Eq. (3) is different from Eq. (2), the liquid velocity fluctuation is proportional to the bubble migration velocity u_c . However, an additional numerical restriction is newly generated to conduct the simulation. The condition is to satisfy $\delta t < V_c^{1/3}/u_c$. This means that the calculation becomes quite time consuming in the case of simulating small bubbles or high-speed moving bubbles.

2.3 Removing the numerical pseudo turbulence

As mentioned above, the numerical pseudo turbulence is caused by an inadequate numerical scheme for the E-L model, which was not considered in the previous investigations. This work focuses on the calculation method of the local void fraction because it is the most important variable in treating the gas-liquid interactions. By considering the following two points, a new definition of local void fraction is proposed for the E-L model.

(a) Continuity: When a bubble moves inside a grid, the local void fraction in the grid must change continuously in time and space. Thus, a limited derivative value of the local void fraction with respect to time and space must be obtained.

(b) Conservativity: The average value of local void fraction over the entire domain must correspond to the actual bulk void fraction. Thus, the local void fraction is conserved and the mass conservation law is satisfied.

When methods (1) and (2) are employed, the void fraction distribution preserves conservativity but not continuity. If a spatial filter such as a scanning averaging filter, is applied to the results of (1) and (2), the continuity is ensured to some degree but the conservativity is not satisfied. Hence, to satisfy conditions (a) and (b), the following method is proposed.

The principle of the new method is illustrated in Fig. 1. First, when a bubble with volume V exists inside a grid, four grid points which enclose the bubble at the shortest distance are selected as the objective grids contributing to the bubble volume V (see P1 to P4 in the figure). Next, a rectangular template with an area of A is set up, whose center of gravity corresponds to the bubble position, and whose area is the same as the grid size. The area of four elements divided by the position of the bubble are calculated ($A_i, i=1$ to 4). The weight factors defined by the four area fractions ($A_i/A, i=1$ to 4) are employed to

P1 ~ P4 : Selected point for delivery of Bubble Volume
[A1] ~ [A4] : Area divided by mesh, [shaded] : Template Area

Fig. 1 TD algorithm for two-way analysis

distribute the bubble volume V to the four grid points, that is, $V \times (A_i/A)$ is attributed to each grid point P_i . By this procedure, the movement of the bubble contributes to the local void fraction even if it migrates inside a grid. Using the same principle is extended to calculate the other space dimensions. For instance, the number of objective grids is two for two-dimensional calculations, and eight for three-dimensional calculations.

By using this method, not only the numerical pseudo turbulence is eliminated because conditions (a) and (b) are satisfied, but also the precision of calculation for mutual interaction between the bubble and the surrounding liquid can be improved down to the subgrid scale because its derivative inside the grid can be treated. From this viewpoint, this new definition is essentially different from either averaging or smoothing filters. We named this method "template delivery method (TD method)" because we use a moving template whose center of gravity traces a bubble. When the TD method is employed to predict the liquid fluctuation velocity due to the bubble migration, the following equation is obtained.

$$u_L = \frac{u_c V_c V_c^{-1}}{2M(1-\alpha)} \quad (4)$$

Equation (4) ensures an exact calculation employing grid-averaged values, since it does not depend on the calculation time interval δt and is proportional to the bubble velocity u_c . Of course, there is no restriction in selecting the time interval. The time-dependent liquid fluctuation due to the bubble migration is explained in Fig. 2. A large pulse of triangular shape is generated when methods (1) and (2) are employed. On the contrary, a continuous wave form is ensured by the TD method, so that unrealistic high frequency fluctuations are not obtained in the predicted results. Furthermore, the velocity fluctuation of

u_L' : liquid velocity fluctuation induced by bubble migration

Fig. 2 Numerical pseudo turbulence

the liquid phase obtained by the TD method is almost in good agreement with the grid-averaged fluctuation of a potential flow around a sphere in the case of small spheres. Therefore, the analysis of microscale flow structures becomes applicable.

3. Turbulence Structure in Rising Bubbly Flow

Following the newly devised method, the structures of turbulence in a bubbly two-phase flow driven by buoyant bubbles have been analyzed. Regarding the governing equations and numerical procedures, we refer to the previous reports^{(1),(7)}. Periodical boundary conditions are applied to all boundaries. The initial condition is that bubbles are randomly dispersed in quiescent liquid.

3.1 Bubble uniformity and turbulence energy

There is a strong relationship between the turbulence structure and bubble distribution pattern. Table 1 lists the simulation conditions, time-averaged turbulence energy E_t and a parameter of "uniformity" η which indicates the state of the bubble distribution pattern. The uniformity is a dimensionless variable which is defined by the equation:

$$\eta = \frac{\frac{1}{N} \sum_{i=1}^N \min(|X_j - X_i|_{i \neq j})}{LN^{1/3}} \quad (5)$$

where X is the center of gravity of each bubble, N is the number of bubbles, and L is the length of physical domain. The subscripts i and j stand for the labeling number of each bubble. This equation is obtained by dividing the averaged value of the shortest bubble-bubble distance with that of the case of uniform distribution. Therefore, the uniformity is zero in the case of clustered bubbles, one in the case of uniform distribution and around 0.5 for a random distribution. The following facts can be found from Table 1: (1) when the void fraction α increases, both turbulence energy and uniformity are enhanced (for comparison between conditions 1 and 2). (2) If, for the same void

Table 1 Simulation conditions and results

System Condition		Calculation Condition	
$g = 9.80665 \text{ m/s}^2$		Time step $dt=10^{-3} \text{ sec.}$	
$L = H = 0.50 \text{ m}$		Tank depth $\delta=0.01 \text{ m}$	
$\rho_L=1000 \text{ kg/m}^3$		Conv. Term : 2-D CIP	
$\rho_a=1.25 \text{ kg/m}^3$		Bubble : Incompressible	
$\nu_L=10^{-6} \text{ m}^2/\text{s}$		Grid Number : 100×100	
Parameter Condition		Turb. Enrgy, $E_t \times 10^3$	Uniformity η
1: $\alpha=0.01, R=0.5\text{mm}$		0.58	0.523
2: $\alpha=0.05, R=0.5\text{mm}$		9.91	0.560
3: $\alpha=0.05, R=0.2\text{mm}$		19.04	0.513
4: $\alpha=0.05, R=1.0\text{mm}$		4.67	0.542
5: $\alpha=0.01, R=0.5\text{mm}$ Without Lift Force		1.22	0.520
6: $\alpha=0.05, R=0.5\text{mm}$ Without Lift Force		17.30	0.493

fraction, the larger bubbles are suspended, a lower turbulence energy is generated (see conditions 2, 3, and 4). The reason is that the vortex-bubble interaction time scale decreases, estimated from the characteristic scale of the vortex and the rising speed of the bubble. On the contrary, small bubbles remain inside the same vortex for longer times, so that the momentum of the vortex is modified by the buoyancy. The reason for the complex change of uniformity with respect to the bubble radius R is that it is a combined phenomenon of the two kinds of bubble diffusion effects, namely, turbulence of the liquid phase and lift forces acting on the bubble. (3) The lift force has the effect to markedly decrease the turbulence energy (see conditions 1, 2, 5, and 6). This is due to the horizontal force component of the lift force that makes the bubble distribution dispersive. The detailed explanation is as follows: first, the upward liquid flow is driven in a dense region of bubbles, next, the bubbles are separated from each other by the lift force which occurs at the surrounding shear layer of the upward flow. Auton's equation⁽¹¹⁾, which is valid for the expression of the inertial lift force in linear-shear flow, was adopted in this simulation as an approximate expression for the actual lift force. However, other types of lift forces still have the same force direction so that similar results can be expected. Moreover, it is well known that much larger lift forces can be generated by contamination of the bubble surface, so that this effect is estimated to be more marked in the experiment.

3.2 Structure of the buoyancy-induced turbulence

Figure 3 shows instantaneous distribution of the kinetic energy of liquids, bubbles and the bubble

Fig. 3 Kinetic energy distribution induced by bubble buoyancy

velocity at $T=10$ s. In the case of low void fraction (condition 1), almost all the bubbles rise up straight, and a weak local flow with a long wavelength in vertical direction occurs. In the cases of high void fractions (condition 2) and small bubbles (condition 3), the bubbles rise up with meandering motion and a strong and isotropic turbulence is induced. When the lift force is ignored (compare conditions 5 and 6), the wavenumber of the turbulence increases and the bubbles concentrate in the region of high kinetic energy.

As mentioned above, the turbulence structure in bubbly flow depends not only on the void fraction but significantly on the translational motion of bubbles as well. The most interesting point in particular, is the "mutual" interaction between rising bubbles and vortex motion. This form of interaction was also elucidated for bubbly two-phase jet flows⁽¹²⁾. In this paper, a more detailed discussion on the liquid-flow field is performed according to the wave-spectrum analysis.

Figure 4 represents the wave spectrum of the instantaneous liquid-flow field obtained by conducting the two-dimensional Fourier transformation. Since the grid number here is 10 000, the wave-number resolution becomes $\log(10\,000/2)=3.91$. The following facts are demonstrated by the figure. (1) The velocity components in the high wavenumber region

decay with constant slope under any condition. (2) The velocity components are enhanced by an increase of void fraction (see A). (3) The velocity components in the low wavenumber region increase slightly in the case of smaller bubbles (see B). (4) The velocity components for all the wavenumbers increase when the lift force is neglected.

Two kinds of slopes are drawn in Fig. 4 as a reference. One is Kolmogorov's $-5/3$ power law and the other is -3 because Esmaeeli and Tryggvason⁽⁶⁾ obtained a slope around -3 from the two-dimensional direct numerical simulation for small Reynolds number bubbles. The present results partially correspond to those slopes. However, the mechanism of turbulence dissipation in this analysis is fundamentally different from Kolmogorov's theory. The turbulence generation in liquid phase through the free-rising bubbles is not due to the inertial effect (or nonlinear effect) but to discretized buoyancy based on heterogeneous media.

Furthermore, the difference of the turbulence structure in free-rising bubbles from single-phase natural convection can be explained as follows. In the single-phase natural convection, body force gradient is generated by a temperature difference given at a boundary or as initial condition. In this case, the buoyancy source is dissipated continuously by the

Fig. 4 Wave spectrum of liquid velocity

thermal diffusion. The flow field therefore does not involve the body force which can continuously produce higher wavenumber fluctuations. On the contrary, continuous fluctuations with higher wavenumbers are supplied since the bubbles do not vanish quickly except in the case where the gas inside the bubbles diffuses quickly or condenses in the liquid

Fig. 5 Liquid-driven ratio (LDR)

phase. Thus, the fluctuation of shorter wavelengths is ordinarily generated in bubbly flow, and it develops into large scales with the vortex-bubble interaction. This process may be called “inverse-energy cascade”⁽⁶⁾.

Although, pseudo turbulence due to volume interaction (or kinematic interaction) is taken into account in this analysis, wake-induced turbulence in the case of high Reynolds number or high Weber number, such as vortex shedding in the downstream of each bubble cannot be considered in this numerical method due to the limitation of resolution. Therefore, the energy spectrum for wavenumbers higher than the maximum wavenumber in Fig. 4 is not resolved. A direct numerical simulation and modeling of the sub-grid scale fluctuation are necessary to examine the bubble wake.

4. Bubble Motion in Buoyant Rising Bubbly Flow

As Wang and Maxey⁽¹³⁾ reported, the average settling velocity of a heavy particle in homogeneous turbulent flow is modified from that in quiescent fluid. In the case of high Reynolds number particle, it decreases owing to nonlinear drag forces caused by the relative velocity between the particle and the fluid. In the case of low Reynolds number particle, it increases due to the local downward flow surrounding the particle. In this section, we investigate whether similar phenomena in bubbly flow are present or not.

4.1 Discussion with three dimensionless parameters

Figure 5 indicates the dimensionless parameter of liquid-driven ratio (LDR) which is defined by the average value of liquid upward velocity at a bubble location divided by the bubble rising velocity. LDR increases as void fraction α increases or bubble radius R decreases. Considering that the average velocity of the liquid phase is zero in the computational domain, a positive value of LDR means that bubbles in a high

Fig. 6 Bubble turbulence ratio

Fig. 7 Turbulence amplification ratio

void fraction region are accelerated by the upward liquid flow.

Figure 6 represents a dimensionless parameter of bubble's turbulence ratio (BTR) which is defined by the RMS fluctuation velocity of bubbles divided by the average bubble rising velocity. Like the LDR, the BTR is enhanced as void fraction α increases, or bubble radius R decreases. In particular, it becomes larger than the one in the case of $\alpha=0.05$ and $R=0.2$ mm. Thus, small bubbles have less linearity in rising motion, and rise up on long paths. A similar phenomenon was observed in a bubble plume⁽⁸⁾, which was considered to be quite an important characteristic of bubbling processes in various industrial applications.

Figure 7 shows a dimensionless parameter of turbulence amplification ratio (TAR) which is defined by the ratio of RMS fluctuation velocity of bubbles against that of liquid. It is well known that in the case of high-frequency vibrating flow, bubbles will cause larger fluctuations than the liquid phase, because of short inertial response times. In the case of spherical bubbles, the TAR ranges from 1.0 to 3.0⁽⁸⁾. In the present analysis, a TAR of 1.3 to 1.5 is obtained.

While the TAR can increase for larger bubbles under the same liquid fluctuation condition, this is not clear in the present results since smaller bubbles generate stronger liquid fluctuations in the free rising bubble two-phase flow. Because of these two combined effects due to two-way interaction, TAR declines with the void fraction but does not depend markedly on the bubble size.

Please note that the so-called "bubble-bubble interaction" was ignored in this research because of treating only the low void fraction. This effect is due to the interaction among the near-interface liquid flows of each bubble. However, its energy level is negligible compared to the large scale turbulence discussed in the last section. In fact, important differences in the turbulence structure are not identified by carrying out the analysis in which the bubble-bubble interaction is considered as a model for the drag modification. In the future, the authors have to analyze the present system by further considering the deformable bubble case, contaminated bubble case and effect of wake-induced turbulence.

5. Conclusion

The two-way coupling procedure of the Eulerian-Lagrangian model is revised by introducing a new definition of local void fraction (template delivery method). By this method, the local void fraction is calculated as a spatially continuous variable so that the liquid fluctuation due to the migration of a bubble inside or through a grid is validly predicted as a grid averaged quantity. Based on this method, the following conclusions on the turbulence structure in buoyant rising bubbles can be made.

(1) Turbulence energy of liquid phase driven by many buoyant bubbles is enhanced when void fraction is high, and bubble size is small (provided that spherical bubbles with Reynolds number below 100 are assumed).

(2) The wave spectrum of the liquid flow field has a constant decaying slope in the high wavenumber region, but a different intensity in low wavenumber region depending on the bubble motion. The slope of the inverse cascade process is correctly predicted by the present Eulerian-Lagrangian model.

(3) The turbulence energy of the liquid phase is markedly reduced by the lift force of bubbles.

This work was supported by the Grant-in-Aid for Scientific Research (C-2: No.10650166 and A: No. 10750121) sponsored by the Japanese Ministry of Education, Science, Sports and Culture.

References

- (1) Matsumoto, Y. and Murai, Y., Trans. Jpn. Soc.

- Mech. Eng., (in Japanese), Vol. 61, No. 588, B (1995), p. 85-92.
- (2) Tomiyama, A., Uegomori, S., Minagawa, H., Fukuda, T. and Sakaguchi, T., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 60, No. 580, B (1994), p. 9-14.
 - (3) Takagi, S. and Matsumoto, Y., Japanese J. Multiphase Flow, (in Japanese), Vol. 10, No. 3 (1996), p. 264-273.
 - (4) Tomiyama, A., Sou, A., Minagawa, H. and Sakaguchi, T., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 57, No. 539, B (1991), p. 2167-2174.
 - (5) Song, X., Yamamoto, F., Iguchi, M., Kato, H., Murai, Y., Ishi, K. and Sakaki, Y., ISIJ International, Vol. 36 (1996), p. 54-57.
 - (6) Esmaeeli, A. and Tryggvason, G., J. Fluid Mech., Vol. 314 (1996), p. 315-330.
 - (7) Murai, Y. and Matsumoto, Y., JSME Int. J., Ser. B, Vol. 41, No. 3 (1998), p. 568-574.
 - (8) Murai, Y. and Matsumoto, Y., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 63, No. 611, B (1997), p. 2283-2288.
 - (9) Lance, M. and Bataille, J., J. Fluid Mech., Vol. 222 (1991), p. 95-118.
 - (10) Tomiyama, A., Higaki, H., Zun, I., Sou, A. and Sakaguchi, T., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 62, No. 599, B (1996), p. 2558-2564.
 - (11) Auton, T.R., J. Fluid Mech., Vol. 183 (1987), p. 199-218.
 - (12) Murai, Y. and Matsumoto, Y., ASME-FED, Vol. 185 (1994), p. 203-208.
 - (13) Wang, L. and Maxey, M.R., J. Fluid Mech., Vol. 256 (1993), p. 27-68.
 - (14) Murai, Y., Matsumoto, Y. and Yamamoto, F., Proc. JSME Centennial Grand Congress, ISAC '97, (1997), p. 121-126.
-