Inverse Energy Cascade Structure of Turbulence in a Bubbly Flow*

(PIV Measurement and Results)

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The inverse energy cascade, which is one of the important phenomena to enhance the large-scale flow instability in bubbly flow, is investigated by measuring a local two-phase flow structure driven by buoyant bubbles using particle imaging velocimetry (PIV). In PIV, the flow field of liquid phase is measured by separating an original image to respective phase images using a statistical thresholding method for separating image parameters of bubbles and particles. The present results obtained in the case where the bubble Reynolds number and average void fraction are less than 30 and 1.5%, respectively, confirm the large energy decay with a slope index steeper than $-5/3$ in the log-log diagram of energy spectrum in a high wavenumber region. An important relationship between the energy spectrum and the bubble-bubble interval distance is also detected.

Key Words: Multiphase Flow, Bubble, Turbulent Flow, Energy Cascade, Particle Imaging Velocimetry

1. Introduction

The performance of various industrial devices which use bubbly flows as the working fluid, such as in agitation, mixing and phase-separation processes, can be improved significantly if detailed information on the turbulence generation mechanism due to bubbles is analyzed clearly. For instance, it will become possible to control the wavelength of the turbulence in chemical plants and bioreactors, so that highly effective mixing and chemical reactions can be obtained. Also, if the dominant wavelength in the flow can be accurately predicted, the prevention of flow instability and resonance phenomena can be

effectively performed.

In order to comprehensively understand the bubbly flow, it is necessary to determine the following two factors: (1) the microscale liquid flow structure around individual bubbles, and (2) the effect of the liquid flow structure on the bubble motion. The details regarding (1) have been reported by many researchers as the basic issue of single bubble motion⁽¹⁾⁻⁽³⁾. The latter issue, in particular the dispersion behavior in a turbulent flow has also been analyzed by a number of researchers^{(4),(5)}.

When the interaction between (1) and (2) is balanced at a quasi-steady state, a microscale turbulence which is continuously generated by individual rising bubbles grows to a macroscale turbulence with increasing time. This phenomenon is called "inverse energy cascade in bubbly flows"⁽⁶⁾ and is observed under the conditions where the buoyancy source in the flow field is not easily dissipated, and a partial kinetic energy given to the liquid phase is not rapidly diffused. Moreover, the low-frequency fluctuation, which is continuously amplified by the bubbles may become the trigger to cause a large flow instability. This phenomenon, however, has only been investigated by two-

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dimensional numerical simulations^{(6),(7)} in the past, and the phenomenological generality has not yet been validated.

The purpose of this study is to validate the presence of the inverse energy cascade in bubbly flows using experimental measurements by image processing, and also by three-dimensional numerical analysis. In this first report, the particle imaging velocimetry (PIV) method to elucidate the detailed flow structure in bubbly flows is demonstrated. The main results obtained from the method are as follows:

① The kinetic energy of liquid phase induced by rising bubbles is decayed in a high wavenumber region with a slope steeper than $-5/3$ when the kinematic viscosity of liquid is $10^{-5} \text{ m}^2/\text{s}$, the void fraction is lower than 1.5%, and the bubble Reynolds number is less than 30.

② The wavelength in which this slope is obtained is much longer than the Kolmogorov scale, and corresponds to that of the bubble-bubble distance.

③ By comparing the two-dimensional direct numerical simulation (DNS) results, it is clarified that the microscale flow is amplified from the bubble-size scale to the bubble-bubble interval distance scale with a constant energy rate.

2. Image Measurement of Bubbly Flows

2.1 Merits of image measurement

When an experiment on multiphase flow is performed, it is necessary to obtain various experimental results with sufficient reproducibility. For bubbly flows, attention should be paid to the many governing factors, such as liquid density, liquid viscosity, gas density, gas viscosity, gas flow rate, liquid flow rate, void fraction, bubble size histogram, gas velocity, liquid velocity, pressure, bubble shape and temperature. The researchers must formulate experimental plans after considering definitely whether these factors are controlled or measured quantities. The measurements based on image processing have the following important advantages for determining the above factors.

① It is contact-free, so that the flow field is not affected by the measurement. If any contact-type measurement is adopted for this study, a small-scale fluctuation may increase to a large one due to the principle of the inverse energy cascade.

② Several measurement quantities can be simultaneously obtained from instantaneous whole-field images. In the present method, the bubble's center of gravity, average bubble radius, standard deviation of the bubble radius, void fraction, bubble velocity and liquid velocity are simultaneously measured.

③ With data processing for time-serial images,

Table 1 Conventional various image analyses for measuring bubbly flows

Variable	Measurement Principle
Void Fraction	Cloudy Liquid: • Brightness Calibration → refs. (8), (9), (10)
	Transparent Liquid: • Gas-Liquid Interface Detection (Obtaining Bubble Size/Shape) → refs. (11), (13)
Bubble Velocity	• Bubble's Pathline Method • PTV (tracking algorithm) → refs. (13)
Liquid Velocity	• Particle's Pathline Method • PTV (tracking algorithm) • Brightness Cross Correlation • Recent other PIV algorithm → refs. (16)
Simultaneous PIV for Both Phases	• Fluorescent Particle Method • Size Identification by MQD • Automatic Identification by Object's Parameter (the present method) → refs. (14), (15), (16)

various statistic properties can be discussed, such as time averaging, spatial averaging, bubble number averaging, ensemble averaging and conditional averaging results. Furthermore, the data can be easily evaluated by converting them to other physical quantities like the vorticity map and the energy spectrum.

2.2 Examples of conventional image processing

Due to the merits mentioned above, many techniques based on image processing have been proposed to measure the bubbly flows. A number of measurement principles are described in Table 1. The methods for measuring the void fraction, interface area concentration or bubble size were proposed due to the issue of turbulent diffusion effects^{(8),(9)}, along with flow visualization inside a nontransparent media using neutron radiography⁽¹⁰⁾, and bubble-size measurement inside a pipe⁽¹¹⁾. In these methods, each quantity is measured by a calibration relationship between the brightness map of the image and the physical quantities.

On the other hand, PIV is available for measuring the flow velocity components from the visualized images, and detecting the motion of dispersion consisting of bubbles and tracer particles. Although PIV is still being developed due to strict limitation of measurement area and precision, detailed information can be obtained by adequate choice of seeding, illumination, and photographic conditions, as well as by selecting a suitable PIV algorithm. For example, one

Fig. 1 Schematic diagram of experimental system

of the authors reported the PIV method for measuring the three-dimensional liquid flow around a bubble⁽¹²⁾ and the bubble motion inside a bubble plume⁽¹³⁾. The application of PIV to the simultaneous measurement of two phases was first reported by Hassan et al.⁽¹⁴⁾ This technique is quite useful to analyze the phase interaction mechanism. Sridar and Katz⁽¹⁵⁾ analyzed the lift force acting on a bubble using the same principle. In these techniques, fluorescence particles were seeded as liquid tracer particles.

When monochrome images contain both bubbles and tracer particles for liquid motion, and have a certain difference in size and brightness (or light reflection ratio) between them, simultaneous measurement of both phases is possible. In fact, usefulness of the size difference for separating images of bubbles and particles was reported by Merzkirch et al.⁽¹⁶⁾ who proposed an algorithm, the so-called minimum quadratic difference (MQD) method. In the present paper we describe a more automatic method for bubble-particle separation from monochrome images based on Otsu's method (or automatic threshold selection method⁽¹⁷⁾) by applying information on both the size and brightness. Detailed procedures are described in section 4.

3. Experimental System

The aim of the experiment is to clarify how the flow field is developed by the growth of bubble-generated turbulence. Therefore, the liquid phase motion is visualized by tracer particles and measured by image processing.

Figure 1 shows the schematic diagram of the present experimental apparatus. A transparent acrylic resin rectangular tank of $100 \times 100 \times 1\,000 \text{ mm}^3$ volume is filled with silicone oil with viscosity of 10 cSt (Shin-Estu Silicone KF96). Nitrogen gas bubbles are injected uniformly from the bottom plane through needle-type capillaries. The gas flow rate is

Fig. 2 Bubble injection system

controlled in the range of 0.33 to $1.67 \times 10^{-5} \text{ m}^3/\text{s}$ using a pressure controller and a gas flowmeter. Detailed structure of the bubble injection device is shown in Fig. 2. Longer needles are effective to restrict the size deviation of the bubbles in this range, since it has a larger pressure loss between both edges of the needles. Tracer particles of white high porous polymer with diameter of 75 to $150 \mu\text{m}$, and density of $1\,010 \text{ kg/m}^3$ and a white light sheet of $2 \times 200 \text{ mm}^2$ area are used to visualize the flow in a vertical cross section of the rising bubbly flow. When the flow is at a well-developed state, the measurement region with an area of $85 \times 65 \text{ mm}^2$ at the position of 500 mm from the bottom plane is recorded using a CCD camera (Sony, DCR-VX1000) at a framing rate of 30 fps. In this region, the lattice style arrangement of bubbles, which is formed by the regular arrangement of the needles, is broken so that the two-phase interaction is confirmed to be at the equilibrium state. Furthermore, since the three-dimensional component of the flow is restricted near the upper free surface, this region is judged as the optimal position. The images are imported to a PC through an image board, then preprocessing, PIV measurement, and postprocessing are performed. The resolution of the image is 640×480 pixels with 256 gray levels (8 bits).

4. Image-Processing Method

The images obtained in the present experiment are mixed images of small tracer particles which are illuminated inside the light sheet of 2 mm thickness, and medium-sized bubbles which are recognized by the light reflection at the bubble interfaces. In order to obtain the liquid flow field from the images, the separation of images into particles and bubbles is first

Fig. 3 Separation of two-phases images

carried out, and then, the PIV measurement is conducted only for particle images. The detailed procedure is given below.

4.1 Image separation of bubbles and particles

Figure 3 shows an original experimental image, a bubble image which is separated from it, and a particle image. The image-separation procedure is as follows:

① The original image is converted to binarized image using an adequate threshold value. Then, the objects which are recognized as clusters of white pixels are labeled.

② Three parameters are calculated for each object, that is, the area, average brightness and the standard deviation of brightness.

③ By applying Otsu's method⁽¹⁷⁾ to the three parameters, the parameter which has the largest statistical mutual deviation is selected, and the threshold value for the parameter is determined (detailed description is given later).

④ Using these threshold parameters, all the objects are separated into bubbles and particles.

Otsu's method is used to determine the threshold value ϕ_t to reasonably classify the histogram of function ϕ based on the following mathematical principle. First, the histogram of ϕ is divided into L levels between the maximum value $\phi_{\max}$ and the minimum value $\phi_{\min}$. Here, the width of a level is given by:

$$\Delta\phi = \frac{\phi_{\max} - \phi_{\min}}{L}, \quad (1)$$

Denoting the number in each level as $n_1, n_2, \dots, n_L$, the normalized histogram is defined by $P_i = n_i/N$, where N is the total number, that is, $N = n_1 + n_2 + \dots + n_L$. The average μ_T value and the total deviation σ_T^2 for the histogram ϕ are calculated by the following equations.

$$\mu_T = \sum_{i=1}^L iP_i, \quad (2)$$

$$\sigma_T^2 = \sum_{i=1}^L (i - \mu_T)^2 P_i, \quad (3)$$

When the k -th level is adopted as the threshold value for the histogram, that is, the histogram which has L levels is classified into two parts by first to k -th levels and $(k+1)$ th to L -th levels, the zeroth and the first-order moments of the histogram are given by:

$$\omega(k) = \sum_{i=1}^k P_i, \quad (\omega(L) = 1) \quad (4)$$

$$\mu(k) = \sum_{i=1}^k iP_i, \quad (\mu(L) = \mu_T) \quad (5)$$

Next, the mutual deviation $\sigma_B^2(k)$ which is defined by the following equation is calculated for all the k (1 to L) values.

$$\sigma_B^2(k) = \frac{(\mu_T \omega(k) - \mu(k))^2}{\omega(k)(1 - \omega(k))} \quad (6)$$

When the value of k_t which introduces the maximum value of $\sigma_B^2(k)$ is determined, the threshold value ϕ_t is given by $\phi_t = \phi_{\min} + \Delta\phi \cdot k_t$.

Figure 3 shows the result of image-separation processing from an original image. In the present experiment, the main factor which separated the bubbles and the particles was "the area of the object" which was automatically determined from the three parameters as demonstrated in procedure ②. On the other hand, Merzkirch et al.⁽¹⁶⁾ used the average brightness of the objects as the parameter. It is an important point to note that a statistically based approach is given to the judgement of the parameter for distinguishing the objects.

When a bubble enters the light sheet partially, the bubble may be identified as a particle, so that the exposure time is intentionally expanded to overcome it. From many examination results like this, it was confirmed that it is very seldom frequent that a bubble is identified as a particle by mistake. Thus, there is no accuracy reduction caused by this image-separation process. However, in the case of a high void fraction of over 2%, the effective measurement region is decreased by heavy overlapping of the bubbles and the particles in an image frame. Therefore, the case of high void fraction is excluded from the measurement target.

4.2 PIV measurement

In this research, the velocity gradient tensor (VGT) algorithm⁽¹²⁾ is adopted as a PIV algorithm for particle-seeded images. The VGT algorithm is expected to have a much better particle-tracking

performance among various PIV algorithms since it can track the particle clusters quite well in the flow field which contains strong local rotation, shearing, and expansion. Although the quantitative performance of the VGT has not yet been investigated, it can at least be said that it has been verified that the correct capturing ratio of the VGT is always higher than the binarized image cross-correlation (BICC) and the four-frame particle tracking velocimetry (4-PTV)⁽¹²⁾. For the present experiment in particular, the VGT is suitable because various vortex scales are mixed in the bubbly flow.

Even though the VGT is adopted for the present case, the ratio of erroneous vectors is nearly 15% since the flow field involves strong spatio-temporal fluctuations. However, this ratio is not considered to be large for the PIV system which is applied to the actual image with noises. The accuracy of the data is usually improved by postprocessing to eliminate the erroneous vectors. For postprocessing techniques, the double-check method⁽¹⁹⁾ and the multiple-step averaging method⁽²⁰⁾ are common. However, these methods are not generalized so that the following two conditions are adopted in order to eliminate the erroneous vectors: 1) one of the coordinates of a particle's displacement vector corresponds to that of the other vector, 2) the length of a particle's displacement vector is more than threefold that of an entire averaged vector. The ratio of eliminated vectors, in which some correctly matched vectors are also included, is nearly 40%. However, it has been confirmed by manual research that more than 95% of the erroneous vectors can be eliminated by these processes.

In addition, when we attempted to apply the brightness distribution cross-correlation (BDCC), the BICC, and the 4-PTV algorithms, good results could not be obtained due to the appearance of too many erroneous vectors. Since the present research is not concerned with development of the PIV algorithm, a detailed comparison of the performance of various PIV methods is omitted.

4.3 Postprocessing to obtain continuous velocity data

Figure 4 (a) shows an example of the velocity vector map of the tracer particles obtained by PIV. The energy spectrum of the flow cannot be derived directly from this result due to the scattered distribution of the velocity vectors. Thus, the data is converted to a continuous distribution by a rearrangement method⁽²¹⁾ which was developed by the authors. Figure 4 (b) shows the rearranged velocity vector map of the liquid phase. The detailed procedures of the rearrangement method are as follows:

- ① The measurement area is divided by iso-interval

Fig. 4 Velocity Vector map of liquid phase

grids (54×72 in this study).

- ② Velocity of all measurement areas is set to zero as the initial value.

- ③ Velocity data obtained by PIV is given to the measurement points.

- ④ Laplace equation given by Eq. (7) for the velocity vector, is solved by utilizing the PIV data as the boundary condition. Since this equation is not applied to the points of the PIV data in this calculation, the PIV data itself is not modified.

$$\nabla^2 u = 0, \nabla^2 v = 0. \quad (7)$$

- ⑤ The amplitude of the velocity vectors obtained by the last step is corrected to ensure the agreement of the kinetic energy between the rearranged velocity map and the original PIV data map.

A continuous velocity distribution is reconstructed by the above postprocessing procedures. This solution is one of multidimensional linear interpolated results for the PIV measurement data and has the following three merits compared with conventional popular interpolation methods like the inverse distance rearrangement (IDR) or polygon interpolation method. First, it has a high generality to treat the various distribution conditions of the PIV data. Second, the computational load is low. Third, the error of kinetic energy which is caused by the rearrangement is prevented by adding the process ⑤.

The grid size in this rearrangement process is set larger than the spatial scale of the flow near each bubble. Therefore, the spatial resolution of the velocity data is the same as the grid size. Wherever the PIV data is not obtained near the bubble image, the above rearrangement method can also be applied. In the same way, the influence of the fluctuation of void fraction and the ambient flow near the bubble interface are not taken into account in the present

Table 2 Experimental conditions

Fixed Conditions : Liquid Density = 960 kg/m ³ , Kinematic Viscosity of Liquid = 1.0×10 ⁻⁵ m ² /s Gas Density = 1.21 kg/m ³ , Environmental Pressure = 101.3 kPa, Temperature = 13.0~17.5 °C Test Tank Size = 0.10m×0.10m×1.00m, Number of Bubble Injection Needle = 16(min)~361(max) Internal Diameter of the Injection Needles = 0.20mm, Liquid Depth (without Bubble) = 0.90m						
Variable Condition		CASE 1	CASE 2	CASE 3	CASE 4	CASE 5
Average Void Fraction	α [%]	0.333	0.612	0.855	0.913	1.062
Average Bubble Radius	R [mm]	0.60	0.64	0.78	0.86	0.89
Standard Deviation of Bubble Radius	R' [mm]	0.11	0.15	0.25	0.28	0.28
Average Rising Velocity of Bubble	Vave[m/s]	0.100	0.109	0.117	0.146	0.157
Average Bubble Reynolds Number	Re [-]	12.0	14.0	18.3	25.1	28.0
Average Bubble Weber Number	We [-]	0.80	1.01	1.42	2.44	2.93

rearrangement process, since the average void fraction used in this study is low.

5. Results and Discussion

5.1 Experimental conditions

Table 2 indicates the detailed experimental conditions. The average value and the standard deviation of the bubble radius are calculated by sampling more than 50 bubble images under each condition. The rising velocity of bubble, bubble Reynolds number and the bubble Weber number are obtained by the PTV measurement results for 50 time-serial bubble images (1.7 s) whose sample is shown in Fig. 3. By considering these dimensionless parameters and the data from conventional investigations⁽¹⁾⁻⁽³⁾, the bubble motion pattern in this experiment can be summarized as follows:

① The bubble does not cause a large deformation of the interface. The bubble shape is almost spherical due to the strong effect of surface tension.

② There is no vortex shedding from the bubble because the bubble Reynolds number is lower than 30. Therefore, the vortex shedding in the wake region of the bubble is not the factor which generates turbulence in the liquid phase. Thus, the bubble-generated turbulence in this experiment is mostly caused by the equation of continuity (or the potential flow component) around the moving bubble interface.

③ The coalescence, the fragmentation, and the bubble-bubble interaction can be considered unimportant because the average void fraction is lower than 1.1%.

④ The compressibility of the bubble is negligible since the volumetric change due to the static pressure gradient in the liquid phase is less than 1% within the measurement area with a height of 85 mm.

5.2 Velocity vector map

Figure 5 shows the instantaneous distribution of the liquid velocity vector and the bubbles for CASE 2. It is recognized from the figure that quite a complex

Fig. 5 Two-phase flow structure measured by PIV (bubble and liquid velocity vectors in CASE 2)

flow, similar to a turbulent flow is formed, depending on the bubble distribution pattern and the liquid viscosity. Also, the flow pattern appears to be influenced by the local bubble concentration resulting in the upward or downward liquid flows. This phenomenon can be related to the increase of the settling velocity of low Reynolds number heavy particles as reported by Wang and Maxey⁽²²⁾, i. e., the rising velocity of low Reynolds number bubble becomes higher than the terminal rising velocity in a quiescent liquid due to the transportation effect of the local liquid's upward flow which is induced by the buoyancy of the bubble. On the other hand, when Reynolds number is higher than 10, as in the present experiment, the rising velocity is decreased by the influence of the turbulent fluctuation of liquid phase⁽⁷⁾. This is because the drag force becomes nonlinear for the relative velocity between

the two phases. The measurement results of the average rising velocity in this experiment indicate that the measured value is lower than the terminal velocities for CASE 1 and CASE 2, and higher than those for CASE 3 to CASE 5. This type of phenomenon is sensitive to the bubble distribution pattern, for instance, when the bubble distribution becomes uniform or clustered to a certain size, the local liquid flow pattern changes. This phenomenon appears quite complicated since it is governed by each component ratio of force acting on the bubbles in the balancing state of the gas-liquid phase interaction. This problem will be discussed in the next report in which a parametric study based on a numerical analysis will be described.

5.3 Measurement of energy spectrum

The kinetic energy spectrum versus the wavenumber is obtained from the rearranged velocity vector map of liquid phase by the following method. First, the absolute amplitude of the velocity vector (u, v) is calculated for the entire measurement area (65 mm × 85 mm) which is divided into 54 × 72 grids. Second, the amplitude is transformed to the energy spectrum by two-dimensional Fourier transformation (2D-FT). The minimum wavenumber $k_{\min}$ is given by $\ln(k_{\min}) = \ln(1/0.065) = 2.73$, and the maximum wavenumber $k_{\max}$ is given by $\ln(k_{\max}) = \ln((54/2)/0.065) = 6.03$. In this case, the shortest wavelength or spatial resolution is $1/k_{\max} = 2.41$ mm which is more than twice the bubble size in this experiment. Although the out-of-plane velocity component w is not considered in the kinetic energy, there is no difference in the slope of the energy spectrum in the log-log diagram since the ratio of the kinetic energy component in each direction, u^2 , v^2 , and w^2 to the total kinetic energy is considered to be constant while the flow is in the quasi-steady state.

Figure 6 shows the time-averaged energy spectra obtained by averaging 50 time-serial sets (1.7 s) of the instantaneous energy spectrum. The abscissa represents natural logarithm values of the wavenumber per meter, the wavenumber of the grid k_G is given by $\ln(k_G) = 6.74$, and that of the bubble size k_B is given by $\ln(k_B) = 7.02$ to 7.42. It is confirmed that the time-averaged energy spectrum and the instantaneous energy spectrum which is obtained by two consecutive images have little discrepancy in the high wavenumber region greater than $\ln(k) = 4.5$. Under all five conditions in this experiment, a energy constant decaying with a slope index of around -3 is identified at the high wavenumber side greater than a critical wavenumber. Furthermore, a gradual decaying with the slope index of less than $-5/3$ is observed in the low wavenumber region which is less than the critical

Fig. 6 Measured energy spectra of liquid phase

wavenumber.

The critical wavenumber is considered to be related to the average bubble-bubble distance L . If the bubbles are assumed to be arranged in a square lattice pattern, L is estimated by $L = (4\pi/3\alpha)^{1/3}R$, where α is the average void fraction and R is the average bubble radius. The wavenumber for L in this experiment ranges from $\ln(1/L) = 5.01$ to 5.18 which agrees well with the critical wavenumber. On the other hand, by assuming that the flow is an isotropic turbulent flow, the equivalent wavenumber for the Kolmogorov scale is calculated by $k_K = (\varepsilon/\nu^3)^{-1/4}$, where ε is the turbulent dissipation coefficient given by $\varepsilon = 2\nu \int k^2 E(k) dk$, and ν is kinematic viscosity. The wavenumber of the Kolmogorov scale for the present experiment is calculated by $\ln(k_K) = 8.28$ which is much higher than the critical wavenumber. The wavelength of the Kolmogorov scale is $\lambda_K = 0.252$ mm so that it is much smaller than the bubble size. Moreover, the evidence that the critical wavenumber is not caused by the post-processing (described in section 4.3) is explained as follows. The average distance among the PIV data before the rearrangement, L_p is estimated by $L_p = (A/N_p)^{1/2}$, where A is the measurement area, and N_p is the number of the PIV data. N_p is greater than 1 000 under all conditions in this experiment so that the wavenumber k_p is greater than $\ln(k_p) = \ln(1/L_p) = 6.05$. The influence of interpolation appears in the region of the wavenumber higher than this wavenumber, but the critical wavenumber is considerably smaller than this wavenumber.

From these discussions, it can be concluded that the constant slope which is detected in the energy spectrum of bubbly flows exists in much lower

wavenumber regions than the cascade process of single-phase turbulent flow. It also has a strong relationship with the bubble-bubble interval distance whose wavenumber corresponds well with the critical wavenumber to divide the spectrum into two parts. Furthermore, a constant slope index of -3 is found in the higher wavenumber region than in the critical wavenumber region, and a calmer slope is observed in the lower wavenumber region because the dispersive effect of the buoyancy of the bubbles does not contribute to the flow with a longer wavelength.

The existence of plural slopes in bubbly flows is also recognized by the numerical results obtained by two-dimensional DNS as reported by Esmaeeli and Tryggvason⁽⁶⁾. Although they did not discuss the bubble-bubble interval distance to explain the physical meaning of the slope, the matching point where two different slopes are connected agrees with the bubble-bubble interval distance in their results.

From the discussion mentioned above, the authors believe that this phenomenon can be constructed by the following dynamic processes. (1) Liquid flow with a scale several times longer than a bubble size is induced around bubble by bubble rising motion. (2) This local flow increases to the flow with a longer wavelength at a constant energy rate. (3) This growth reaches the scale of average bubble-bubble interval distance of the dispersed bubbles. (4) The flow with a longer wavelength than the bubble-bubble interval distance does not grow rapidly, but slowly.

6. Conclusion

In the present study, the inverse energy cascade in bubbly flows, which was reported in the past only by two-dimensional numerical simulations, has been validated experimentally by means of particle imaging velocimetry. The following matters have been clarified by the present results.

1) By the experimental measurement based on image processing, it is confirmed that the local liquid flow pattern generated by rising bubbles has a constant slope in the energy spectrum in a higher wavenumber region than the critical wavenumber. The slope index is around -3 in the present experimental conditions, that is, the kinematic viscosity of liquid is $10^{-5} \text{ m}^2/\text{s}$, the bubble Reynolds number is lower than 30, and the average void fraction is less than 1.1%. The wavenumber range in which this phenomenon appears is confirmed to be much lower than those of the bubble size and the Kolmogorov scale.

2) The minimum wavenumber at which a constant slope appears in the energy spectrum has a close relationship with the bubble-bubble interval distance.

Therefore, the inverse energy cascade in bubbly flow starts from a scale of turbulence generation around each bubble, and increases to the scale of the bubble-bubble interval distance at a constant energy rate. The growth rate of the flow with a longer wavelength than the bubble-bubble interval distance is low because the dispersive effect of the buoyancy is not significant for such a long wavelength.

All the conclusions mentioned above are limited under the present experimental conditions, so that it is not definite that the critical wavenumber and the slope index are invariant. In the next report, the generality of this phenomenon will be examined by a parametric study based on numerical analysis.

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