Motion of the Piston in Piston Pumps and Motors*

(Theoretical Discussion in Relation to Solid Contact Force and Power Loss)

Atsushi YAMAGUCHI**

Tribological conditions between the piston and cylinder wall are very important criteria to realize high efficiency and reliability in piston pumps and motors, in particular for swash plate types. In this paper we theoretically analyze the solid contacting forces and power losses under medium- and high-speed operations. Three calculating methods are discussed with regard to cavitation of the fluid film. The effects of piston mass, differences between the male and female pistons, rotating cylinder block types and rotating swash plate types are also considered.

Key Words: Fluid Power System, Piston Pump, Piston, Theory, Cavitation, Tribology, Solid Contact, Power Loss

1. Introduction

Pistons are the principal operating elements of piston-type machines and their fundamental performance invariably depends on the suitable motion of pistons in the cylinder bore. The author has previously reported studies reflecting this viewpoint. The motion of pistons is unstable in the case of cylindrical pistons and bores¹⁾ and to stabilize the motion the exponential function-type pistons with smaller head diameters (equivalent to tapered or stepped pistons) are effective in cases of bent axis types because of the small lateral force acting on pistons. For the swash plate types, however, these pistons are not effective because of large lateral forces²⁾. A design method for pistons with hydrostatic pads to realize fluid film lubrication between the piston and cylinder wall³⁾ was confirmed experimentally⁴⁾. A theoretical method to obtain the conditions in the case where the piston

contacts the wall⁵⁾, and the characteristics of low-speed operating ranges were experimentally checked⁴⁾.

Recently, the test results of the frictional forces acting on a piston and the contacting conditions have been reported⁶⁻¹⁵⁾. However, the tests were carried out with rotary swash plate types under low speeds and constant pressure operations.

In this report, we theoretically analyze the case of conventional swash-plate-type pumps and motors under middle- and high-speed operations and the effects of inertia forces of pistons and piston dimensions on the solid contacting forces and power losses around pistons. Solving methods using the Reynolds equation to determine the pressure distribution around the piston, such as the direct numerical solution and the perturbation method considering the second-order terms are compared. Moreover, the solutions of rotary swash plate types are compared with conventional types and the differences between the female and male piston types are discussed.

2. Nomenclature

$\bar{A}_c$: sectional area of piston $= A_c/R^2$

C : radial clearance

$\bar{I}$: principal moment of inertia of piston
 $= I\omega/(6\mu R^3(R/C)^3)$

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** Department of Mechanical Engineering and Materials Science, Faculty of Engineering, Yokohama National University, 79-5 Tokiwadai, Hodogaya-ku, Yokohama 240-8501, Japan. E-mail: yama@post.me.ynu.ac.jp

- $\bar{l}$: piston length $= l/R$
 $\bar{M}$: moment load-carrying capacity
 (moment of force due to fluid pressure) $= M(6\mu\omega R^3(R/C)^2)$
 $\bar{m}$: piston mass $= m\omega/(6\mu R(R/C)^3)$
 $\bar{p}$: fluid pressure $= p/(6\mu\omega(R/C)^2)$
 $\bar{p}_c$: cylinder pressure
 $\bar{p}_{co}$: $\bar{p}_c$ on high-pressure side $= 1/S_0$
 $\bar{P}$: load-carrying capacity (force due to fluid pressure) $= P/(6\mu\omega R^2(R/C)^2)$
 $\bar{P}_{th}$: theoretical power $= P_{th}/(6\mu\omega^2 R^3(R/C)^2)$
 $\Delta\bar{P}_q/\bar{P}_{th}$: power loss due to leakage
 $\Delta\bar{P}_f/\bar{P}_{th}$: power loss due to friction
 R : piston radius
 $\bar{R}_A, \bar{R}_B$: contact force on piston
 $= R_A/(6\mu\omega R^2(R/C)^2), = R_B/(6\mu\omega R^2(R/C)^2)$
 $\bar{R}_c$: pitch circle radius of cylinder bores $= R_c/R$
 $\bar{R}_s$: radius of piston end sphere $= R_s/R$
 S_0 : $= 6\mu\omega(R/C)^2/p_{co}$
 t : time $= \tau/\omega$
 $\bar{W}$: force load $= W/(6\mu\omega R^2(R/C)^2)$
 x, y, z : coordinates
 $\bar{x}, \bar{y}$: piston displacements $= x/C, = y/C$
 $\bar{z}_A$: z coordinate of piston head $= \bar{l}/2$
 $\bar{z}_B$: z coordinate of piston end or end of cylinder block $= z_B/R$
 $\bar{\alpha}$: inclination angle of piston $= aR/C$
 δ : swash plate angle (for the swash-plate-type case)
 λ_A, λ_B : frictional coefficient on contact point
 λ_s : frictional coefficient between piston and slipper
 μ : viscosity
 τ : $= \omega t$
 φ : angle coordinate from x -axis on o_b - xyz system
 $\bar{\Phi}_x, \bar{\Phi}_y$: angle between centerline of piston and xz -plane $= \Phi_x R/C, yz$ -plane $= \Phi_y R/C$
 ω : angular velocity of shaft
 Ω : (spinning angular velocity of piston $\Omega + \omega$)/ ω
 subscripts
 • : differentiation with respect to τ
 - : nondimensional quantity

3. Theory

We consider the conventional rotary cylinder block types. We define the coordinate systems as shown in Fig. 1. The O_c - XYZ system is an inertial system whose origin is the center of the cylinder block, the o_b - xyz is a moving system whose origin is the center of the cylinder bore, and o_p is the center of the piston. We obtain equations of motion of the piston as follows (with more detailed expression than those in Ref. (4)).

Fig. 1 Schematic diagram

Fig. 2 Explanation of α, Φ_x and Φ_y

Translation:

$$\begin{aligned} \bar{m}\ddot{\bar{x}} &= \bar{R}_c(R_c/C)\bar{m}\cos\tau + \bar{P}_x \\ &\quad - \bar{R}_A\cos\varphi_A - \bar{R}_B\cos\varphi_B \\ &\quad - \text{sign}(\bar{\nu}_{Ex})\lambda_s\bar{W}|\bar{\nu}_{Ex}|/\bar{\nu}_E \end{aligned} \quad (1)$$

$$\begin{aligned} \bar{m}\ddot{\bar{y}} &= \bar{R}_c(R_c/C)\bar{m}\sin\tau + \bar{P}_y \\ &\quad - \bar{R}_A\sin\varphi_A - \bar{R}_B\sin\varphi_B - \bar{W}\sin\delta \\ &\quad - \text{sign}(\bar{\nu}_{Ey})\lambda_s\bar{W}\cos\delta|\bar{\nu}_{Ey}|/\bar{\nu}_E \\ \bar{m}\ddot{\bar{z}} &= -\bar{A}_c\bar{P}_c - (\lambda_A\bar{R}_A + \lambda_B\bar{R}_B)\text{sign}(\dot{\bar{l}}_2) \\ &\quad + \bar{W}\cos\delta \\ &\quad - \text{sign}(\bar{\nu}_{Ez})\lambda_s\bar{W}\sin\delta|\bar{\nu}_{Ez}|/\bar{\nu}_E \end{aligned} \quad (3)$$

Rotation:

$$\begin{aligned} \bar{I}_1\ddot{\bar{\Phi}}_x - \bar{I}_0(\Omega/\omega)\dot{\bar{\Phi}}_y &= -\bar{M}_x - \bar{z}_A\bar{R}_A\sin\varphi_A \\ &\quad + |\bar{z}_B|\bar{R}_B\sin\varphi_B + (\bar{l} - \bar{z}_A)\bar{W}\sin\delta \\ &\quad + (\lambda_A\bar{R}_A\sin\varphi_A + \lambda_B\bar{R}_B\sin\varphi_B)\text{sign}(\dot{\bar{l}}_2) \\ &\quad + \text{sign}(\bar{\nu}_{Ey})\lambda_s\bar{W}(\bar{\alpha}\cos\delta \pm \bar{R}_s)|\bar{\nu}_{Ey}|/\bar{\nu}_E \end{aligned} \quad (4)$$

$$\begin{aligned} \bar{I}_1\ddot{\bar{\Phi}}_y + \bar{I}_0(\Omega/\omega)\dot{\bar{\Phi}}_x &= \bar{M}_y - \bar{z}_A\bar{R}_A\cos\varphi_A \\ &\quad + |\bar{z}_B|\bar{R}_B\cos\varphi_B \\ &\quad + (\lambda_A\bar{R}_A\cos\varphi_A + \lambda_B\bar{R}_B\cos\varphi_B)\text{sign}(\dot{\bar{l}}_2) \\ &\quad + \text{sign}(\bar{\nu}_{Ez})\lambda_s\bar{W}(\bar{\alpha} \pm \bar{R}_s\cos\delta)|\bar{\nu}_{Ez}|/\bar{\nu}_E \end{aligned} \quad (5)$$

where the sliding velocities between the spherical surface on the piston and slipper are given by

$$\begin{aligned} \bar{\nu}_{Ex} &= \dot{\bar{x}} - (\bar{\alpha} \pm \bar{R}_s\cos\delta)\dot{\bar{\Phi}}_y \\ \bar{\nu}_{Ey} &= \dot{\bar{y}} - (\bar{\alpha} \pm \bar{R}_s\cos\delta)\dot{\bar{\Phi}}_x \\ \bar{\nu}_E &= (\bar{\nu}_{Ex}^2 + \bar{\nu}_{Ey}^2)^{1/2} \end{aligned} \quad (6)$$

where, $\bar{\alpha} = \bar{l}/2$, the plus sign is for the male piston and the minus sign is for the female one. Illustration of Φ_x and Φ_y is given in Fig. 2 ($\Phi_x = \alpha \sin \beta$, $\Phi_y = \alpha \cos \beta$, β : maximum azimuth angle). We assume that the z position of the piston is known and its spin velocity is

a given constant.

The fluid film thickness is

$$\bar{h} = 1 - (\bar{x} + \bar{z}\bar{\Phi}_y)\cos\varphi - (\bar{y} + \bar{z}\bar{\Phi}_x)\sin\varphi \quad (7)$$

We must solve the Reynolds equation

$$\frac{\partial}{\partial\varphi}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\varphi}\right) + \frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right) = \bar{Q}\frac{\partial\bar{h}}{\partial\varphi} - \bar{w}_0\frac{\partial\bar{h}}{\partial\bar{z}} + 2\frac{\partial\bar{h}}{\partial\tau} \quad (8)$$

for $\bar{P}_x, \bar{P}_y, \bar{M}_x$ and $\bar{M}_y$ ⁵⁾.

Here, we compare the following three solving methods of Eq. (8), based on the rationalization of cavitation treatment in the film and the calculation time required.

(1) Numerical calculation method

The film pressure is given by

$$\begin{aligned} \bar{p}(\varphi, \bar{z}, \tau) = & \bar{p}_1(\varphi, \bar{z}, \tau) \\ & + \bar{p}_2(\varphi, \bar{z}, \tau)\dot{\bar{x}} + \bar{p}_3(\varphi, \bar{z}, \tau)\dot{\bar{y}} \\ & + \bar{p}_4(\varphi, \bar{z}, \tau)\dot{\bar{\Phi}}_x + \bar{p}_5(\varphi, \bar{z}, \tau)\dot{\bar{\Phi}}_y \end{aligned} \quad (9)$$

where $\bar{p}_1$: $\bar{p}$ is due to hydrostatic and wedge effects and $\bar{p}_2 \sim \bar{p}_5$: is due to squeeze effects. The boundary conditions are

$$\left. \begin{aligned} \bar{p}(0, \bar{z}, \tau) &= \bar{p}(2\pi, \bar{z}, \tau) \\ \frac{\partial}{\partial\varphi}\bar{p}(0, \bar{z}, \tau) &= \frac{\partial}{\partial\varphi}\bar{p}(2\pi, \bar{z}, \tau) \\ \bar{p}(\varphi, \bar{z}_A, \tau) &= \bar{p}_c(\tau) \\ \bar{p}(\varphi, \bar{z}_B, \tau) &= \bar{p}_a \end{aligned} \right\} \quad (10)$$

For the numerical method I: $\bar{p}$ is obtained by the direct numerical method and instantaneous negative pressure is assumed zero for calculating the load-carrying capacities and power losses ($\bar{p}_a = 0$ indicates atmospheric pressure).

For the numerical method II: $\bar{p}_1$ is obtained by the direct numerical method the same as Eq. (8), and the negative value of $\bar{p}_1$ is assumed to be zero for the load-carrying capacities. The following perturbation solutions are used for $\bar{p}_2 \sim \bar{p}_5$.

(2) Perturbation method

We apply the perturbation method with respect to parameter, $\bar{a}$, that is,

$$\bar{p}(\varphi, \bar{z}, \tau) = \bar{p}_0^* + \bar{a}\bar{p}_1^* + \bar{a}^2\bar{p}_2^* + \dots \quad (11)$$

For considering cavitation, we use terms up to the second order and put the negative value into zero.

On the other hand, for the squeeze-effect terms we take the negative values into consideration, because the bubbly liquid film exists even in the negative-pressure area and the apparent viscosity is larger than that of the liquid without bubbles. Therefore, the squeeze-effect terms are orthogonal functions. We term this treatment the second-order perturbation.

The conditions of solid contacts are as follows:

Occurrence:

$$\left. \begin{aligned} (\bar{x} + \bar{z}_A\bar{\Phi}_y)^2 + (\bar{y} + \bar{z}_A\bar{\Phi}_x)^2 &\geq 1 \\ (\bar{x} + \bar{z}_B\bar{\Phi}_y)^2 + (\bar{y} + \bar{z}_B\bar{\Phi}_x)^2 &\geq 1 \end{aligned} \right\} \quad (12)$$

Dissolution:

$$\bar{R}_A \leq 0, \quad \bar{R}_B \leq 0 \quad (13)$$

where φ_A and φ_B are φ of contact points A and B, respectively.

The power losses around the piston are composed of losses due to the solid and fluid frictions and due to leakage⁵⁾.

For rotating swash plate types, the direction of loads acting on the piston is a function of the shaft rotational angle. The spinning velocity would be different than that of the conventional types. Since the cylinder block is at rest, $\bar{Q}$ of Eq. (8) corresponds to the spin ($\bar{Q} = \Omega/\omega$), and the value can be from unity to -1 based mainly on the contact between the slipper and swash plate (for uniform contact, $\bar{Q} = 1$). For the conventional rotating cylinder block type, since the point o_b rotates around the z -axis at angular velocity ω , the value of $\bar{Q}$ in Eq. (8) ranges from unity (without spin) to 2 (without peripheral slip between wall and piston) ($\bar{Q} = 1 + \Omega/\omega$).

4. Numerical Results

Table 1 presents the piston dimensions and parameters for the standard numerical examples. The operating condition is expressed by S_0 and the cylinder pressure shown in Fig. 3. We present male pistons case without note.

The solid contact forces and power losses are mainly selected for evaluation. In the case of the former, the maximum contact forces $\bar{R}^* = \bar{R}/(\pi\bar{p}_{c0})$ (ratio to pressure force acting on the piston head), the contact angles, and the ratio of contact time to a periodic time (approximation, t_A, t_B) are given.

Table 2 presents the results for numerical

Table 1 Dimensions and parameters

$\bar{z}_A = \bar{l}/2 = 3.180$	$\bar{z}_B = -1.538 (\delta = 0^\circ)$
$\bar{R}_C = 4.205$	$R/C = 1 \times 10^3$
$\bar{m} = 0.1308 \times 10^{-4}$	$\bar{I}_0 = 0.654 \times 10^{-5}$
$\bar{I}_1 = 0.473 \times 10^{-4}$	$\delta = 20^\circ$
$\lambda_A = \lambda_B = \lambda = 0.02$	$\lambda_S = 0.0$
$\bar{R}_S = 0.6818$	$\bar{\Omega} = 1 + \Omega/\omega = 1.5$

Fig. 3 Pressure $\bar{P}_c$ in cylinder

Table 2 Piston performance (inertia terms included, method I)

S_0	0.01	0.1	1.0	2.0
$\bar{R}_A^*$	0.463	0.462	0.233	0.056
φ_A rad	1.576	1.62	2.00	2.00
t_A	0.55	0.50	0.40	0.21
$\bar{R}_B^*$	0.269	0.135	0.0	0.0
φ_B rad	-1.56	-1.52	—	—
t_B	0.13	0.11	—	—
$\Delta \bar{P}_T / \bar{P}_{th}$	0.0064	0.0062	0.0065	0.0099
$\Delta \bar{P}_Q / \bar{P}_{th}$	0.066	0.0040	-0.0022	-0.0025

Table 3 Piston performance (inertia terms included, method II)

S_0	0.01	0.1	1.0	←	←
$\bar{R}_A^*$	0.649	0.705	0.234	0.216	0.908
φ_A rad	1.58	1.63	1.95	1.97	2.16
$\bar{R}_B^*$	0.319	0.437	0.0008	0.052	2.290
φ_B rad	-1.56	-1.42	-1.16	-1.28	-1.40
$\Delta \bar{P}_T / \bar{P}_{th}$	0.0096	0.0121	0.0073	0.0067	0.0135
$\Delta \bar{P}_Q / \bar{P}_{th}$	0.0911	0.0059	-0.0014	-0.0012	-0.0012
$\bar{m} (\times 10^4)$	0.1308	←	←	$\times 10$	$\times 100$
$\bar{I}_0 (\times 10^5)$	0.654	←	←	$\times 10$	$\times 100$
$\bar{I}_1 (\times 10^4)$	0.473	←	←	$\times 10$	$\times 100$

method I. Equations of motion (Eqs. (1)–(5)) were solved by the Runge-Kutta method. The maximum contact force at piston head $\bar{R}_A^*$ is larger than $\bar{R}_B^*$ (at the cylinder block end). For S_0 larger than unity, the contact forces decrease considerably because of hydrodynamic effects. Although the contact forces decrease, the power loss due to friction does not decrease. The reason for this is that the frictional loss based on fluid friction increases with S_0 . In contrast, the power loss due to leakage decreases with S_0 , and the minus in the table indicates the flow rate gain based on the piston motion.

The results for numerical method II, listed in Table 3, agree qualitatively with the results in Table 2. For $S_0=1.0$ a quantitative agreement is obtained. Numerical method II is superior to method I in terms of both convergency and computing time. This is due to the fact that for method II, the squeeze-film effects are given as analytical solutions. Even in the negative-pressure film, bubbly liquids exist and have a damping effect on the piston. As a result, method II may be superior to method I.

In the nondimensionalization adopted in this paper, $\bar{m}$, $\bar{I}_0$ and $\bar{I}_1$ are proportional to ω , thus we must consider this point for the numerical examples, if changes in S_0 depend on those in ω . Therefore, in Table 3, the numerical results for the case of $\bar{m}$, $\bar{I}_0$ and $\bar{I}_1$ ranging from 10 to 100 times are given for $S_0=$

Fig. 4 Radial displacement of piston end at points A and B (inertia terms included, method II)

Fig. 5 Time change of fluid film parameters (inertia terms included, method II)

1.0. In the case of 10 times, changes in power losses are rather small, but $\bar{R}_B$ increases. In the case of 100 times, the contact forces, particularly, $\bar{R}_B$, increase rapidly and power losses due to friction also increase.

The piston displacements of the case given in Table 3 are shown in Figs. 4 and 5. Figure 4 shows the changes in film thicknesses at the piston head (A) and cylinder block end (B). Generally, during the delivery stroke (high-pressure side), solid contact occurs at the piston head ($H_A=1.0$), and the displacement becomes small during the suction stroke. The solid contact at the cylinder block end occurs immediately after the cylinder pressure changes from low to high. Figure 5 shows the change in fluid film parameters ($\bar{x}$, $\bar{y}$, $\bar{\Phi}_x$, $\bar{\Phi}_y$). These parameters change significantly at the transient period from low to high pressure. $\bar{R}_A^*$ and $\bar{R}_B^*$ occur after the changing period.

Table 4 presents the results of the case where the Reynolds equation is solved by method II and only centrifugal forces are considered as inertia terms in equations of motion of the piston (treatment by neglecting the left side terms in Eqs. (1)–(5)). Comparing the results with those in Table 3, where only the treatment of inertia terms is different, a good agreement in the contact forces is recognized for small S_0 ,

Table 4 Piston performance (only centrifugal force included, method II)

S_0	0.01	0.1	1.0	0.01	1.0
$\bar{R}_A^*$	0.738	0.762	0.826	0.738	0.827
φ_A rad	1.58	1.69	1.80	1.58	1.76
$\bar{R}_B^*$	0.309	0.593	0.466	0.309	0.497
φ_B rad	-1.57	-1.41	-1.39	-1.57	-1.38
$\Delta\bar{P}_T/\bar{P}_{th}$	0.0137	0.0143	0.0140	0.0130	0.0137
$\Delta\bar{P}_Q/\bar{P}_{th}$	0.0222	0.0015	-0.0004	-0.0215	-0.0004
Type	Male			Female	

Table 5 Maximum contact force of pistons (perturbation method)

method	second-order			first-order			
S_0	0.5	0.75	1.0	0.01	0.1	0.5	0.75
$\bar{R}_A^*$	0.529	0.499	0.546	0.761	0.745	0.532	0.515
φ_A rad	1.96	2.04	1.59	1.57	1.69	1.95	2.01
$\bar{R}_B^*$	0.987	1.02	1.06	0.353	0.449	1.02	1.17
φ_B rad	-1.38	-1.34	-1.31	-1.57	-1.77	-1.45	1.36

however, disagreement is observed with large S_0 . This could be due to some differences in power losses. In Table 4, the comparison between female and male pistons is also listed. The female pistons are constructed so as to tighten the spherical bearing of the slipper from the piston end by caulking. Then, even if the line of action of the force (load) acting on the piston through the slipper passes through the cylinder wall at the bottom dead center, the contact forces are small. For the numerical examples given in the paper, for large swash plate angles, the difference between the female and male pistons is hardly recognizable.

Table 5 presents the results obtained by the perturbation method, and the term first order indicates the case considered up to $\bar{a}$ terms for hydrostatic effects. The same treatments for squeeze effects and inertia terms are used as for the second-order perturbation method. Good agreement between these two methods is obtained up to $S_0=0.75$. The second-order method is superior to the first-order method in terms of both convergency and computation time. For $S_0=1.0$ and 2.0 , however, half-order subharmonic oscillation appears (the value of $\bar{R}^*$ in the table is the maximum in one period). Comparing the results shown in Tables 2 and 3, we must be careful to adopt the perturbation method for large S_0 values.

The treatment for small S_0 , where the contact occurs in yz -plane and the negative pressure is permitted, was reported in a previous paper⁴⁾. Applying this treatment to the case of $S_0=0.01$, we obtain $\bar{R}_A^*=0.753$ and $\bar{R}_B^*=0.330$. These values coincide well with the results by the first-order perturbation method listed in Table 5. However, for $S_0=0.75$ we obtain $\bar{R}_A^*=1.003$

Table 6 Comparison of contact force of rotation swash plate type and rotation cylinder block type ($S_0=0.75$, second-order perturbation method)

type	swash plate				cylinder block		
$\bar{\Omega}$	-0.5	0.0	0.5	1.0	1.0	1.5	2.0
$\bar{R}_A^*$	1.03	1.05	0.725	0.676	0.394	0.499	0.670
φ_A rad	-2.82	-2.58	-2.14	-2.36	2.20	2.04	1.67
$\bar{R}_B^*$	0.451	0.358	0.962	0.965	1.03	1.02	1.06
φ_B rad	0.826	0.033	0.380	0.234	-1.44	-1.34	-1.45

and $\bar{R}_A^*=0.347$. The differences are considerable.

Table 6 shows the results for the rotating swash plate type, by the second-order perturbation method. To evaluate the effects of $\bar{\Omega}$ on the contact forces, we select four kinds of $\bar{\Omega}$. Comparing the results with the rotating cylinder block type, it is seen that the maximum contact forces are nearly equal and the maximum forces are apt to occur in $\bar{R}_A$ for the rotating swash plate type.

5. Conclusion

1. In this report, three methods to calculate the pressure distribution around the piston are compared. The following results were obtained:

a) For the case where the hydrodynamic effect affects the pressure distribution around the piston ($S_0 \geq 0.1$), the pressure distribution based on the hydrostatic and wedge-film effects should be solved by direct numerical methods using the Reynolds equation.

b) For the squeeze-film effects, although a comparison with experiments is essential, treatment with the perturbation method may be superior to the direct numerical method, because of the ease of calculations.

c) For the case where the hydrodynamic effect scarcely affects the pressure distribution ($S_0 < 0.1$), the perturbation method is reasonable to obtain approximate values.

2. On the basis of the numerical results of the solid contact forces and power losses around the pistons, it is concluded that:

a) The contact forces increase with the hydrodynamic effects (S_0), and then decrease. The contact forces at the end of cylinder block ($\bar{R}_B$) increase with the piston mass.

b) The power loss ratio due to the solid contact forces decreases with S_0 . Also, the power loss ratio due to leakage decreases with S_0 , and then a gain in fluid power occurs.

c) In the numerical examples reported in this paper, no merit of the female pistons is recognized. For the rotating swash plate types, the contact forces are larger at the piston head and smaller at the cylinder block end than those for the conventional rotating cylinder block.

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