Numerical Analysis of Passive Control of Pseudo-Shock*

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Oscillation of the pseudo-shock in the rectangular duct with the porous wall was investigated numerically. In the simulation, 2nd-order accurate, Harten-Yee's upwind TVD scheme was used for high resolution and numerical stability, and Chien's $k-\epsilon$ two equation model was used as the turbulence model. To simulate the effect of the blowing and suction on the porous wall, Darcy's law was used on the porous wall boundary. The boundary layer thickness under the first shock wave oscillates due to the shock oscillation, which changes the strength of the first shock wave, so that it suppresses the pressure fluctuation downstream of the first shock wave. The effects of some parameters of the passive control on the pseudo-shock oscillation were investigated.

Key Words: Supersonic Flow, Shock Wave, Boundary Layer, Passive Control, Fluid Vibration, Numerical Analysis

1. Introduction

It is well known that the pseudo-shock wave in the supersonic channel is liable to oscillate, because the position of the pseudo-shock depends on the upstream and downstream pressure gradient produced by the wall shear stress and on the inlet to outlet pressure ratio, and the oscillation of the pseudo-shock causes the oscillation, noise and damage of the channel. To avoid such difficulties, passive control method, which was invented for the stabilization of the shock wave on the transonic airfoils⁽¹⁾⁻⁽⁶⁾ was successfully

applied to the pseudo-shock⁽⁷⁾⁻⁽⁹⁾.

In the previous researches, the effect of passive control on the steady-state structure of the pseudo-shock wave was studied experimentally and numerically as the basis of this problem⁽¹⁰⁾, and the oscillation of the pseudo-shock was investigated experimentally⁽¹¹⁾. As the results, the pressure gradient induced by the first shock is alleviated, and the total pressure loss is reduced⁽¹⁰⁾, suggesting that the passive control is effective on the oscillation of the pseudo-shock, especially for the low frequency oscillation⁽¹¹⁾. However, the mechanism of the control and the effects of parameters remain unclarified.

In the present research, the passive control of the pseudo-shock oscillation in the supersonic rectangular duct was investigated numerically, and the mechanism of the control and the effects of parameters on the pseudo-shock wave oscillating with the various frequencies were studied.

Nomenclature

A_s : amplitude of shock wave oscillation
 c : sonic speed
 f_b : frequency of back pressure fluctuation

* Received 8th November, 1999. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 65, No. 632, B (1999), p. 1192-1199 (Received 22nd July, 1998)

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- L_p : length of porous wall
 P_r, P_{rt} : laminar or turbulent Prandtl number
 p : pressure
 Re : Reynolds number $\equiv (\rho c D / \mu)^*$
 γ : ratio of specific heats
 μ, μ_t : viscosity or eddy viscosity
 σ : porosity
 θ : flow direction
 ϕ_1 : phase of first shock
 ϕ_{1j} : phase lag of j -th shock
 Subscript
 c : cavity
 o : main channel
 $*$: dimensional quantity

2. Numerical Analysis

2.1 Governing equations

The numerical simulation was made, referring to the experimental conditions. The height of the channel D^* is the characteristic length, and ρ^* , U^* and μ^* at the inlet are the characteristic quantities. The other quantities are non-dimensionalized as

$$p = p^* / \rho^* c^{*2}, \quad f_b = f_b^* D^* / c^*, \quad t = t^* c^* / D^*.$$

The basic equations are the time-averaged, two-dimensional Navier-Stokes equations that are given in the non-dimensional conservative form in the Cartesian coordinate as follows.

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{1}{Re} \left(\frac{\partial R}{\partial x} + \frac{\partial S}{\partial y} \right), \quad (1)$$

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e + p)u \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e + p)v \end{pmatrix}, \quad (2)$$

$$R = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \\ R_4 \end{pmatrix}, \quad S = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ S_4 \end{pmatrix}, \quad (3)$$

$$R_4 = (\tau_{xx}u + \tau_{yx}v) + \frac{1}{\gamma - 1} \left(\frac{\mu}{P_r} + \frac{\mu_t}{P_{rt}} \right) \frac{\partial}{\partial x} (c^2), \quad (4)$$

$$S_4 = (\tau_{xy}u + \tau_{yy}v) + \frac{1}{\gamma - 1} \left(\frac{\mu}{P_r} + \frac{\mu_t}{P_{rt}} \right) \frac{\partial}{\partial y} (c^2),$$

where e is the total energy per unit volume, that is the sum of internal and kinetic energy. The pressure p was derived by the equation of state for a perfect gas. The viscosity μ which depends on temperature was calculated by the Sutherland's law. The eddy viscosity μ_t was computed by the Chien's low-Reynolds number $k-\varepsilon$ turbulence model⁽¹²⁾.

The Harten-Yee's second-order-accurate TVD scheme⁽¹³⁾ was used for the convection terms and the second-order central difference method was used for the viscous terms. Explicit method was used to the equations.

Fig. 1 Computational domain

Fig. 2 Computational mesh

2.2 Computational region and grid

Figures 1 and 2 show the flow field and the mesh for the numerical computation. The flow is assumed to be symmetric in terms of the centerline of the channel, and the lower-half was calculated. The streamwise length of the computational region was eight times the height of the channel D , was considering the pseudo-shock length. The calculation was carried out, changing the cavity length and the depth. Figure 1 shows the typical shape of the channel with the length of cavity $0.8D$ and the depth $0.5D$.

The number of the nodes were 201×32 in the main channel and 21×19 in the cavity as shown in Fig. 2. The size of the cell was uniform in x -direction, while exponentially stretched away from the wall surface in y -direction in the region nearer to the wall with a minimum spacing of $y^+ \cong 3$ inside of the boundary layer.

2.3 Boundary conditions

The boundary conditions on the solid wall were non-slip and adiabatic, and on the upper boundary the flow was symmetric. The inlet condition was the uniform supersonic flow with the turbulent boundary layer of 1/7th power law velocity profile and the viscous sublayer, and the boundary layer thickness is $0.1D$. Enthalpy was assumed uniform across the boundary layer. Turbulent quantities at the inlet were given by values of the flat plate turbulent boundary with the same thickness. The first-order extrapolation was used at the outlet boundary layer, except that the pressure was given so as to adjust the position of the first shock wave. In order to produce the oscillation of the pseudo-shock wave, the back pressure fluctuation was assumed sinusoidal with the amplitude of 2.5% of mean back pressure p_b referring to the previous experiment⁽¹⁴⁾.

In the porous region, the blowing or suction velocity was represented, using the following Darcy's type equation:

$$\lambda = \frac{64}{Re_p} + 0.3164 Re_p^{-0.25}, \quad Re_p = \frac{\rho_o v_o d_p}{\mu_o \sigma},$$

$$S_p = \text{sign}(p_c - p_o),$$

$$v_o = \sigma S_p \sqrt{\frac{2d_p}{\lambda \rho_o l_p} |p_c - p_o|}, \quad (5)$$

where λ is the friction coefficient of the pore, Re_p is the Reynolds number in the pore, l_p is the porous wall thickness, and p_c and p_o are the pressure on the porous wall at the cavity and the main channel side, respectively. These pressure can be obtained by solving the following momentum equation on the porous wall:

Fig. 3 Pressure distribution near the first shock wave

$$\frac{\partial p}{\partial y} = -\frac{\partial \rho v}{\partial t} - \frac{\partial \rho v^2}{\partial y} + \frac{1}{Re} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right). \quad (6)$$

However, in this research, the Reynolds number is very high, and the blowing or suction velocity on the porous wall is no more than 1~2% of main flow velocity, so that the right hand side of Eq.(6) is negligible. Then the pressure at the porous wall surface was given as follows:

$$\frac{\partial p}{\partial y} = 0. \quad (7)$$

Figure 3 shows the pressure distribution calculated by Eqs.(6) and (7). In Fig.3 arrowheads denote the leading and the trailing edge of porous wall. The difference in the shape of pressure distributions is negligible, because the first grid point from the wall is extremely close to the wall. Non-slip and adiabatic conditions were assumed on the wall.

Boundary conditions on the wall of the cavity were non-slip and isothermal. The mass flux to or from the cavity was equal to that of the main channel side obtained by Eq.(5).

Solid wall

Porous wall

Fig. 4 Iso-density contours

Fig. 5 Positions of shock waves

3. Results and Discussion

3.1 Oscillation mechanism of pseudo-shock

In order to understand the oscillation mechanism of pseudo-shock wave under the passive control, the typical parameters are selected, and the pseudo-shock wave on the porous wall is compared with that on the solid wall. Mach number of the uniform inlet flow is $M=1.80$, the depth of the cavity is $D_c=0.5$, the diameter of the hole is $d_p=0.015$, the length and the thickness of the porous wall are $L_p=0.8$ and $l_p=0.23$, respectively, and the frequency of the back pressure fluctuation is $f_b^*=60$ Hz. These values are basically fixed in the following section, unless mentioned otherwise.

The iso-density contours of the oscillating pseudo-shock wave in each phases of a cycle are shown in Fig. 4. It is confirmed that the pseudo-shock wave on the solid wall does not change in the structure at all, and it oscillates as if it is a rigid body, keeping the distances same through the first shock wave to the tailing shock wave. On the other hand, the pseudo-shock wave on the porous wall varies in the strength of the each shock wave, expanding and contracting the spacing between each shock wave.

Figure 5 shows the positions of the first, second and third shock waves, corresponding to Fig. 4. The shock waves on the solid wall oscillate in parallel and in phase, and the shock waves on the porous wall also oscillate in phase, but the amplitude of the first shock is fairly reduced, and it gradually becomes larger, then the amplitude of the third shock being the same as that of the solid wall.

Figures 6 and 7 show the pressure distribution along the centerline of the channel and the flow direction near the first shock wave at $y=0.18$. Compared with the results of the solid wall, the shape of the pressure distribution of the porous wall varies in

(a) Solid wall

(b) Porous wall

Fig. 6 Pressure distribution

(a) Solid wall

(b) Porous wall

Fig. 7 Flow direction near the first shock

Fig. 8 Wall pressure fluctuation

phase, and the flow direction, that depends on the displacement thickness of the boundary layer, also varies due to the oscillation of the first shock, which is also confirmed by the variation of the boundary layer thickness in Fig. 4. These results show the fact that as the first shock shifts on the porous wall, the first shock strength is modified by the change of the adverse pressure gradient caused by the blowing and suction mass flux, and this modification absorbs the pressure fluctuation behind the first shock. Thus the oscillation of the first shock is suppressed.

The wall pressure fluctuations are shown in Fig. 8, where the pressure fluctuation is defined as the difference between maximum and minimum pressures through a cycle at an arbitrary position. It is found that the pressure fluctuation at the first shock is reduced by the passive control, but this effect becomes weak at downstream from the second shock, and the fluctuation level is same at the second shock wave and slightly larger in the downstream. It is considered that such enhancement is attributed to the variation of the pressure distribution, but it is negligible compared with the reduction at the first shock. Thus it can be concluded that this passive control is quite effective.

3.2 Effects of parameters

Unlike the oscillation of the rigid body, the oscillation of the pseudo-shock wave has wide spread of frequencies due to many degrees of freedom. On the other hand, if the pseudo-shock wave is forced to oscillate by the back pressure fluctuation, the frequency may be locked to that of the back pressure fluctuation.

In this section, the effects of parameters are studied to realize the optimum passive control for the various boundary conditions.

3.2.1 Effects of back pressure frequency The back pressure frequency was changed in the range of $fb^* = 60 \sim 180$ Hz. Figure 9 shows the amplitude of the first, second and third shock waves in terms of the frequency. By the passive control, the amplitude of the first and the second shock waves are reduced,

(a) Solid wall

(b) Porous wall

Fig. 9 Amplitude of shock waves

Fig. 10 Phase delay of the shock waves

especially in the low frequency region of less than 120 Hz, but there is no effect on the third shock wave through the whole frequency region.

Figure 10 shows the phase lag of the second shock wave against the first shock. The phase lag is very small in both cases, and it increases with the increase in the frequency. It is considered that there may be some time lag between the time at which the first shock senses the pressure fluctuation and the time at which it begins to move due to the pressure rise in the cavity.

Figure 11 shows the fluctuation of the flow direction near the first shock wave at $y=0.18$, which is

Fig. 11 Flow direction near the first shock wave

Fig. 12 Change of wall pressure fluctuation

given in the similar way as Fig. 8. The fluctuation on the porous wall is very large, especially in the low frequency region, corresponding to that of the amplitude in Fig. 9. This result shows that the variation of the flow direction strongly affects the pseudo-shock amplitude.

Figure 12 shows the wall pressure fluctuation at the first shock wave. The wall pressure fluctuation is reduced even in the high frequency region where the amplitude is not so suppressed. This shows that in the high frequency region, the effect of the weakened wall pressure gradient⁽¹⁰⁾ is predominant, while in the low frequency region the effect of the passive control is slightly decreased, because the pressure distribution considerably varies with the oscillation of the pseudo-shock wave.

3.2.2 Effects of length of porous wall In order to study the effects of the porous wall length, the calculation was carried out with two types of porous wall, that is, $L_p=0.8$ and 1.0 , and the frequency range of the back pressure fluctuation is $f_b^*=60\sim 180$ Hz.

The amplitude of the first shock wave is shown in Fig. 13. In the previous research⁽¹⁰⁾, it is confirmed that the optimum length for the steady pseudo-shock is $L_p \leq 1.0$, however, it is not the case for the oscillat-

(a) First shock wave

(b) Second shock wave

Fig. 13 Amplitude of shock waves

Fig. 14 Pressure distribution ($f_b^*=60$ Hz)

ing pseudo-shock. In the case of $L_p=1.0$, the length of the porous wall is just comparable with the distance between the foots of the first shock wave. Then if the amplitude is relatively large, the rear-foot shock shifts upstream of the trailing edge of the porous wall and the first shock wave moves upstream of the porous wall. Then the oscillation is not suppressed. On the other hand, the amplitude of the second shock decreases because of the pressure feedback from back-side of the first shock.

The pressure distributions at $\phi=\pi$, corresponding to the time when the first shock reaches most

Fig. 15 Change of wall pressure fluctuation

Fig. 17 Change of pressure fluctuation

Fig. 16 Amplitude of first shock wave

Fig. 18 Amplitude of first shock wave

upstream point, is shown in Fig. 14, and the wall pressure fluctuation at the first shock wave is in Fig. 15. The first shock wave of $L_p=1.0$ becomes stronger than that of the solid wall, and the pressure fluctuation at the first shock is higher, because the off-design pressure feedback through the porous wall increases the pressure under the rear-foot shock. Therefore, $L_p=0.8$, that is slightly shorter than the spacing between the fore-foot shock and the rear-foot shock, is adequate.

3.2.3 Effects of cavity depth In order to study the effects of the cavity depth, $D_c=0.3, 0.5$ and 0.7 were selected, and the calculation was carried out for the same frequency as the previous subsections.

The amplitude of the first shock is shown in Fig. 16. The amplitude is almost independent of the cavity depth, but only in the case of $f_b^*=60$ Hz and $D_c=0.3$, the amplitude is slightly larger than others. It is considered that when cavity is shallow, the apparent Reynolds number in the cavity decreases, and in the case of $f_b^*=60$ Hz, when the mass flux through the porous wall is maximum, the effect of the cavity depth appears. Figure 17 shows the wall pressure fluctuation at the first shock. The pressure fluctuation is independent of the cavity depth throughout the whole frequency.

(a) $\phi_1=0$ (b) $\phi_1=\pi$

Fig. 19 Pressure distribution near the first shock

Fig. 20 Change of pressure fluctuation

3.2.4 Effects of porosity In this section, under the condition of $\sigma=4\sim 16\%$, the effect of the porosity on the pseudo-shock oscillation is studied.

The amplitude of the first shock is shown in Fig. 18. It is confirmed in the steady-state results⁽¹⁰⁾ that under the condition of $\sigma \leq 6\%$ the resistance of the porous wall becomes strong, so that the suction mass flux in the downstream region of the porous wall decreases. Same effect is also seen in the unsteady result of Fig. 18. The pressure feedback is not sufficient, so that the amplitude increases more than others. On the other hand, there is no remarkable difference from the other porosity cases, but there is a tendency that the amplitudes decrease with the increase in the porosity in the low frequency region of less than 120 Hz.

Figure 19 shows the pressure distribution near the first shock along the centerline in the case of the back pressure frequency of $f_b^* = 60$ Hz. The figure shows that under the condition of $\sigma=4\%$, the strength of the rear-foot shock does not vary at $\phi_1=0$ and the first shock is pushed away to downstream.

The wall pressure fluctuation at the first shock is shown in Fig. 20. The fluctuation is also small at $\sigma=4\%$ because of the small change of the pressure distribution. But in this case, the large amplitude of the first shock elongates the high pressure fluctuation region near the first shock wave, which has serious effect on the channel wall. As the results, the wall pressure fluctuation is most suppressed at $\sigma=12\%$.

4. Conclusion

Passive control of the pseudo-shock oscillation in the supersonic rectangular duct was numerically investigated, and the following conclusions were derived.

(1) The pseudo-shock wave under the passive control oscillates, varying the structure. It is caused by the change of the boundary layer thickness at the first shock due to the first shock oscillation on the

porous wall.

(2) With the porous wall, the variation of the first shock strength suppresses the oscillation, and the wall pressure fluctuation decreases.

(3) Each shock wave that consists the pseudo-shock oscillates in phase, and the frequency is the same as that of the back pressure fluctuation even under the passive control.

(4) With the passive control, the oscillation of the first shock is suppressed in the low frequency region, while the wall pressure fluctuation is suppressed in relatively wide frequency range, because of the reduced adverse pressure gradient at the first shock.

(5) Under the condition of $L_p \cong 0.8$ and $\sigma=12\%$, the oscillation is successfully suppressed. The effect of the cavity depth is very small.

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