A Dynamic Analysis of Free Piston Vuilleumier Cycle Heat Pumps*

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A dynamic analysis of a free piston Vuilleumier cycle heat pump was performed by a time-stepping integration method. The nonlinear relationship between displacement and force for pistons was taken into account for the motion of reciprocating components, in addition to the pressure change of working gas, nonlinear viscous dissipative force due to an oscillating flow and discontinuous damping force caused by solid friction. The displacement of pistons and pressure changes in the Vuilleumier cycle heat pump were integrated by an ideal isothermal thermodynamic relationship. It was assumed that the flow friction was proportional to the kinematic pressure of working gas, and that the solid friction at the seals was due to the functions of the working gas pressure and the tension of seal springs. In order to investigate the dependence of the characteristics of a machine on various dynamical parameters, some calculations were performed for a proposed case study machine and discussed. The roles of working gas pressure, the displacer piston rod and dissipative force due to flow and solid friction were clarified, and the proposed method was found to be very effective in predicting dynamic behavior of free piston Vuilleumier cycle heat pumps and in designing the machine components.

Key Words: Forced Vibration, Nonlinear Vibration, Numerical Analysis, Stirling Engine, Heat Pump, Vuilleumier Cycle, Simulation Model, Free Piston

1. Introduction

The concern about issues regarding chloro-fluorocarbon compounds has renewed interest in alternative refrigeration cycles. One of these is the Vuilleumier cycle, which is expected to offer performance parameters comparable to the Rankin cycle. This cycle is well known for its application at cryogenic temperatures. In the early 1980s, it was proposed in Europe that this cycle could have a high probability for applications in air conditioning^{(1),(2)}. Since then, many researchers have attempted to apply the Vuilleumier cycle heat pumps to air conditioning machines.

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One of the authors was in the process of developing a kinematic type of Vuilleumier cycle heat pumps for industrial and residential use^{(3),(4)}. Since this type of machines has a crank mechanism for synchronizing two displacer pistons, they weigh more than the conventional air conditioning machines. In recent years, the free piston mechanism, which originates from the technology of Stirling engines, has been applied to the Vuilleumier cycle heat pumps for fabricating more compact machines. The free piston mechanism consists of displacer pistons as masses, with springs supporting them, and hence constitute a vibration system. Alternating force exerted on the displacer pistons due to pressure variations of the working gas maintains the oscillation of pistons. This operates the cycles, which induces heat exchange.

Thomas⁽⁵⁾, Kawajiri⁽⁶⁾ and Sekiya⁽⁷⁾ reported the analyses of free piston Vuilleumier cycle heat pumps. In their analyses, one or two of the following assumptions were made for the sake of simplification: linearizing of pressure variation, disregarding vibrations of

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a casing and ignoring solid friction.

In the present paper we describe an analysis without taking any of these assumptions into account. The nonlinear dynamic analysis of a free piston Vuilleumier cycle heat pump was performed by the time-stepping integration method for a proposed case study machine, and the results were partly compared with the experimental data and those of linearized analysis. The effects of friction due to an oscillating flow in the hot and cold side heat exchangers, along with the solid friction at the seals and rod diameters, on the dynamic behavior of the machine, were discussed in detail.

2. Main Nomenclature

- A : area
 F : force
 $\bar{F}$: solid friction force
 f : frequency
 K : spring constant
 m : mass of reciprocating components
 P : pressure
 T : temperature
 t : time
 V : volume of each space
 W : work and power
 X : amplitude
 x : displacement
 ϕ : phase angle

subscripts

- b : buffer
 c : cold space or cold side
 e : casing
 f : flow friction
 h : hot space or hot side
 k : solid friction
 m : medium space or medium side
 r : rod
 s : spring

3. Modeling and Formulation

3.1 Equations of motion for dynamic analysis

Figure 1 shows an analysis model for the proposed free piston Vuilleumier cycle heat pump with some nomenclatures. Assuming concentrated masses and weightless springs, constant pressure in buffer space, and no solid friction except at seals, the equations of motion for a hot displacer piston, a cold displacer piston, and a casing are:

$$\begin{aligned} m_h \frac{d^2 x_h}{dt^2} &= K_h(x_e - x_h) - P_h A_h + P_m(A_h - A_r) \\ &\quad + P_b A_r + K_c(x_c - x_h) + \bar{F}_h + \bar{F}_r, \\ m_c \frac{d^2 x_c}{dt^2} &= P_c A_c - P_m(A_c - A_r) - P_b A_r \end{aligned} \quad (1)$$

$$-K_c(x_c - x_h) + \bar{F}_c - \bar{F}_r, \quad (2)$$

and

$$\begin{aligned} m_e \frac{d^2 x_e}{dt^2} &= -K_h(x_e - x_h) + P_h A_h - P_c A_c \\ &\quad - P_m(A_h - A_r) + P_m(A_c - A_r) - \bar{F}_h - \bar{F}_c, \end{aligned} \quad (3)$$

respectively.

3.2 Working gas pressure distribution

The calculation of working gas pressure distribution in the Vuilleumier cycle heat pump, which varies with the displacements of the two displacer pistons and a casing, is based on a multispace isothermal thermodynamic model. Figure 2 shows the control volumes for this model. Here, the term ht indicates the heater, hrg the hot side regenerator, hra the hot side radiator, dct the connecting duct, cra the cold side radiator, crg the cold side regenerator and cl the cooler. First, a baseline pressure is obtained from an initially assumed uniform pressure throughout the system. Second, the pressure losses due to the friction of an oscillating working gas flow are calculated at each time step. Finally, these values are added to the baseline pressure to obtain the pressure distribution at various times. For the present model, the following are assumed: time variation, but uniform pressure

Fig. 1 Dynamic model of a free piston type machine

Fig. 2 Thermodynamic model

Fig. 3 Seal model

throughout all control volumes, no working gas leakage to the outside from the system, uniform property in each control volume, constant temperature distribution in control volumes at each time step and perfect heat regeneration in regenerators. By applying the conservation law of mass and the equation of state, one can obtain the baseline pressure P_m in the medium space as

$$P_m = P_n \cdot \left(1 - \frac{A_h(x_h - x_e)}{S \cdot T_h} + \frac{A_m(x_h - x_c)}{S \cdot T_m} + \frac{A_c(x_c - x_e)}{S \cdot T_c} \right)^{-1}, \quad (4)$$

$$S = \frac{V_{hn}}{T_h} + \frac{V_{ht}}{T_{ht}} + \frac{V_{hrg}}{T_{hrg}} + \frac{V_{hra}}{T_{hra}} + \frac{V_{mn}}{T_m} + \frac{V_{cra}}{T_{cra}} + \frac{V_{crg}}{T_{crg}} + \frac{V_{cl}}{T_{cl}} + \frac{V_{cn}}{T_c}, \quad (5)$$

where P_n is the pressure when both the reciprocating masses are at a neutral position and is assumed to be an average pressure, and the subscript n indicates the neutral position.

In order to obtain the pressures in the hot space P_h and in the cold space P_c , pressure losses due to flow friction must be calculated. For cylindrical tubes such as heater tubes, and elements in the shell-and-tube type of heat exchangers and connecting ducts, the following expression of pressure loss ΔP is applied as a function of density ρ and velocity u .

$$\Delta P_i = \left(\lambda_i \frac{L_i}{d_i} + H_i \right) \cdot \frac{1}{2} \rho_i |u_i| u_i \quad i = ht, hra, cra, cl, \quad (6)$$

where L is the length of each control volume, λ is the flow friction factor and is calculated using the Blasius' equation for a cylindrical tube, and H , which is taken from Heames' report⁽⁸⁾, is the loss factor for flow expansion and reduction. For the pressure drop for porous metals at the hot and cold side regenerators, we can apply the following formula

$$\Delta P_i = \lambda_i \frac{L_i}{d_{hi}} \cdot \frac{1}{2} \rho_i |u_i| u_i \quad i = hrg, crg, \quad (7)$$

The friction factor in the above equation was obtained from the report of Tanaka et al.⁽⁹⁾ The flow velocities in Eqs. (6) and (7) are calculated at each

Table 1 Specification of case study machine

Working gas	Helium
Mean pressure	8 MPa
Gas temperature (T_h, T_m, T_c)	873, 333, 273
Displacer diameter (Hot/Cold)	75 mm
Design amplitude (Hot/Cold)	10 mm
Rod diameter	20 mm
Spring constant (Hot/Cold)	24000/2400 N/m
Mass (Hot/Cold/Casing)	1/0.5/20 kg
Solid friction factor at seals	0.1
Breadth of seal (Hot/Cold/Rod)	3/3/3 mm
Seal tension (Hot/Cold/Rod)	2/2/3 N
Stage of seals	2
Effective volume (S in eq.5)	$1.25 \times 10^{-6} \text{ m}^3/\text{K}$

time step, using mass flow rates through the boundaries at both sides of the control volume. The mass flow rate is estimated from the mass change in each control volume. The density in each control volume is also calculated from the equation of state at each time step.

3.3 Solid friction loss

Figure 3 shows a solid seal model⁽¹⁰⁾ and pressure conditions surrounding the seal. The same model used for our kinematic Vuilleumier machine was applied to the present analysis. A linear pressure distribution was assumed between contact surfaces. Considering an extra solid friction force by seal tension, the solid friction force by the seal is given by

$$\bar{F} = \pi \mu B D \{ 0.5(P_1 - P_2) + P_t \}, \quad (8)$$

$$P_t = 2F_t / BD, \quad (9)$$

where B is the breadth of the seal, D the diameter of contact surface, μ the coefficient of solid friction. The subscripts 1 and 2 indicate the high pressure side and the low pressure side, respectively, and t indicates the seal tension. Figure 3 shows the case of a single-stage seal, but in practical use, multistage seals are popular. In such cases, $(P_1 - P_2)$ in Eq. (8) should be replaced by the pressure difference between the pressures at the extreme ends of the seals, and P_t by NP_t . Here, N indicates the number of seal stages.

4. Numerical Analysis

Specifications of a Vuilleumier cycle heat pump proposed as a case study are given in Table 1. The Runge-Kutta-Gill method was used to integrate the differential equations of motion, Eqs. (1) to (3), coupled with the isothermal thermodynamic equation, Eq. (4), along with the equations for flow friction losses and solid friction force. Convergence characteristics and the accuracy of the present analysis were examined in detail for the case study machine. It was

determined that time steps less than 0.0001 s gave satisfactory converged results, and that the transient behavior caused by an initial displacement of the hot displacer piston, which was assumed arbitrarily at the start of the calculations, ended within 0.2 s. Subsequently, the calculations were conducted with a time step of 0.00005 s for 20 periods.

5. Comparison with Experiments

Experimental investigations of a free piston Vuilleumier cycle heat pump were carried out by Thomas⁽⁵⁾ and Schulz et al.⁽¹¹⁾ Dynamic specifications of their machine are listed in Table 2. In Table 3 we show a comparison of the present analysis with the experimental data and a linearized closed-form analysis by the same authors⁽¹¹⁾. In the present analysis, the effect of the solid friction is excluded because of the absence of seals in their experiment. It is apparent that the present nonlinear calculations can predict the experimental dynamic behavior better than the closed-form analysis.

6. Results and Discussions

Table 4 shows the calculated results for the

Table 2 Specifications of test machine

Working gas	Helium
Mean pressure	2 MPa
Gas temperature (T_h, T_m, T_c)	748, 318, 253
Displacer diameter (Both)	66 mm
Rod diameter	17 mm
Spring constant (Hot/Cold)	4603/375 N/m
Mass (Hot/Cold/Casing)	0.74/0.34/15 kg
Seals	No seal

Table 3 Comparison with experimental data

	Exp. data ⁽¹¹⁾	Linear analysis ⁽¹¹⁾	Present analysis
Frequency (Hz)	10.0	10.1	10.7
Phase diff. (deg.)	84.1	91.6	85.4
Amplitude-Hot (mm)	14.0	11.4	14.1
Amplitude-Cold (mm)	14.7	12.4	14.7
Amplitude ratio (C/H)	0.95	0.92	0.96

Table 4 Basic performance

Frequency at free running	19.7 Hz
Amplitude-Hot	6.96 mm
Amplitude-Cold	8.95 mm
Amplitude ratio (C/H)	1.29
Phase difference	78.7 deg.

machine with the specifications listed in Table 1. Once, a machine piston is excited, vibrational motion continues stably at a frequency of 19.7 Hz. The phase difference shown in Table 4, is defined as the difference between the phase angle of the minimum volume point of a hot space (which corresponds to the maximum displacement of a hot displacer piston) and the minimum volume point of a cold space (which corresponds to the minimum displacement of a cold displacer piston) in a unit period.

6.1 Effect of pressure variations

Figure 4 shows the relationship of frequency, amplitude and phase difference of the vibration system, and the effective volume S in Eq. (5). It is seen that the amplitude decreases and the frequency increases moderately with effective volume. As observed from the equation, the effective volume is an important factor to determine a pressure change. Since it becomes larger with decreasing effective volume, vibration systems with a small effective volume have low stiffness and large amplitude.

The effective volume is defined as the sum of the volume of each space divided by its temperature, and is a specific value for a machine. Temperature at each space is given as a design factor and it depends on the appliances of the machine, atmospheric temperature and the component materials used. The effective volume is largely a volume of the heat exchangers. Therefore, the data shown in Fig. 4 is basically useful in designing their geometries correlated to the vibrational characteristics of the system. The frequency values shown in Fig. 4 are about 20 Hz, which are significantly different from the natural frequencies (10.6 and 26.6 Hz) of the 2 degrees of freedom system without pressure change as given in Table 1. This means that the pressure changes play an important role in determining the operating frequency of such a system. The two natural frequencies mentioned above are obtained to solve the characteristic equation

Fig. 4 Effect of effective volume

Fig. 5 Force on displacer pistons

(13) of the following equations of motion without pressure change and solid friction.

$$m_h \frac{d^2 x_h}{dt^2} = K_h(x_e - x_h) + K_c(x_c - x_h), \quad (10)$$

$$m_c \frac{d^2 x_c}{dt^2} = -K_c(x_c - x_h), \quad (11)$$

$$m_e \frac{d^2 x_e}{dt^2} = -K_h(x_e - x_h), \quad (12)$$

$$\omega^6 - \left(\frac{K_h + K_c}{m_h} + \frac{K_c}{m_c} + \frac{K_h}{m_e} \right) \omega^4 + K_h K_c \left(\frac{1}{m_h m_c} + \frac{1}{m_h m_e} + \frac{1}{m_c m_e} \right) \omega^2 = 0. \quad (13)$$

Here, ω is the natural angular frequency.

Figure 5 shows the relationship between the forces acting on the pistons and their displacements at the operating frequency mentioned in Table 4. The forces are given by

$$F_h = K_h(x_e - x_h) + A_r(P_b - P_m) + K_c(x_c - x_h), \quad (14)$$

$$F_{sh} = K_h(x_e - x_h) + K_c(x_c - x_h), \quad (15)$$

$$F_c = -A_r(P_b - P_m) - K_c(x_c - x_h), \quad (16)$$

$$F_{sc} = -K_c(x_c - x_h). \quad (17)$$

The inclinations of the distributions of F_h and F_c , shown by open circles and squares, are different from those of F_{sh} and F_{sc} marked with solid circles and squares, respectively. An inclination of the forces with respect to the displacement gives an effective spring constant. It is obvious that the pressure change decreases the hot side spring constant and increases the cold side spring constant. This is the reason why the calculated operating frequencies are different from the natural frequencies. Pressure change acts as if it were a part of the spring action of the vibration system.

6.2 Effect of rod diameter

The dynamic behavior in each space are shown for rod diameters in Fig. 6. It is seen from Eqs. (1) to (3) that a rod plays an important role in maintaining the vibration because the vibrational exciting force depends on the cross-sectional area of the rod as well

Fig. 6 Effect of rod diameter

Fig. 7 Power and coefficient of performance

as the pressure difference between the medium and buffer spaces. For rods with diameters of under 16 mm such as 15 mm, 14 mm and so on, stable operation was not obtained, because such rods could not produce an exciting force sufficient to maintain a vibration. The phase difference becomes smaller for larger diameter rods. The small phase difference reduces the amplitude of pressure vibration and the pressure difference between the working space and the buffer space, and hence the exciting forces. This is the reason why the amplitudes tend to decrease and the frequency to increase sharply for over 28 mm in diameters.

Figure 7 shows the thermal power done by the hot, cold and medium spaces and the coefficient of performance in the cooling. They are defined as follows.

$$W_h = f \int_t^{t+\bar{T}} P_h A_h \left(\frac{dx_h}{dt} - \frac{dx_e}{dt} \right) dt, \quad (18)$$

$$W_c = f \int_t^{t+\bar{T}} P_c A_c \left(\frac{dx_c}{dt} - \frac{dx_e}{dt} \right) dt, \quad (19)$$

$$W_m = f \int_t^{t+\bar{T}} P_m A_m \left(\frac{dx_h}{dt} - \frac{dx_c}{dt} \right) dt, \quad (20)$$

Fig. 8 Energy balance

$$COP_c = W_c / W_h, \quad (21)$$

Here, $\bar{T}$ is the period. The each thermal power shows the same trend of amplitudes in Fig. 6 and the coefficient of performance decreases with rod diameter. Since the cooling power W_c and the heating power W_m and the cooling efficiency COP_c decrease for rods over 28 mm in diameter, they are not suitable for practical use.

Figure 8 shows the work done by the rod and the damping force, due to solid and flow frictions. They are defined as follows.

$$W_r = \int_t^{t+\bar{T}} (P_b - P_m) A_r \left(\frac{dx_h}{dt} - \frac{dx_c}{dt} \right) dt, \quad (22)$$

$$W_{fh} = \int_t^{t+\bar{T}} (P_m - P_h) A_h \left(\frac{dx_h}{dt} - \frac{dx_e}{dt} \right) dt, \quad (23)$$

$$W_{fc} = \int_t^{t+\bar{T}} (P_c - P_m) A_c \left(\frac{dx_c}{dt} - \frac{dx_e}{dt} \right) dt, \quad (24)$$

$$W_{kh} = \int_t^{t+\bar{T}} \bar{F}_h \left(\frac{dx_h}{dt} - \frac{dx_e}{dt} \right) dt, \quad (25)$$

$$W_{kc} = \int_t^{t+\bar{T}} \bar{F}_c \left(\frac{dx_c}{dt} - \frac{dx_e}{dt} \right) dt, \quad (26)$$

$$W_{kr} = \int_t^{t+\bar{T}} \bar{F}_r \left(\frac{dx_h}{dt} - \frac{dx_c}{dt} \right) dt. \quad (27)$$

The sum of the work is zero for a given rod diameter. This obviously means that the amplitude is so determined that the work done by the rod and the damping force of the friction balance each other.

6.3 Damping by flow friction

It is seen from Fig. 8 that the damping of the present system is mainly caused by the friction of the oscillating working gas flow in the hot and cold sides. Let us check the damping effect on the dynamic behavior by changing the hot side and cold side friction factors individually. The magnitude of damping is assessed using the following equivalent viscous damping coefficient

$$C_i = \frac{|W_i|}{\pi X_i^2 \omega} \quad (i = fh, fc), \quad (28)$$

where C is the equivalent viscous damping coefficient

Fig. 9 Effect of hot side damping due to flow friction

Fig. 10 Effect of cold side damping due to flow friction

and ω is the angular frequency.

The influence of the hot side damping on the dynamic behavior is plotted in Fig. 9. Flow friction factors of the cold side are assumed to be constant for the calculations. Equivalent viscous damping coefficients at the hot and cold sides for reference purposes are 92 Nm/s and 71 Nm/s, respectively. As the damping increases, the balancing point of the work moves to the point for a smaller amplitude. The amplitude of the hot displacer piston decreases when the damping coefficient is steeper than that of the cold one. This means that the hot side damping has a larger effect on the amplitude of the hot displacer piston than on the amplitude of the cold one.

Figure 10 shows the effect of the cold side damping on the dynamic behavior. In this case, the hot side flow friction factors are kept constant. The distribution of the parameters shown in Fig. 10 is opposite to that in Fig. 9, except for the amplitude. Namely, when the cold side damping increases, the frequency decreases and the phase difference increases, while the situation is inverse for the hot side damping. In both of the figures, it is noted that the effect of damping on the amplitudes is larger than the effects on the

Table 5 Effect of each seal

Calculation case	1	2	3	4	5
Seal	Hot dis. piston	○	○	×	×
	Cold dis. piston	○	×	○	×
	Rod	○	×	×	○
Frequency Hz	19.7	19.6	19.6	19.7	19.6
Amplitude (Hot) mm	6.96	7.62	7.78	7.38	7.9
Amplitude (Cold) mm	8.95	10.1	9.98	9.51	10.3
Amplitude ratio (C/H)	1.29	1.32	1.28	1.29	1.30
Phase difference deg.	78.7	75.9	74.9	78.9	75.5

frequency and phase difference.

6.4 Damping by solid friction

The solid friction due to the seals occurs at displacer piston seals and the rod seal. The influence of seals on the dynamic behavior is summarized in Table 5. The marking ○ in the column indicates which seal is taken into account for the calculations and the marking × indicates which one is not taken into account. The effect of all the seals is included in case 1, the effect of the hot side displacer piston seal only is included in case 2, the cold side displacer piston seal only in case 3, the rod seal only in case 4, and the effect of no seal in case 5.

Comparing case 5 with cases 1 to 4, the amplitudes are found to decrease when the damping effect of the solid friction is included. Comparing cases 2 and 4, it is seen that the hot side displacer piston seal increases the amplitude ratio, whereas the cold side displacer piston seal reduces it. The rod seal reduces the amplitudes more than the other seals do.

The influence of the displacer piston seal on the phase difference is small, but the hot side seal increases it and the cold side one decreases it. The phase difference in case 4 is close to that in case 1, which indicates that the rod seal has a comparatively large effect on the dynamic behavior in comparison with the other seals.

The frequency remains almost constant. The distribution of the discontinuous solid friction force with time is nearly a rectangular wave and is almost independent of the piston's displacement. Hence, the frequency does not change with the coefficient of solid friction, because if that happened, the stiffness of a system would vary and the frequency would not remain constant.

7. Conclusions

The nonlinear dynamic analysis of a free piston Vuilleumier cycle heat pump was performed by the time-stepping integration method for the proposed case study machine, and was partly compared with the experimental data and linearized analysis. The effects of effective volumes, friction due to an oscillat-

ing flow in the hot and cold side heat exchangers, solid friction at the seals and rod diameters on the dynamic behavior of the machine were discussed in detail. The main conclusions are drawn as follows:

(1) The effective volume has a large effect on the dynamic behavior due to pressure variation.

(2) The designing of heat exchanger geometries is highly correlated to the vibrational characteristics of a system.

(3) The pressure variation acts as a part of the spring action of the vibration system and therefore, the running frequency is significantly different from its natural frequencies.

(4) The rod plays an important role in maintaining the vibration, because the magnitude of exciting force depends on its cross-sectional area.

(5) Owing to the dissipative work done by the flow and solid friction, the amplitudes of both the displacer pistons are reduced. This can be explained on the basis of the energy balance between the work done by the rod and the dissipative force.

(6) The effect of the rod seal on the dynamic behavior is large as compared with that of the displacer piston seals.

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