Buoyant Convection in an Enclosure under Time-Dependent Boundary Conditions*

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An overview is made on time-dependent buoyant convection in an enclosure with time-periodic thermal or mechanical boundary conditions. The classical models for resonant convection are reviewed. Recent numerical efforts to determine the resonance frequency and the physical rationalizations are described. Extensions and practical utilizations of resonant convection are discussed. Resonant enhancements of transport phenomena in mixed convection are illustrated, and explanations are given for physical mechanisms.

Key Words: Natural Convection, Time-Dependent Conditions, Periodic Excitations, Lateral Heating, Resonant Convection, Resonance Frequency

1. Introduction

In most practical thermo-fluid systems, buoyancy-induced flows and associated heat transfers are inherently time-dependent. However, as pointed out by Kwak and Hyun⁽¹⁾, the great majority of the studies on natural convection in an enclosure have been for steady convection. The information acquired from these steady problems is useful for evaluating performance in design phase of a system. For optimal operation and control, prediction of the dynamic behavior is a crucial requirement. With the advances in experimental and computational capabilities, the research front has marched toward this important realm of unsteady phenomena.

Unsteady natural convection in an enclosure encompasses a broad spectrum of topical issues. Among others, a separate class of unsteady problem is thermal convection responding to time-varying external conditions. An obvious situation is due to the step change in thermal conditions. Applications can be found in startup and shutdown of thermal systems, and heat-up and cool-down of a contained fluid. The essential task is to describe the scenarios and relevant time scales for dominant evolution processes of the flow and temperature fields. Considerable research efforts have been given to examine this type of unsteady convection, which established the fundamental physical concepts pertinent to unsteady buoyant convection of a contained fluid. A general review on this subject is available⁽²⁾.

Another relevant time-dependent condition is the periodic variation of thermal boundary condition. Typical examples are solar heating varying on a daily basis, and periodic energizing of electronic devices by on-and-off heating (or cooling) modes. The question here is how the imposed periodicity affects the flow and attendant heat transfer. A significant aspect is the possibility of resonance; the external forcing with the

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correct natural frequency may give rise to a substantial amplification of eigenmodes of the system.

In 1990s, a series of studies⁽³⁾⁻⁽²¹⁾ explored this subfield of buoyant convection in an enclosure. A most remarkable achievement was that the presence of resonance was demonstrated in the context of confined buoyant convection. Subsequent efforts have elaborated the physical mechanism of resonance. There have also been serious attempts to search for practical utilities. It is an opportune juncture to scan the major accomplishments. In this paper, some highlights of these recent works are briefly touched on.

2. Basic Models of Resonant Natural Convection

A cavity with lateral heating is the canonical flow configuration for buoyant convection of a contained fluid. Consider two conceptual flow models with time-periodic lateral heating as illustrated in Fig. 1. A Boussinesq fluid is contained in a rectangular cavity of height H and width L . The fluid properties, e.g., conductivity k , thermometric expansion coefficient α , thermal diffusivity κ , and kinematic viscosity ν , are assumed to be constant except for density. The cavity is aligned to the gravity g . The horizontal walls are insulated. The one sidewall is kept at a constant temperature T_c . On the opposing sidewall, two types of time-periodic thermal conditions are considered.

In one flow configuration (hereinafter, referred to as model I), a periodic heat flux fluctuating in a square wave fashion, $q_H(t)$, is applied,

$$q_H(t) = q [1 + \varepsilon \text{sign}(\sin ft)]. \quad (1)$$

The other configuration (model II) is characterized by a complementary time-periodic thermal condition; the sidewall temperature varies sinusoidally with time about the mean value T_h ,

$$T_H(t) = T_h + \Delta T \varepsilon \sin ft, \quad (2)$$

Fig. 1 Flow configuration of a rectangular cavity with time-dependent lateral heating.

where ΔT is the mean temperature difference between the two sidewalls, i.e., $\Delta T = T_h - T_c$. ε represents the amplitude of the periodic heat flux (sidewall temperature oscillation) relative to the corresponding steady heat flux $q(\Delta T)$ for model I (model II).

The relevant parameters are the Rayleigh number, Ra ; the Prandtl number, $Pr = \kappa / \nu$, and the amplitude of periodic excitations ε , and the forcing frequency f . Due to the difference in thermal boundary conditions, Ra is defined, respectively: $Ra = \alpha g q H^4 / k \nu \kappa$ for model I and $Ra = \alpha g \Delta T H^3 / \nu \kappa$ for model II.

Here, it is advantageous to define two operators for assessing the impact of periodic heating

$$A(\phi) = [\max(\phi) - \min(\phi)] / \phi_s, \quad (3)$$

$$G(\phi) = (\phi_m - \phi_s) / \phi_s, \quad (4)$$

where ϕ_m is the time-averaged value of an oscillating property $\phi(t)$ over a cycle and ϕ_s denotes the corresponding value in the case of non-oscillating condition ($\varepsilon=0$). Consequently, $A(\phi)$ and $G(\phi)$ represent the fluctuating amplitude and the gain of the time-mean value of ϕ relative to ϕ_s , respectively.

Lage and Bejan⁽³⁾ (referred to as L-B), by using model I, was the first to demonstrate the presence of resonance of natural convection of an enclosed fluid for high values of Ra . The numerical computations encompassed a broad range of f for $\varepsilon=0.2$ and $H/L=1$. In an effort to analyze time-dependent heat transfer rate in the cavity, they monitored temporal behavior of the Nusselt number at the vertical center plane, Nu ,

$$Nu = k / q H \int_0^H (\partial T / \partial x - u T / \kappa)_{x=L/2} dy, \quad (5)$$

where u is the horizontal velocity and T is the temperature. The principal finding was that the fluctuations of Nu were substantially amplified at certain intermediate frequency. At this frequency, the fluctuation of heat transfer in the interior region was more vigorous than at the hot sidewall at which oscillatory heating was applied. Based on these findings, L-B defined resonance as the maximal amplification of fluctuations of Nu , which is closely related to the occurrence of maximum fluctuations in local velocity and temperature of the enclosed fluid.

Several authors⁽⁴⁾⁻⁽⁵⁾ examined unsteady thermal convection in model II, however, resonance was not the issue. The resonance in model II was described by a more recent work of Kwak and Hyun⁽⁶⁾ (referred to as K-B). K-H conducted extensive numerical calculations by varying f for several sets of (Ra, Pr) with $\varepsilon=0.1$ and $H/L=1$. The time-dependent behavior of heat transfer in the interior reconfirmed the existence of resonance. The representative results are displayed in Fig. 2 which exhibits the variation of $A(Nu)$ with f scaled by the

Fig. 2 Variation of $A(Nu)$ with f . $Pr=0.7$, $\varepsilon=0.1$.

Brunt-Väisälä frequency, $N=(\alpha g \Delta T/H)^{1/2}$. It is clear that the fluctuations of Nu are substantially amplified with a peak at the resonance frequency.

3. Physical Mechanism of Resonance

An intrinsic question is the physical mechanism of resonance, which calls for identification of the resonance frequency f_r . From a practical point of view, an accurate prediction of f_r is a premise for utilization and regulation of resonance phenomenon, since the occurrence of resonance is sensitive to the forcing frequency.

In an effort to identify f_r , L-B proposed the concept of rotation of a fluid wheel; resonance is thought to occur if the period of the oscillatory external heating coincides with the time required for its effects to travel through the full cavity. The period of one rotation of a fluid wheel was estimated as

$$P = 2(H+L)/V. \quad (6)$$

The interpretation is that the fluid rotates like a wheel of circumference $2(H+L)$ with velocity V . In the subsequent analysis, L-B adopted the longitudinal velocity in the vertical boundary layer for V . Antohe and Lage⁽⁷⁾ suggested a modified scaling, which incorporated the independent effect of the transverse velocity U in the horizontal boundary layers, i.e.,

$$P = 2(H/V + L/U). \quad (7)$$

These scaling arguments underscore the fact that the principal transport is done by convective currents mainly confined in the boundary layers. The predictions of f_r based on eqs. (6) and (7) showed qualitative agreement with the numerical results.

K-H searched for f_r on a different basis. Unsteady natural convection in a sidewall-heated cavity has been extensively studied during the past two decades⁽¹⁾⁻⁽²⁾. Several flow instability mechanisms have been reported to be responsible for oscillatory modes in the laterally-heated cavity: (i) the internal gravity wave mode prevailing in the interior, (ii) the

thermal boundary layer instability developing on the heating sidewalls, (iii) the instability related to intrusion flow in the horizontal boundary layers, and (iv) the Rayleigh-Benard type instability due to local gravitationally-unstable density profiles. The assertion of K-H was that resonance is associated with one of these eigenmodes.

In order to find the resonating mode, K-H analyzed the time-dependent flow and temperature fields. Resonance was characterized by a periodic tilting of the isotherms in the interior. The flow fluctuations were more vigorous in the interior than in the boundary layers. Another result was that f_r scaled by the Brunt-Väisälä frequency N was almost independent of Ra and Pr . All these results suggest that the flow resonates with the internal gravity waves in the stably-stratified interior. K-H estimated f_r based on the principal modes of internal gravity waves,

$$f_r/N = S/[(H/L) + (n_x/n_y)^2 (L/H)], \quad (8)$$

where S is the parameter indicating the overall strength of stratification, and n_x and n_y are the wavenumbers in the horizontal and vertical directions, respectively. S was evaluated by using the numerical solution for the corresponding case with $\varepsilon=0$. The predicted values of f_r were in quantitative agreement with the numerically-acquired values.

Here, it is worth noting that the physics of buoyant convection in model I is generally similar to that in model II. There exists an analogy between these two models. The difference is only the type of external forcing, and this does not affect the occurrence of resonance. This implies that f_r of these two analogous models may be predicted on a similar ground. Based on this notion, Kwak et al.⁽⁸⁾ attempted to estimate f_r of model I by employing the scheme of eq.(8), which gave an improved prediction. This exercise confirmed again that resonance is primarily associated with internal gravity wave oscillations.

The influences of the principal parameters, i.e., Ra , Pr , and ε , have been scrutinized. The numerical results of L-B and K-H illustrated that the resonant amplification becomes more vigorous as Ra increases. K-H and Antohe and Lage⁽⁹⁾ disclosed that resonance produces most drastic consequences for $Pr=O(1)$. The interpretation was that the two damping mechanisms for propagation of the effects of periodic external heating, i.e., thermal diffusion and viscous diffusion, are comparable in magnitude for $Pr \approx 1$. More recently, Antohe and Lage⁽⁷⁾ examined the effects of ε for model I; $A(Nu)$ is a linear function of ε , while f_r is nearly independent of ε . This argument was reconfirmed by Kwak et al.⁽¹⁰⁾ for model II.

4. Extensions and Utilizations

The basic analyses of L-B and K-H have been extended to more complex and realistic situations. Antohe and Lage⁽¹¹⁾ numerically investigated unsteady natural convection in model I filled with porous medium. Lee et al.⁽¹²⁾ dealt with a modified version of model II; the working fluid has density inversion characteristics like water near 4°C. All these studies demonstrated that the flow can resonate with internal gravity wave oscillations. Chung et al.⁽¹³⁾ adopted a realistic modification of model II; the perfectly-conducting vertical sidewall was replaced by a wall of finite thickness and finite conductivity. The finite-wall effects on occurrences of resonance and amplification characteristics were described. Bae et al.⁽¹⁴⁾ designed a flow configuration based on model II; a vertical baffle was placed in the cavity to screen the interior core without distorting the horizontal boundary layer flows. The aim was to suppress resonant convection by blocking internal gravity waves in the stratified interior core. This attempt turned out to be successful.

The foregoing discussions were concerned with time-dependent features of resonant convection. In view of practical utility, the time-mean heat transfer rate is of importance. A particular consideration is that the time-mean heat transfer rate may be enhanced by oscillatory heating. Ho and Chu⁽¹⁵⁾ exemplified this idea for the melting process in a sidewall-heated cavity; the sinusoidal temperature variation achieved a faster melting than the constant heating. Antohe and Lage⁽¹⁶⁾ conducted an experimental study for model I. The time-mean heat transfer rate was measured. The periodic heating gave about 20% enhancement of the time-mean heat transfer coefficient in comparison to that obtained by steady heating with the same time-mean heat flux. A similar trend was reported for model II. Kwak et al.⁽¹⁰⁾ reconsidered the problem of K-H for the cases of large values of ε . The key finding is illustrated in Fig. 3. It is noted that $G(Nu)$ represents the gain of the time-mean value of Nu relative to the corresponding value for $\varepsilon=0$. The time-mean heat transfer is augmented measurably. An important notion is that the maximum gain is found at the resonance frequency at which a maximal fluctuation of Nu occurs. For $\varepsilon=1$, the periodic thermal excitation gives about 13% increase of the time-mean Nusselt number.

The above descriptions suggest potentially useful applications; the imposition of time-periodic thermal conditions with a proper frequency brings forth the resonant convection resulting in an enhancement of the time-mean heat transfer. A requirement in this attempt is an easy and precise control of the external forcing.

Fig. 3 Variation of $G(Nu)$ scaled by ε with f . $Ra=10^7$, $Pr=0.7$, $H/L=1$.

However, it is difficult to control the time-varying thermal condition in real thermo-fluid systems. A physical contention is that the occurrence of resonance depends on the forcing frequency, not on the type of external forcing. The implication is that the thermal forcing can be replaced by a relatively simple way of time-periodic mechanical forcing.

The resonant convection of an enclosed fluid has also been discussed in the context of mixed convection⁽¹⁷⁾⁻⁽²⁰⁾. Among others, Fu and Shieh⁽¹⁸⁾ examined the periodic convection in a sidewall-heated cavity vibrating vertically in a sinusoidal manner. The numerical results disclosed the existence of a resonant convection regime in which flow and heat transfer strongly interact with the external vibration. The mechanical vibration led to a substantial enhancement of heat transfer at the resonance frequency.

The remaining issue is the qualitative and quantitative analogy between thermal forcing and mechanical forcing. Kim et al.⁽²⁰⁾ investigated the unsteady convection under three types of external forcing; time-periodic sidewall temperature (model II), a horizontal vibration, and a vertical vibration of the system. It is noted that Fu and Shieh⁽¹⁸⁾ simulated the vibration by using a form of displacement, $b \sin ft$. The influence of vibration was modeled by gravity modulation, $g^*(t)$,

$$g^*(t)/g = b f^2 \sin ft. \quad (9)$$

Consequently, the effect of f was not decoupled from the effect of the forcing amplitude. To resolve this defect, Kim et al. adopted a modified formulation, i.e.,

$$g^*(t)/g = \varepsilon \sin ft, \quad (10)$$

where ε represents the amplitude of external forcing as in models I and II. The results are summarized in Fig. 4 showing $A(Nu)/\varepsilon$ multiplied by a factor β vs. f . It is visible that resonance takes place at the principal mode of internal gravity wave oscillations for all cases. A

Fig. 4 Variation of $A(Nu)/\varepsilon$ with f . $Ra=10^7$, $Pr=0.7$, $H/L=1$. The values of the multiplication factor β are given in the figure. ■, horizontal vibration; ●, vertical vibration; ▲, model II.

more important observation is that the curves for three types of external forcing are almost identical near the resonance frequency. Recalling that the $A(Nu)$ is a linear function of ε , this points to the quantitative proportionalities among three types of external forcing. Namely, the sidewall temperature oscillations can be modeled by the horizontal or vertical vibrations with appropriate amplitudes.

A more straightforward means of time-periodic external conditions is acoustic pulsation. Recently, Kim et al.⁽²¹⁾ designed a resonant cooling device using a speaker system which generates acoustic pressure waves. They applied this device for cooling of a Pentium CPU. The experimental results demonstrated that, when the external acoustic forcing was tuned with a specific natural frequency, the surface temperature of CPU was drastically reduced without a convectational cooling fan.

5. Concluding Remarks

The pioneering treatise of L-B demonstrated the presence of resonance in the context of natural convection, which opened a new subfield of resonant buoyant convection of an enclosed fluid. A series of subsequent studies have elucidated the physical mechanism. Parallel efforts have been given to searching for the practical utilities of resonance natural convection phenomena.

There are many unresolved technical issues. A relevant issue is the resonance interacting with other eigenmodes of oscillation found in a laterally-heated enclosure. Control and/or suppression of occurrence of resonance should be considered. A conceptual extension of resonance may be using spatially-periodic conditions to match the wavelength of the eigenmodes. Another consideration is that the major studies of resonant buoyant convection have been mostly

numerical. Experimental efforts should be strengthened.

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