Gas Flow Characteristics in Microtubes*

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Microchannel is one of the essential components that construct various micro systems. However, it has been reported that the flow and heat transfer behavior in microchannel deviates from predictions based on the conventional assumptions generally accepted in macro scale. In this study, frictional characteristics of nitrogen (N_2), argon (Ar) and helium (He) flowing through microtubes whose diameter ranges from 5 to 100 μm have been investigated experimentally. Inlet/outlet pressure difference and volumetric flow rate were measured. In the range of Reynolds number ($Re=0.03\sim29.7$) tested in this study, the measured friction constant was observed to take the values around 50, which is about 20% less than 64, the value regarded to be correct for macro scale tube predicted by the incompressible flow assumption. Transition from incompressible to compressible flow regimes was observed experimentally. The onset of compressibility effect was dependent on the inlet/outlet pressure difference (or the pressure ratio) as well as on the Mach number. The frictional resistance of nitrogen flow showed a Knudsen number dependence, which is in rough agreement with the first-order slip model.

Key Words: Microtube, Gas Flow, Frictional Resistance, Compressibility, Rarefaction Effect

1. Introduction

During the last three decades, striking development in microelectromechanical systems (MEMS) technology has made it feasible to fabricate complex micro devices⁽¹⁾. Recently, micron size mechanical devices become more prevailing in commercial products as well as in scientific research because they can offer advantages such as reduced size and fast response time of the system⁽²⁾.

Microchannel is one of the essential components that construct any micro systems. For example, the advent of systems employing very-large-scale integrated (VLSI) circuits requires effective and compact heat removal. Advances in micromachining technology allowed the heat exchangers with multiple micro flow channels to be fabricated. Applications of such micro heat exchangers can be found in the integrated cooling of IC chips^{(3),(4)} and the Joule-Thomson refrigerators for infrared detectors and diode

lasers^{(5),(6)}.

On the other hand, in order to design such micro devices properly, it is necessary to establish the physical laws governing the fluid flow and heat transfer in microgeometry. However, it has been reported that phenomena in microgeometry may differ from those in macroscopic counterparts⁽⁵⁾⁻⁽¹²⁾. Several factors that might be dominant in micro scale have been identified through a number of experimental, analytical and numerical works. Among them, non-continuum effect, compressibility effect and various surface effects have been under vigorous investigation.

2. Brief Review of Previous Studies

Table 1 summarizes previous experimental works on gas flow in microchannels. In Wu & Little⁽⁵⁾, friction factors of nitrogen flow were measured for both laminar and turbulent regions in miniaturized channels etched in silicon and glass. The values of friction factor they measured were much larger (e.g., 10 to 30% in silicon channels and 3 to 5 times in glass channels) than those predicted by the correlations for a smooth circular tube. They attributed their anomalous results to the large relative (and asymmetric) roughness of test channels.

Choi *et al.*⁽⁷⁾ measured friction factors of nitrogen flow in microtubes of diameter ranging from 3 to 81 μm . The measured friction factors for laminar and turbulent flows were found to be consistently smaller

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Table 1. Experimental works on gas flow in microchannels

Researchers	Channel (Material) D_h [μm]	Working fluid	Frictional resistance	Remarks
Wu & Little ⁽⁶⁾	Trapezoidal (Si, Glass) 45.46~83.08	N ₂ , H ₂ , Ar	Increase	Large relative roughness, $\epsilon=0.2\sim0.3$ For glass channels with heat treatment $f=(110\pm8)/\text{Re}$ ($\text{Re}<900$) $f=0.165(3.48\text{-}\log\text{Re})^{2.44}+(0.081\pm0.007)$ ($900<\text{Re}<3000$) $f=(0.195\pm0.017)/\text{Re}^{0.11}$ ($3000<\text{Re}<15000$)
Choi <i>et al.</i> ⁽⁷⁾	Circular (Silica) 3, 7, 10, 53, 81	N ₂	Decrease	19~27% lower for laminar flow, $f=53/\text{Re}$ ($\text{Re}<2300$) 10~30% lower for turbulent flow, $f=0.140\text{Re}^{-0.182}$ for 100 μm tube ($4000<\text{Re}<18000$)
Yu <i>et al.</i> ⁽⁸⁾	Circular (Silica) 19, 52, 102	N ₂ (& Water)	Decrease	19% lower for laminar flow, $f=50.13/\text{Re}$ ($\text{Re}<2000$) 5% lower for turbulent flow, $f=0.302\text{Re}^{-0.25}$ ($6000<\text{Re}<20000$)
Pfahler <i>et al.</i> ⁽⁹⁾	Rectangular (Si) 0.96~8.66 Trapezoidal (Si) 8.47~39.7	N ₂ , He	Decrease	For larger channels, slightly decreasing trend with decreasing Re for helium flow and about 15% lower for nitrogen flow For 5 μm deep channel, consistently decreasing trend with decreasing Re for both helium and nitrogen flows
Harley <i>et al.</i> ⁽¹⁰⁾	Trapezoidal (Si) 1.01~35.91	N ₂ , He, Ar	Decrease	Good agreement with the first-order single coefficient slip model
Arkilic <i>et al.</i> ⁽¹¹⁾⁻⁽¹²⁾	Rectangular (Si) 2.59	He	Decrease	Good agreement with the zeroth-order perturbation solution subject to slip velocity boundary condition

than those predicted by the correlations in macro tube. Utilizing the same experimental apparatus, nitrogen and water flows in microtubes have been investigated in Yu *et al.*⁽⁸⁾. They could obtain friction factors falling into approximately the same range as in the previous research⁽⁷⁾. However, like in Choi *et al.*⁽⁷⁾, they didn't attribute this reduction in the friction constant to the slip effect.

In Pfahler *et al.*⁽⁹⁾, flow resistance of nitrogen and helium flows through silicon microchannels was measured to be consistently smaller than that predicted by the conventional incompressible theory. On the other hand, Harley *et al.*⁽¹⁰⁾ measured friction factors of gas flows in silicon channels ranging in hydraulic diameters from 1.01 to 35.91 μm , under the Knudsen numbers less than 0.38. The measured friction constant was in rough agreement with theoretical predictions based on the isothermal, 'locally fully developed,' first-order slip flow model.

In Arkilic *et al.*⁽¹¹⁾⁻⁽¹²⁾, helium flow through microchannel of 1.33 μm deep, 52.25 μm wide and 7500 μm long was investigated under pressure ratios of 1.6 to 4.2, and the relationship between mass flow rate and pressure drop was obtained. Through a perturbation expansion, it was also shown that the zeroth-order perturbation solution for the streamwise mass flow corresponds well with the experimental results⁽¹²⁾.

3. Experimental Apparatus and Procedure

The experimental apparatus used in this study is shown in Fig. 1. All the experiments were carried out in a clean room, where ambient temperature is under the careful control. The experimental apparatus consists of a pressurized gas cylinder, two (coarse and fine) pressure regulators, a filter, a microtube test section, and a flow meter. Inlet pressure was controlled by two pressure regulators (Yutaka Co.) upstream to the test section. The test gas is supplied from a pressurized gas cylinder and flows through the coarse

pressure regulator. The pressure of the test gas is reduced down to a proper pressure level (~100 kPa) before going to the fine pressure regulator, where the flow rate is controlled precisely. To avoid clogging, gas was then made to pass through a filter (GL Science Co., FI-061), which can keep number density of the particles larger than 0.01 μm under $1/\text{cm}^3$. Commercially available capillary tubes for gas chromatography (GL Science Co.), made of quartz glass, were used as test microtubes. After leaving the test section, the gas passes through a volume flow rate measuring section. A closer view of microtube test section is depicted in Fig. 2. Epoxy adhesive was used to prevent leakage from the connection parts.

To measure a volumetric flow rate, a pipette was

Figure 1. Schematic description of the experimental apparatus for a microtube friction test.

Figure 2. Close view of microtube test section.

connected to the exit of test section. Cross-section of the pipette was chosen to be sufficiently large compared to that of the microtube, according to the range of volumetric flow rate to be measured. A liquid drop was inserted to form a meniscus inside the pipette. The meniscus moves with a speed proportional to the volumetric flow rate of gas coming out of the test section. The moving speed of meniscus was measured using a stopwatch and a digital camera (Epson Co., CP-500). The recorded images were processed in a PC to determine the volumetric flow rate. The volumetric flow rate was obtained by averaging the readings from the images that were captured repeatedly after confirming that the steady state was reached. Although there might exist a pressure difference on either side of the moving meniscus [$O(1 \text{ Pa})$], it is negligible compared to the total pressure drop along the microtube test section [$O(1 \text{ to } 100 \text{ kPa})$].

Pressure difference between inlet and outlet of the test section was measured using a differential pressure transducer (Validyne Co., DP-15TL). Pressure losses in the feeding line other than in the microtube can be neglected due to much larger cross-section of the line and thus due to negligibly small flow velocity there, compared to those in the microtube. In this study, therefore, the measured value from the differential pressure transducer was taken as the pressure difference along the microtube. Several diaphragms were used to cover a relatively wide range of pressure drop tested in this study. Calibration of the pressure transducer was carried out with a water manometer for the pressure up to 14 kPa (140 cmH₂O) and with a weight-type standard pressure gauge for higher pressure. Accuracy of the pressure transducer, given by the supplier, is 0.5% of a full scale.

Nitrogen, argon and helium (Sumitomo Co.) were used as working fluids (Table 2). Helium has quite different physical properties (e.g., much smaller density) from nitrogen and argon. In addition, nitrogen is a diatomic molecule, while argon and helium are monatomic molecules. Experiments were performed on four tubes of different diameter. Table 3 shows the ranges of dimensionless numbers; Knudsen number (Kn), Reynolds number (Re) and Mach number (Ma) in the experimental conditions considered in this study.

4. Data Reduction

If the flow is assumed to be incompressible, friction factor, f_{incomp} , is given as a function of inlet/outlet pressure difference, ΔP , in the following form⁽¹³⁾,

$$f_{incomp} = \frac{2\Delta P D}{\rho U^2 L}, \quad (1)$$

where U represents the cross-sectionally averaged velocity and ρ is the fluid density. D and L denote diameter and length of the microtube, respectively. Equation (1) has been used as a working equation for the experiments with liquid and sometimes gas as working fluid.

On the other hand, if the flow is assumed to be

isothermal compressible, the friction factor, f_{comp} , is given as⁽¹⁴⁾

$$f_{comp} \frac{L}{D} = \left(f \frac{L_{max}}{D} \right)_1 - \left(f \frac{L_{max}}{D} \right)_2, \quad (2)$$

$$f \frac{L_{max}}{D} = \frac{1 - \gamma Ma^2}{\gamma Ma^2} + \ln \gamma Ma^2, \quad (3)$$

where subscripts 1 and 2 refer to the inlet and outlet conditions, respectively, and γ is the specific heat ratio.

In the case of an *adiabatic* process, the counterpart of Eq. (3) is expressed as

$$f \frac{L_{max}}{D} = \frac{1 - Ma^2}{\gamma Ma^2} + \frac{\gamma + 1}{2\gamma} \ln \frac{(\gamma + 1) Ma^2}{2 + (\gamma - 1) Ma^2}. \quad (4)$$

Equations (2), (3) and (4) have been used as working equations for gas experiments (e.g., Pfahler *et al.*⁽⁹⁾; Choi *et al.*⁽⁷⁾).

When the flow is assumed to be incompressible, but to *slip* at the solid walls⁽¹¹⁾, derivation starts from the streamwise momentum equation for a fully developed, laminar, incompressible flow

$$0 = -\frac{dP}{dx} + \mu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right). \quad (5)$$

The first-order velocity slip boundary conditions are given as⁽¹⁵⁾

$$\frac{\partial u}{\partial r} = 0 \text{ at } r=0 \text{ and } u = -\sigma \lambda \frac{\partial u}{\partial r} \bigg|_{r=D/2} \text{ at } r=D/2, \quad (6)$$

where λ is the molecular mean free path. Here, $\sigma = (2 - \sigma_v)/\sigma_v$ and $0 < \sigma_v \leq 1$. σ_v denotes the tangential momentum accommodation coefficient implying the fraction of the molecule's tangential momentum lost through collisions with the solid surface⁽¹⁰⁾.

The friction factor based on the first-order slip model is obtained as⁽¹⁶⁾

$$f_{slip} = \frac{64}{Re} \frac{1}{1 + 8\sigma Kn}. \quad (7)$$

Table 2. Physical properties at 295 K and 1 atm. of the gases used in flow experiments.

Working fluid	Density, ρ [kg/m ³]	Viscosity, μ [μ Pa·s]	Molecular weight, M [g/mole]
Nitrogen	1.1580	17.755	28.013
Argon	1.6505	22.430	39.948
Helium	0.1654	19.610	4.003

Table 3. Range of the values of nondimensional groups in flow experiments using four different diameter microtubes.

Inner diameter, D [μ m]	Length, L [mm]	Reynolds number, Re	Average Knudsen number, Kn	Average Mach number, Ma
5	14.8, 30.5	0.041~3.508	0.00302~0.01555	0.00047~0.03073
15	50.2, 100	0.074~29.74	0.00100~0.00842	0.00026~0.08685
50	100	0.030~3.183	0.00120~0.00378	0.00005~0.00279
100	100, 200	0.072~18.25	0.00061~0.00190	0.00008~0.00799

Figure 3. Darcy friction factor based on the *isothermal* compressible assumption, f_{comp} , of nitrogen, argon and helium flows in four microtubes as a function of Reynolds number, Re . Numbers in the above legend denote the inner diameter of microtubes in micrometer.

where Kn is the Knudsen number defined as $Kn = \lambda/D$. In this study, complete momentum accommodation ($\sigma=1$) was assumed at the solid wall¹.

5. Results and Discussion

Comparison of two friction factors reduced from Eq. (1), and Eqs. (2) and (3), respectively, indicated that the compressible assumption is more appropriate than the incompressible one for data reduction in gas flow experiments. In addition, there was no significant difference in two friction factors depending on the assumption of isothermal or adiabatic flow. The Darcy friction factor based on the *isothermal* compressible assumption (Eqs. (2) and (3)), f_{comp} , is depicted in Fig. 3 as a function of Reynolds number. In Fig. 3, almost all the experimental data obtained with different gas species in different diameter tubes fall below the line of theoretical prediction, $f=64/Re$, for a conventional smooth tube. This indicates that the frictional resistance in microtubes becomes smaller than that in conventional sized tubes.

The product of friction factor (f) and Reynolds number (Re) is often referred to as friction constant (C). From a simple analysis of the fully developed, laminar,

¹ For almost all calculations (e.g., Arkilic *et al.*⁽¹¹⁾⁻⁽¹²⁾; Harley *et al.*⁽¹⁰⁾; Piekos & Breuer⁽¹⁷⁾; Beskok & Karniadakis⁽¹⁸⁾), complete momentum accommodation at the solid wall ($\sigma=1$) has been assumed, although values other than unity have been reported by some researchers. Albertoni *et al.*⁽¹⁹⁾ evaluated the slip coefficient as $\sigma=1.1466$ from a solution of the linearized Boltzmann equation. From an experimental observation, Sreekanth⁽¹⁶⁾ proposed the possibilities that the first-order slip coefficient (σ) may be a function of Kn rather than a constant simply related to specular reflection alone, and that its value changes from 1.0 to 1.1466 as Kn increases. In silicon microchannels, Arkilic *et al.*⁽²⁰⁾ measured σ_r to be 0.80 for argon and 0.88 for nitrogen, respectively. Based on the simplified molecular kinetic theory, Li *et al.*⁽²¹⁾ derived the friction factor for microtubes as $f=64/Re(1+4.018Kn)$, which corresponds $\sigma \approx 0.5$.

(a) Nitrogen

(b) Argon and Helium

Figure 4. Normalized friction constant, C^* , of gas flows as a function of Reynolds number, Re . (a) Nitrogen and (b) Argon and Helium.

incompressible Navier-Stokes equations, the theoretical value of C for a conventional smooth tube is reduced to $64^{(13)}$. Figures 4(a) and (b) show normalized friction constants of nitrogen, argon and helium, respectively, as functions of Reynolds number. The normalized friction constant, C^* , is obtained by dividing the measured friction constant by 64. It can be clearly seen from the figures that friction constants were reduced significantly in microtubes. The amount of decrease in friction constant was larger for nitrogen flow than for argon and helium flows. Friction constant of nitrogen flow decreases as the tube diameter decreases as seen in Fig. 4(a) (also in Fig. 5). The friction constants of nitrogen flow in microtubes of diameter less than $50 \mu m$ fall around the value of 50, which corresponds to about 80% of the theoretical value, 64 for conventional macro tubes (see Table 4). The results from $100 \mu m$ diameter tube even showed a slightly decreasing trend of friction constant with increasing Reynolds number, while those from the other tubes were kept nearly constant. The trend of decreasing friction constant with increasing Reynolds number could be also observed for helium flow in four different diameter tubes (even though we don't have sufficiently large number of data in this case) as depicted in Fig. 4(b). This is in striking contrast to the results of helium and nitrogen flow experiments in $0.51 \mu m$ deep microchannel of Pfahler *et al.*⁽⁹⁾, wherein

Table 4. Averaged friction constants of three gas flows in four different diameter microtubes, together with standard deviations. The parenthesized figures indicate the number of data sets. Each data set was obtained by averaging more than 10 readings in one experimental run.

Tube diameter [μm]	Nitrogen	Argon	Helium
5	50.142 $\pm$ 3.080 (22)	60.567 $\pm$ 3.065 (8)	60.559 $\pm$ 1.935 (4)
15	49.028 $\pm$ 1.470 (26)	56.286 $\pm$ 3.056 (6)	59.232 $\pm$ 2.446 (2)
50	52.919 $\pm$ 1.856 (23)	52.217 $\pm$ 1.016 (8)	54.566 $\pm$ 3.692 (3)
100	59.285 $\pm$ 3.289 (38)	61.116 $\pm$ 3.272 (6)	63.165 $\pm$ 3.490 (4)

the increasing friction constant has been reported with increasing Reynolds number.

Table 4 summarizes the average friction constants of three gas flows in four different diameter microtubes, together with their standard deviations. The same data are plotted in **Fig. 5**, which shows the averaged normalized friction constant of each gas flow as a function of tube diameter. A decreasing trend with decreasing tube diameter is seen clearly for nitrogen flow, while the relationship between friction constant and tube diameter is vague for the cases of argon and helium flows.

The ratio of incompressible (Eq. (1)) to compressible (Eqs. (2) and (3)) friction constants, C_{incomp}/C_{comp} , is depicted as a function of inlet/outlet pressure difference, ΔP , in **Fig. 6**. It is clearly seen that the flow inside microtubes starts to exhibit compressibility effect, identified with a transition from incompressible to compressible flow regimes. All the data from different gas flows in different diameter tubes fall on the same curve, indicating that the transition to compressible flow occurs at an inlet/outlet pressure difference of around 10 kPa, or at a pressure ratio (P_{in}/P_{out}) of around 1.1. Data points for $\Delta P < 10$ kPa in **Fig. 6** correspond to those from 50 and 100 μm diameter tubes. However, we couldn't obtain such a beautiful curve when we plotted the same data against the average Mach number. This may imply that the compressibility effect in microtubes cannot be described by only the Mach number, which is in contrast to the practice in the conventional scale.

Figures 7(a) and (b) depict normalized friction constants of nitrogen, argon and helium, respectively, as functions of average Knudsen number. The theoretical variation of friction constant with the Knudsen number, given by the first-order slip model (Eq. (7)), is shown together for comparison. It is seen from **Fig. 7(a)** that the friction constant of nitrogen flow decreases as the Knudsen number increases, especially evident in the data obtained with 5 μm diameter tube. However, the absolute amount of decrease in friction constant is not in agreement with the first-order slip model, implying a possible underestimation in the reduction of friction constant by the slip model. Argon and helium flows, on the other hand, showed the scattered distribution in **Fig. 7(b)**, making it difficult to find a consistent relation

Figure 5. Averaged normalized friction constant, C^*_{avg} , of gas flows as a function of tube diameter, D .

between the variation of friction constant and the Knudsen number.

A possible explanation to the reduced frictional resistance observed in microtubes would be condensed into two effects from current experimental investigation. First, the frictional resistance in microtubes was affected by compressibility effect, which was identified by the fact that the friction constant starts to deviate from the prediction by the incompressible theory even at very low pressure ratio (e.g., $P_{in}/P_{out} \sim 1.1$ in this study). Secondly, the flow in microtubes exhibited the rarefied phenomena due to extremely small dimension of flow passages. It has been identified experimentally that as the tube diameter decreases (or as the Knudsen number increases), the flow behavior in microtubes deviates much more from that based on the conventional continuum assumption. Existence of significant rarefaction effect would be a plausible scenario for the reduced frictional resistance in microtubes. Therefore, slip boundary condition should be introduced at the wall, where no-slip boundary conditions have been usually assumed in the conventional analysis. However, Reynolds number and Knudsen number dependences of the friction constant obtained from nitrogen, argon and helium flow experiments are not quite consistent. More experimental investigation is needed in order to answer that question.

6. Concluding Remarks

The frictional resistance of gas flow in microtubes of diameter ranging 5 to 100 μm was observed to be smaller than that in the conventional sized tubes. The reduced frictional resistance in microtubes seems to be caused by the *compressibility* effect due to a significant pressure drop along relatively short flow passages and the *rarefaction* effect due to extremely small dimensions of flow passages. Further investigation is needed to clarify the thermofluidic phenomena in microgeometry.

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Figure 6. The ratio of incompressible to compressible friction constants, C_{incomp}/C_{comp} , of gas flows as a function of inlet/outlet pressure difference, ΔP . Gas densities in Table 2 were assumed in the evaluation of C_{incomp} .

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(a) Nitrogen

(b) Argon and Helium

Figure 7. Normalized friction constant, C^* , of gas flows as a function of average Knudsen number, Kn . The first-order slip model (Eq. (7)) is shown together for comparison. (a) Nitrogen and (b) Argon and Helium.