Measurement of Transmission Error Including Backlash in Angle Transmission Mechanisms for Mechatronic Systems*

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The characteristics of angle transmission mechanisms exert a great influence on the servo performance in the robotic or mechatronic mechanism. Especially, the backlash of angle transmission mechanism is preferable the small amount. Recently, some new types of gear reducers with non-backlash have been developed for robots. However, the measurement and evaluation method of the backlash of gear trains has not been considered except old methods which can statically measure at only several meshing points of gears. This paper proposes an overall performance testing method of angle transmission mechanisms for the mechatronic systems. This method can measure the angle transmission error both clockwise and counterclockwise. In addition the backlash can be continuously measured in all meshing positions automatically. This system has been applied to the testing process in the production line of gear reducers for robots, and it has been effective for reducing the backlash of the gear trains.

Key Words: Gear, Servomechanism, Measurement, Mechatronic System, Angle Transmission Mechanism, Transmission Error, Backlash

1. Introduction

Accuracy of angle transmission is one of the important characteristics of angle transmission mechanism. In the case of gear reducers, this accuracy is evaluated by single flank gear meshing error. The meshing error will be also called as transmission error in this paper. Several types of single flank gear meshing test systems have been developed for measuring the error⁽¹⁾⁻⁽⁶⁾. By these systems, only meshing error of the rotation in one direction can be measured. The meshing error of the rotation in another direction must be measured separately. Besides, the measurement and evaluation method of the backlash of gear trains has not been considered except old methods for static measurements at only several meshing points of gears.

On the other hand, angle transmission mecha-

nisms used in mechatronic systems such as robot usually rotate in both directions (clockwise and counterclockwise) and accelerate/decelerate frequently. The characteristics of angle transmission mechanisms, especially the transmission error and backlash exert a great influence on the servo performance in the mechatronic system. For this reason, it is necessary to develop a new measurement system to evaluate the transmission characteristics of angle transmission mechanism in mechatronic applications. As a measurement system for mechatronic application, it should be available in various dynamic conditions both for measurement of transmission error including backlash and measurement of motion parameters in time domain simultaneously, which is important for analysis of dynamic behavior due to the transmission error. With these considerations, a composite single flank gear meshing test system for measurement of transmission error including backlash in gear reducers has been developed by authors⁽⁷⁾ and it has been also applied to the inspecting process of reducers in the production line⁽⁸⁾.

In this paper, the principle of measurement and the construction of the developed measuring system

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for angle transmission mechanisms are described. Examples of measured results on one pair of spur gear, a parallel axis gear reducer, a planetary gear reducer and a simple mechanism with backlash performed by the system are shown.

2. Measurement System for Transmission Error Including Backlash

2.1 Principle of measurement

Consider a simple model of angle transmission mechanism shown in Fig. 1. If the angular positions of both input axis and output axis of angle transmission mechanism at the starting moment of measurement are set to zero, transmission error of angle transmission mechanism can be defined by Eq. (1).

$$\varepsilon(\theta_2) = \theta_2 - \frac{1}{k} \theta_1 \quad (1)$$

where

θ_1 : Measured angular position of input axis of angle transmission mechanism

θ_2 : Measured angular position of output axis of angle transmission mechanism

k : Reduction ratio (=input angle/output angle) of angle transmission mechanism

Here, the history that the gear rotates to the angular position now is not considered. However, the transmission error of angle transmission mechanism depends on the rotation directions. To make difference between the values of different rotation directions, we use "+" for clockwise rotation and "-" for counterclockwise rotation. Then, the transmission errors in two rotation directions can be written as Eqs. (2) and (3).

$$\varepsilon^+(\theta_2) = \theta_2^+ - \frac{1}{k} \theta_1^+ \quad (2)$$

$$\varepsilon^-(\theta_2) = \theta_2^- - \frac{1}{k} \theta_1^- \quad (3)$$

By subtracting Eq. (3) from Eq. (2), Eq. (4) can be obtained.

$$\varepsilon^+(\theta_2) - \varepsilon^-(\theta_2) = \theta_2^+ - \theta_2^- - \frac{1}{k} (\theta_1^+ - \theta_1^-) \quad (4)$$

Fig. 1 A model of angle transmission mechanism

By measuring θ_2^+ , θ_2^- at the condition of $\theta_1^+ = \theta_1^-$ in measurement, following equation can be derived.

$$\varepsilon^+(\theta_2) - \varepsilon^-(\theta_2) = \theta_2^+ - \theta_2^- \quad (5)$$

It is obvious that the right side of Eq. (5) represents the backlash of angle transmission mechanism. Therefore, the deviation between transmission error in clockwise rotation and that in counterclockwise rotation at any angular position is just the backlash of angle transmission mechanism at the angular position.

2.2 Configuration of developed measurement systems

Figure 2 shows the basic configuration of prototype of composite single flank meshing test system for gear reducers developed by authors. As shown in Fig. 2, two rotary encoders are attached on the input axis and output axis of gear reducer to measure the angular positions θ_1 and θ_2 . The measured angular positions are inputted to computer, and transmission errors are calculated by Eqs. (2), (3) and (5). The input axis is driven by servo motor and its angular position is measured by an encoder (1 500 P/R) built in the motor. The angular position of output axis is measured by an encoder (5 000 P/R), and a resolution of 50 000 P/R (=25.92"/P) is realized by using interpolating circuit of 10 times. Pulse from each encoder is counted by up/down counter so that the value of counter always shows the angular position from the starting point, independent of rotation direction. The values of counter are read by computer at every count of the counter of the input axis, that is, at a sampling angle equal to the resolution (14.4') of the input axis. The transmission error is calculated and displayed on CRT by computer in real time. The rotation direction of input axis can be changed at arbitrary pre-assigned angular position. After the measurement is completed, the result of transmission errors including backlash is outputted to a plotter.

Figure 3 shows the detailed construction of the system being used for inspecting gear reducers in production line. As shown in Fig. 3, the reducer is

Fig. 2 Composite single flank gear meshing test system

Fig. 3 Detailed construction near the setup of reducer

Fig. 4 Photograph of developed measurement system

supported by two vertical centers. A prony brake or a fly wheel can be attached to the output axis of reducer to provide loads to the system. The motion of servo motor, that is, the motion of input axis, can be controlled to realize an arbitrary velocity pattern by computer. For this reason, it is also possible to measure the dynamic behavior of reducers by the system.

As an improvement of the system shown in Fig. 3, a general measurement system of transmission error in angle transmission mechanism has been developed by authors (Fig. 4). Figure 5 shows the configuration of the system. It consists of input unit, output unit, and angle transmission mechanism for test between the two units. In input unit, the spindle is driven by AC servo motor and can be controlled to implement an arbitrary motion by computer. Two rotary encoders (6 000 P/R) are used for measuring the angular positions of input axis and output axis of angle trans-

(a) Construction of mechanism

(b) Measuring and control units

Fig. 5 Configuration of general measurement system

mission mechanism. To reduce the errors introduced by other mechanisms, two encoders are mounted just on the ends of both spindles near the angle transmission mechanism. The resolution of angular position measuring in both input axis and output axis is 360 000 P/R ($=5.4''/P$) by using a interpolator of ten times and quad mode of counter. Maximum speed of the system is 2 000 rpm and natural frequency of it is 200 Hz. In output unit, fly wheel and brake are mounted on the spindle to provide various types of loads to the system. Input unit and output unit can be moved along x -axis and y -axis, so that different types of angle transmission mechanisms (Gear, toothed belt, chain, etc.) can be measured by the system.

In this system, two measuring modes are prepared for different purposes. One is the measuring based on the angular position of input axis, which is the same with that of systems shown in Figs. 2 and 3. The angular position of output axis is recorded at each sampling angular position of input axis. Another mode is time-based measuring, in which the angular positions of input and output axes of angle transmission mechanism is recorded at each sampling time. Transmission error including backlash can be

calculated as the deviation of angular positions between input and output axes at each sampling time. Besides the transmission error, motion parameters (velocity and acceleration) of input and output axes can be obtained also.

3. Examples of Experimental Results

3.1 Measuring example on one pair of spur gears

As the preliminary experiment, one pair of spur gears with pre-set backlash is tested. The measured result is shown in Fig. 6. As described above, the deviation between transmission errors in clockwise rotation and that in counterclockwise rotation is just the backlash. From this figure, we can know that the backlash of the reducer is about 12' at all meshing positions. It is confirmed that this result is same with that by static measuring method.

3.2 Measuring example on a parallel axis gear reducer

A parallel axis gear reducer with two pairs of spur gears shown in Fig. 7 is tested. The reducer is developed for the usage in robot and has small backlash. The measured results with different pinion shafts are shown in Figs. 8(a) and (b). Comparing to Fig. 6, the backlash in Fig. 8 varies greatly on the angular position of input axis. Large transmission error occurs in Fig. 8(a) because of large offset of the pinion shaft. By exchanging the shaft to one with

Fig. 6 Measured result on one pair of spur gears

Fig. 7 Structure of a parallel axis gear reducer

small offset, the transmission error decreases (Fig. 8(b)).

Figure 9 shows the variation of measured transmission error and backlash while reducing the backlash by adjusting the screw which is used to determine the value of backlash in the parallel axis gear reducer. A backlash near zero is achieved in Fig. 9(c).

Same parallel axis gear reducer ($k=10$, backlash = 3') is tested under the condition that fly wheel is mounted on the output axis, when input axis is driven by a velocity pattern of symmetrical trapezoid. Figure 10 shows the measured results in three different speeds. In Fig. 10, input axis first rotates in clockwise direction (ab =acceleration period, bc =period of constant speed, cd =deceleration period) and then rotate in counterclockwise direction immediately (dc =acceleration period, bc =period of constant speed, cd =deceleration period). Ten waves related with the reduction ratio, and fifty-six waves related with the number of teeth of final gear in one revolution of output axis can be observed on all curves of transmission errors. However, those waves occurs in different ways in the cases of different speeds. Figure 11 shows the measured results of the reducer with same construction but different parameters ($k=15$,

(a) In the case of pinion shaft with large offset

(b) In the case of pinion shaft with small offset

Fig. 8 Measured results on the parallel axis gear reducer

(a)

(b)

(c)

Fig. 9 Variation of measured transmission error and backlash with backlash adjusting mechanism

backlash=25°). In both figures, the transmission error increases and larger vibrations occur with the increase of speed. By comparing Fig. 10 with Fig. 11, we can know that the rotational vibrations in the deceleration period due to the collision into opposite face of teeth occurs obviously in the case of large backlash. Especially in Fig. 11, the characteristics of transmission error in the speed of 1000 rpm is quite different from that of 50 rpm. The dynamic behavior of transmission error becomes very complicated in the case of high speed rotation.

Fig. 10 Influence of speed and backlash on transmission errors —In the case of small backlash—

Fig. 11 Influence of speed and backlash on transmission errors —In the case of large backlash—

Fig. 12 Influence of speed and backlash on transmission errors in a planetary gear reducer

3.3 Measuring example on planetary gear reducer

A planetary gear reducer with the construction shown in Fig. 3 is used for test. The reducer consists of four planet gears, and its reduction ratio is 5. Figure 12 shows the results of measured transmission errors with different speeds (50 rpm and 1000 rpm). Fly wheel is used in both conditions. In the case of low

Fig. 13 Schematic view of meshing in planetary gear

speed (Fig. 12(A)), the transmission error is independent of the states of motion (acceleration, constant speed, deceleration). But in the case of high speed (Fig. 12(B)), a step of 3' occurs and then disappears on the curve of transmission error at the starting point (b and c) of deceleration in both clockwise rotation and counterclockwise rotation.

The dynamic behavior of transmission error in this case can be explained according to Fig. 13. As shown in Fig. 13, the face L of teeth of planet gear is driven by the face L of teeth of sun gear when sun gear rotates in increasing speed or in constant speed. At this state, when motor decelerates, planet gear cannot decelerates rapidly because of the inertia of flywheel in the case of high speed. Then, the planet gear departs from L face of teeth of sun gear and rotates by the angle equal to the backlash relative to the sun gear.

3.4 Measuring example on a simple mechanism with backlash

To make a good understanding of the dynamic behavior of transmission error occurred above, it is necessary to use simple mechanisms for testing instead of reducer with many teeth. For simplicity, a simple transmission mechanism with backlash shown in Fig. 14 is used for the experiment. The value of backlash can be adjusted by using rods of different diameters.

Figure 15 shows transmission error and variation of angular velocity of input and output axes during the deceleration of reference velocity (From 500 rpm to 350 rpm) of input axis. In the experiment, the backlash of the mechanism is set to 32'. In Fig. 15(a),

Fig. 14 Simple transmission mechanism with backlash

(a) Transmission error

(b) Angular velocity

Fig. 15 Measured results on simple mechanism with backlash

large vibrations of transmission error can be observed. From Fig. 15(b), we can know that the relation between the rod on input axis and the groove on the output axis is in three states, that is, contact, no-contact and collision. By looking at Figs. 15(a) and (b) together, the dynamic behavior of transmission error can be explained as follows. The input axis is decelerated rapidly from point A by servo control, while the output axis cannot decelerate rapidly due to the inertia of fly wheel. Then, the speed of input axis becomes lower than that of output axis. The rod will

departs from face R of the groove and moves toward face L of the groove relatively. After the rod moves by a relative displacement equal to the backlash of the mechanism, the rod arrives at face L and collides with the face (point B). By the collision, the rod, that is, the input axis is accelerated and the speed of input axis becomes larger than that of output axis. Then, the rod departs from face L, and moves toward face R relatively. Because the input axis is decelerated by servo control, the speed of input axis becomes lower than that of output axis at point C before the rod arrives at face R. Then, the rod moves toward face L relatively, arrives at face L and collide with the face again (point D). Collisions of rod with same face L of the groove occur in this case.

4. Conclusions

A new measurement system of transmission error of angle transmission mechanism for mechatronic systems has been developed and its availability is shown by experimental measurements and industrial use. By this system, the measurement of transmission error and backlash at all meshing positions becomes possible. Besides, it is useful for inspecting and evaluating the dynamic characteristics of angle transmission mechanism.

Following results are obtained from experimental measurements.

(1) The value of backlash varies with the meshing point in gear reducer. It is difficult to evaluating true value of backlash by measuring only at several meshing points in one revolution according to old measuring method.

(2) Dynamic transmission error is influenced greatly by the value of backlash and test conditions. Complicated behavior including contact, non-contact and collision due to backlash is observed.

Further investigation will be done on the dynamic behavior and its control by this measurement system.

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