Temperature Distribution in Teeth and Blanks of Ultra-High-Speed Gears*

(2nd Report, Calculation of Temperature Distribution in Teeth and Blanks)

Yasuyoshi TOZAKI**, Susumu MATSUMOTO**
and Masatoshi FUKUTOMI***

Currently in industries, high-speed turbo machineries such as compressors and steam or gas turbines are required to have increased power capacities. Due to this, a large gear size is required which leads to nonuniform temperature distribution. As the result of nonuniform temperature distribution, nonuniform thermal distortion occurs. Change in load distribution due to this thermal distortion is considered to be closely related to scoring and pitting failures. In order to ensure reliability of high-speed gear units, it is necessary to improve the following techniques. (1) The method of predicting the temperature rise of gears and thermal distortion. (2) Tooth correction against thermal distortion. In this paper, a three-dimensional model for calculating the temperature of teeth surfaces and blanks is presented. This model includes the effects of convective cooling by lubricating oil and the change in load distribution due to mechanical deformation. The calculated results of gear teeth temperature and bending stress are in reasonable agreement with the experimental results. It is confirmed that the analytical method is practical and useful.

Key Words: Gear, Calculation, Temperature Distribution, Bending, Machine Element

1. Introduction

The establishment of a design method for ultra-high-speed gears has become increasingly necessary to deal with the recent trend of increased capacities and operational speeds of high-speed turbo machinery such as compressors and steam or gas turbines. In a previous paper⁽¹⁾, the authors verified the temperature distribution of a helical gear with L/D (tooth face width/pitch circle diameter of pinion) = 1.3, the number of modules 6, the number-of-teeth ratio 28/83 and a helical angle of 15 degrees, under test conditions of 1 100 kW delivered horsepower and pitch line velocity

up to 150 m/s using a newly made test stand. It has already been confirmed that the temperature distribution shows the highest temperature at a position slightly away from the face width center and that there is an apparent effect of thermal distortion.

In this paper, we present a comprehensive calculational model developed by us, which adds the calculational technique of thermal distortion to the conventional calculational model of tooth contacts considering the mechanical deformation. This model can cover the load distributions considering the gear temperature distribution, the thermal distortion and the tooth contacts if the operational conditions of the gear unit are known.

2. Current Status of Theoretical Prediction Methods for Thermal Distortions in Gears

Because it is very difficult to measure thermal distortion due to the manufacturing error of gears, measurements of the gear proper or its temperature distribution have been attempted to predict the thermal distortion due to temperature. It has been verified

* Received 17th March, 2000. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 65, No. 630, C (1999), pp. 276-281 (Received 29th January, 1998)

** Faculty of Engineering, Nagasaki Research and Development Center, Mitsubishi Heavy Industries, Ltd., 5-717-1 Fukahori-machi, Nagasaki 851-0392, Japan. E-mail: tozaki@ngsrdc.mhi.co.jp

*** Choryo Engineering Ltd., 5-717-1 Fukahori-machi, Nagasaki 851-0392, Japan

Fig. 1 Measured temperature distribution and tooth root stress of wheel (pitch line velocity 150 m/s)

that nonuniform temperature distribution can arise, depending on the loading conditions and the behavior of oil lubrication, in the temperature measurement⁽²⁾ of 50 000 kW delivered horsepower ($n=1\,000$ rpm, pitch line velocity 137 m/s).

The measurements of temperatures and tooth root stresses in the case of 1 100 kW delivered horsepower and pitch line velocity up to 150 m/s, conducted by the authors⁽³⁾, are shown in Fig. 1. As shown in the figure, it has been found that the temperature and the tooth root stress exhibit their highest values at the center of the face width, showing a nonuniform temperature distribution along the face width.

Regarding the thermal distortion, a two-dimensional model of Patir and Cheng⁽⁴⁾ using the Finite Element Method (FEM) analysis is available which deals with spur gears having a smaller face width. This model assumes various heat fluxes along the engaging tooth surfaces to calculate temperature distributions corresponding to them but it does not take into consideration the temperature distribution and the flow of heat in the axial direction (along the tooth face width).

Although no experimental results are available for this model, there exists a chart based on it to predict the bulk temperature if the coefficient of tooth surface friction and the coefficient of boundary heat transfer are known.

Table 1 Specifications of the gear and the bearing

	Pinion	Wheel
Number of teeth	28	83
Module	6	
Normal pressure angle, degree	20	
Helical angle, degree	15.161	
P. C. D., mm	174.054	515.946
Face width, mm	244	246
Material and heat treatment	Carburized Ni · Cr Alloy steel	
Finishing of teeth	Ground	
Maximum speed, rpm	24,158	8,150
Maximum pitch line velocity, m/s	220	
Type of journal bearing	Tilting pad type	Two lobe type
Peripheral velocity of journal, m/s	101	38.4
Bearing pressure, MPa	110	80

Furthermore, in Japan, Ito et al.⁽⁵⁾, have demonstrated their calculations of thermal distortions in gears using an approximate FEM analysis of 149 nodes and 16 elements to calculate the difference in the thermal distortion between the center and the side of the teeth; however, they have not given any specific solution to the thermal distortion distribution effect, particularly its distribution or the temperature distribution along the face width because the mechanical deformation has not been taken into consideration.

In order to determine the detailed temperature distributions along the face width or in the radial direction, it is necessary to calculate the gear proper three-dimensionally. Grekousis et al.⁽⁶⁾ conducted the first three-dimensional FEM calculation. In their calculations, however, changes of the load distribution on tooth surfaces due to thermal distortions and mechanical deformations have not been taken into consideration.

3. Calculation

3.1 Purpose of this calculational technique development

For the thermal calculation model of gears, the analytical technique and experimental verification have not yet been established regarding gears with smaller tooth face widths and no established technique for analysis of ultra-high-speed gears with large tooth face widths is available.

The purpose of this study was to develop a comprehensive calculational model of load distributions taking into consideration temperature distributions, thermal distortions and tooth contacts in large gears

operated at high-speeds by linking the mechanical deformation to the thermal distortion in simultaneous equations. Table 1 presents the principal particulars of the gear and the bearing used for this calculation⁽¹⁾.

3.2 Calculation of mechanical elastic deformation in the gear

The tooth contact is calculated by determining the gear contact, its bending deformation and the distortion and bending deformation of the shaft, using, as input terms, the shapes and sizes of the gear and the shaft, the delivered horsepower, its rotating speed and the initial correction of tooth surfaces (relative clearance between engaging tooth surfaces under no-load condition, including the gear fabrication error). This calculation technique has already been developed by the authors and details are given elsewhere⁽⁷⁾.

3.3 Calculation of thermal distortion in the gear

Although the gear configuration is nearly axially symmetric, a helical gear showing that its size is practical in the field must be handled as a three-dimensional deformation problem mainly for the following reasons:

(1) The tooth contact along the face width is not uniform and (2) because there are cooling effects due to the ambient lubricating oil splash or air at the gear side, friction heat of engaging tooth surfaces will flow radially and axially.

The symbols used for the present calculations are shown below.

Symbols

P_{ar}	: mean Hertz's contact pressure (MPa)
μ	: coefficient of friction
V_s	: relative sliding velocity (m/s)
q_F	: friction heating at tooth contact surface (W/m ²)
a	: width of Hertz's contact (m)
ω	: velocity of rotating angle (rad/s)
v	: velocity of tooth surface (m/s)
n	: rotating velocity (1/s)
Q'	: quantity of accelerating oil (m ³ /s)
Q_0	: confineable volume (m ³ /s)
Q	: oil feed rate (m ³ /s)
ρ	: density (kg/m ³)
v_p	: pitch line velocity (m/s)
l	: length of line of action (m)
r	: radius of pitch circle (m)
h_k	: addendum (m)
Z	: number of teeth
B	: tooth face width (mm)
Nu	: Nusselt's number
Re	: Reynolds number
Pr	: Prandtl number
d	: diameter (mm)

η : kinematic viscosity (m²/s)

k : coefficient of thermal conductivity (W/mK)

α_t : coefficient of heat transfer (W/m²·K)

F_n : normal force (N)

α : pressure angle (degree)

h_d : total tooth height (mm)

3.4 Temperature rise of gear teeth due to gear operation

There are three terms of heat sources as shown below, which contribute to the temperature rise of the gear teeth during its operation. (1) Friction heat exists over the engaging tooth surfaces because relative sliding exists thereon, except at the pitch points. As shown in the following equation, the work of acting friction is expressed by the integral of distribution of friction factor μ , contact pressure P_{ar} and sliding velocity V_s in the region covering the line of action.

$$q_F = \mu P_{ar} V_s \quad (1)$$

Friction factor μ and sliding velocity V_s are functions along the line of action, which change according to the tooth engagement region. Because the oil film formed between tooth surfaces is extremely thin, the quantity of heat carried away by the oil film over the tooth surfaces is so slight that most of the friction heat generated over the tooth surfaces is thermally conducted through the gear proper. The friction heating generated from the teeth engagement is divided, respectively, to the pinion and the wheel according to the dividing ratio Λ :

$$q_1 = \Lambda q_F \quad q_2 = (1 - \Lambda) q_F \quad (2)$$

If the temperature difference in both tooth surfaces between the pinion and the wheel does not exist, the dividing ratio Λ is governed by the sliding velocity and the physical properties of both gears. The theory of Blok⁽⁸⁾ states that the dividing ratio under the present conditions will be 0.5. Equations (1) and (2) shows a momentary heat flux at the Hertz's contact width along the contact line of action. It takes a time of a/v_1 for gear 1 to cross this Hertz's width. Therefore, the quantity of heat flowing during its crossing the Hertz's width in one rotation is q_1 . Accordingly, a/v_1 , the mean quantity of heat flowing during one rotation can be obtained by dividing the total quantity of heat by the time $2\pi/\omega_1$ required for one rotation. If the width of the Hertz's contact over the tooth surface is denoted by a , the quantity of heat which flows through the contact part during one rotation of the pinion can be determined by:

$$q_1 = \frac{a}{v_1} \frac{\omega_1}{2\pi} \frac{q_F}{2} \quad (3)$$

$$q_2 = \frac{a}{v_2} \frac{\omega_2}{2\pi} \frac{q_F}{2} \quad (4)$$

Fig. 2 The relationship between calculated results and measured results

3.5 Friction loss due to tooth engagement

The engaging tooth surfaces are in rolling and sliding contact, and a sliding friction loss due to the tooth engagement can exist. The friction loss between a pair of engaging teeth L_0 can be expressed by:

$$L_0 = \frac{\mu F_n}{\cos \alpha} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \frac{l_1^2 + l_2^2}{2} \quad (5)$$

where

$$l_1 = \{ (r_2 + h_k)^2 - r_2^2 \cos^2 \alpha \}^{0.5} - r_2 \sin \alpha$$

$$l_2 = \{ (r_1 + h_k)^2 - r_1^2 \cos^2 \alpha \}^{0.5} - r_1 \sin \alpha$$

If Z is used to denote the number of teeth in the pinion, then the friction work of the pinion during one rotation is ZL_0 , and the friction loss due to tooth engagement, L_M , is as follows:

$$L_M = 8.34 \times 10^{-9} \frac{\mu F_n Z n}{\cos \alpha} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) (l_1^2 + l_2^2) \quad (6)$$

As seen from Eq.(6), accurately determining the friction coefficient between tooth surfaces is necessary to accurately evaluate the friction work of the tooth engagement. The authors⁽¹⁰⁾ have previously determined the technique for the evaluation of the friction coefficient under the rolling-sliding condition using a function of the D value (total of maximum tooth surface roughnesses/minimum oil film thickness). Figure 2 shows the relationship between the friction coefficients due to the teeth engagement of a spur gear, measured by Terauchi et al.⁽¹¹⁾ and their results calculated from Eq.(7). Because the difference between the calculated and measured results was within about 10%, it has been verified that Eq.(7) is practical and useful.

$$D < 1 \quad \mu = 0.01$$

Fig. 3 Oil acceleration volume

$$1 \leq D < 10 \quad \mu = 0.01(5 \log D + 1) \quad (7)$$

$$10 \leq D \quad \mu = 0.01(2.5 \log D + 3.5)$$

3.6 Acceleration of lubricating oil

If lubricating oil is fed to a pair of rotating gears, the oil confined in the tooth space, its clearance and its backlash, are momentarily accelerated up to the peripheral velocity of the rotating gears with an increase of their rotation. This accelerating direction includes the radial and axial directions, and the oil loss resulting from this acceleration is characteristically determined by factors such as the elements of the tooth profile, the number of rotations and the oil feed rate, but not by the delivered power. If Q' is used to denote the quantity of the accelerated oil, the resultant work to give momentum to the lubricating oil is expressed by:

$$l_{AR} = \rho Q' v_p^2 \quad (8)$$

In this section, it is important to set the quantity of the accelerated oil. The quantity of the accelerated oil is based on the quantity of oil staying in the space, the clearance and the backlash. Figure 3 shows the model of the operating tooth in action. From this figure, the oil accelerator volume is obtained. This calculation method may be found in a previous paper⁽⁹⁾. The quantity of oil, Q' , confined and accelerated in the space, is simultaneously accelerated axially (along the tooth trace). Considering the case where the torsional angle β becomes smaller, the quantity of the axially accelerated oil, Q'' , is expressed by:

$$\left. \begin{aligned} Q'' &= K Q' \sin \beta \\ \text{where, } K &= h_1 S_1 B / Q' \end{aligned} \right\} \quad (9)$$

Considering that the moving velocity along the tooth trace of the lubricating oil, v_A is, $v_p / \tan \beta$, the axial accelerating work of the lubricating oil confined in the oil groove can be expressed by the product of the force ($\rho Q'' v_A$) to give the moving velocity v_A to the $\rho Q''$ accelerated oil and the pitch line velocity v_p .

$$\begin{aligned} l_{AA} &= \rho Q'' v_p / \tan \beta \cdot v_p \\ &= \rho K Q' v_p^2 \sin \beta \frac{\cos \beta}{\sin \beta} = K \cos \beta l_{AR} \end{aligned} \quad (10)$$

3.7 Shaft bearing loss

Sliding bearings working in a perfect fluid

Table 2 Tested bearings

Marks	○	△	◇
Bearing type	2 lobes	4 lobes	Til.Pads
Diameter × width	φ 100 mm×50 mm		
Preload ratio	0.6		
Diameter clearance	0.157 mm	0.166 mm	0.146 mm
Direction of load	ON		between pads
Metal back cooling	NON		
Bearing material	LBC3 + Pb-Overlay

Fig. 4 The relationship between calculated power loss and measured loss

lubricating condition are used for the gear shaft bearings in high-speed gears. We used the established method⁽¹²⁾ regarding the power loss of the sliding bearing. This method also includes considerations of viscosity changes due to the shear heating of the lubricating oil. Here, we tried to correlate the values from the present analytical method to the results experimentally measured. Table 2 shows three different types of tested bearings.

These bearings showed no damage within the tested range of the peripheral velocity of the shaft up to 100 m/s and the bearing pressure up to 5 MPa. The correlation between the calculated and measured power losses is shown in Fig. 4. Since a reasonable correlation between them could be confirmed, the bearing loss is theoretically predictable.

To reduce the power loss of the gear unit, it is also necessary to fully study how to decrease the bearing loss (i.e., bearing type, dimensions, viscosity of lubricating oil used, oil feed rate, etc.).

3.8 Cooling conditions

In gears rotating at a high speed, their heat transfer coefficient, can vary from tooth crest to root. Furthermore, it can differ, depending on the type of the cooling medium. To obtain the heat transfer coefficient, the following empirical formulas are used. The equation of Hartnett and Deland⁽¹³⁾ was used for

Table 3 Windage loss of a gear rotating in oil splash

Velocity m/s	100	125	150
Measured loss	3.3 kW	8.0 kW	11.0 kW
Equation (13)	3.1 kW	6.1 kW	10.5 kW

the laminar flow state ($Re < 3 \times 10^5$).

$$\alpha_t = Nu k \sqrt{\frac{\omega}{\eta}} \quad (11)$$

where, k and η are, respectively, the coefficient of thermal conductivity and kinematic viscosity of the cooling. Nu is the Nusselt number decided by the Prandtl number Nu .

If the condition of flow becomes turbulent ($Re > 3 \times 10^5$), then, the equation of Dorfmann⁽¹⁴⁾ is used.

$$\alpha_t = 0.0297 Re^{0.3} Pr^{0.6} k \sqrt{\frac{\omega}{\eta}} \quad (12)$$

To roughly estimate what medium was used as the coolant for the gear under high-speed rotations, its windage losses were calculated from the following equation⁽¹⁵⁾ and the calculated results are given in Table 3.

$$l_w = 3.80 \times 10^{-22} n^3 d^3 \times \{0.006 + 0.2(B/d)^{0.8} + (h_a/d)\} \rho \quad (13)$$

Because the calculated results of windage losses in Table 3 show that the major element of the cooling media for the gear was air, the cooling medium in the present study was taken as air, except for the parts supported by bearings.

If the air temperature is assumed to be 80°C, the Reynolds number is 7×10^6 from $d = 0.516$ m, $\omega = 581$ rad/s and $\mu = 0.217 \times 10^{-4}$ m²/s, and then, Eq.(12) is used. Using the Prandtl number = 0.71 and $k = 0.03$ W/m²·K, determined from Eq. (2), then, the thermal conductivity coefficient at the pitch point, $\alpha_t = 422$ W/m²·K.

3.9 FEM analysis method

The helical gear must be handled as a three-dimensional problem mainly for the following reasons:

(i) The heat source is distributed along the tooth trace and the tooth contact (load distribution) is also nonuniform.

(ii) The cooling part due to lubricating oil or air is located at the gear side and the heat generated at the tooth operating part flows radially and axially.

(iii) The friction heat at the sliding bearing to support the gear shaft exists as the heat source to the shaft end, and therefore, a study of the three-dimensional heat flow is necessary to investigate the effect of the friction heat.

The flow chart of this calculational method is shown in Fig. 5, and the finite-element meshing of the wheel in Fig. 6. In the calculations, the shapes and

Fig. 5 Calculation flow chart

Fig. 6 Finite-element mesh of wheel

sizes of the gear and its shaft, the delivered power, the rotating speed and the relative clearance, are first given as the input terms to calculate the contact

Fig. 7 Comparison between measured results and calculated results of temperature

between the gears, the gear bending deformation and the shaft distortion and bending deformation. The heat sources and the cooling conditions at the relevant parts are then set up, based on the calculated tooth contact (load distribution) results according to the calculational procedures already given in sections 3.4 through 3.8, followed by the three-dimensional FEM analysis to calculate the temperature distribution and the thermal distortion of the gear entity. Because the tooth contact keeps changing according to the level of thermal distortion, the calculation must be repeated up to convergence of the tooth surface clearance and temperature.

4. Calculated Results

The calculated results of temperatures of the gear entity using the calculational conditions of 150 m/s and 2 000 N·m as already described were compared with the measured results.

The broken lines in Fig. 7 indicate the calculated results and the solid lines to the measured results. A good correlation was obtained regarding the trend of the temperature distribution along the tooth face width except for the tooth edges of the pinion. This temperature difference at such tooth edges is estimated to have been caused by the difference in the heat transfer coefficient. Regarding the heat source from the accelerating work of lubricating oil, the following assumptions were taken: (1) Most of the heat generated from the accelerating work flows into the gear entity, and (2) The level of the heat flowing into the gear entity is proportional to the respective surface areas of the pinion and the wheel. These

Fig. 8 Comparison between measured results and calculated results of bending root stress

assumptions are thought to be appropriate because the calculated maximum temperature in the pinion agreed well with the measured result. The reason that the calculated results at the wheel side were lower than the measured results may probably be due to the fact that the difference in the heat transfer coefficient increased due to the larger surface area of the tooth edges, as described above.

Because the cooling medium is a mixture of lubricating oil and air, the difference between the calculated and measured results would be further reduced if its mixing ratio is assumed; however, because the predicted maximum temperature agreed well with the measured results, the assumptions used in this paper could be practically and useful if one uses the results obtained here to predict the possibility of seizures and other problems.

Figure 8 compares the measured tooth root stresses with the calculated results. It has been shown that the tooth root stress distribution along the face width and its absolute values exhibited a good agreement between the measured and calculated results.

5. Conclusions

In order to investigate the temperature distribution characteristics of the gear operating in an ultra-high-speed region, the authors have developed a comprehensive calculation model for load distributions covering considerations of the gear temperature distribution, the thermal distortion and the gear tooth contacts if the operating conditions are known by adding the thermal distortion calculation technique to the conventional gear tooth contact calculation model taking the mechanical deformation into consideration. The results obtained from the present developmental work are as follows:

(1) Comparison of the gear entity temperatures between calculated and measured results under the conditions of 150 m/s and 200 N·m showed that there

was a good correlation regarding the trend of temperature distribution along the tooth face width except for the tooth edges in the pinion. Furthermore, a good agreement between calculated and measured maximum temperature distribution was verified and it has been confirmed that the analytical method used in this paper is practical and useful when this method is applied for the prediction of seizures and other problems.

(2) The tooth root stress distribution along the face width and its absolute values showed a good agreement between the calculated and measured results.

References

- (1) Matsumoto, S., Tozaki, Y. and Fukutomi, M., *Trans. Jpn. Soc. Mech. Eng.*, Vol. 65, No. 629, C (1999), p. 277.
- (2) Bloch, P., *Machinery*, No. 515 (1973), p. 5.
- (3) Matsumoto, S., Tozaki, Y. and Sudoh, K., *Proc. of MPT91* (1991), p. 496.
- (4) Patir, N. and Cheng, H.S., *ASLE Trans.*, Vol. 22, No. 1 (1979), p. 25.
- (5) Ito, M., Ishikawa, J. and Umezawa, K., *Proc. the Japan Society of Mechanical Engineers*, No. 770-15 (1978), p. 76.
- (6) Grekoussis, R. and Retzepis, J., *Antribstechnik*, Vol. 29 (1990), p. 67.
- (7) Matsumoto, S., Asanabe, S., Tanaka, Y. and Sudoh, K., *Proc. 21st Symposium of the Japan Society of Mechanical Engineers*, No. 83-8 (1984), p. 166.
- (8) Blok, H., *Advance Proc. NASA Symp.*, 1, 4.1 (1968), p. 153.
- (9) Matsumoto, S., Asanabe, S., Takano, K. and Yamamoto, M., *Proc. of the JSLE Int. Trib. Conf.* (1985), p. 1165.
- (10) Matsumoto, S., Asanabe, S., Kanamaro, M. and Sudoh, K., *Proc. Japanese Society of Trybologists*, (1984).
- (11) Terauchi, Y., Kumamoto, Y., Kimoto, I. and Sasaki, Y., *Trans. Jpn. Soc. Mech. Eng.*, (1966), p. 1571.
- (12) Taniguchi, S., Saki, K. and Morohoshi, S., *Preprint of JSME*, No. 759-19 (1975), p. 45.
- (13) Hartnett, J.P. and Deland, E.C., *ASME, J. Heat Transfer*, C83 (1961), p. 95.
- (14) Dorfmann, L.A., *Soviet Physics*, Vol. 3 (1958), p. 249.
- (15) Matsumoto, S., Asanabe, S., Sudoh, K., Takano, K. and Yamamoto, M., *Mitsubishi Technical Review*, Vol. 24, No. 2 (1987).