A Formulation for Automated Computer-Aided Design of Machine Parts*

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A mathematical formulation for the automated design of parts and assemblies is presented as a numerical algorithm and its accompanying experimental code. A mechanical part is modeled as an assembly of links connected via joints. Constraints are imposed by the designer such that some dimensions are specified as variables while others are allowed to propagate due to a design change. Graph theory is used to model the assembly where the method of cut-joint constraints is used to obtain a spanning tree. Joint constraints between the various links are introduced and their variations are computed. A velocity transformation matrix is introduced and the Jacobian of for each constraint is developed. The Jacobian of the constraint is transformed to joint coordinate space. Finally, because the resulting system of equations has a non-square Jacobian, the Moore-Penrose inverse is used to compute an assembled configuration, hence, redesigning the assembly. The formulation is extended to include various methods for computing an assembled configuration. While the accompanying numerical code is only experimental, the formulation presented herein is believed to form the basis of a new methodology for automated computer-aided design of parts and systems. The significance of this work is characterized by the formulation's ability to automatically design and redesign parts.

Key Words: Constraint Propagation, Automated Computer Aided Design, Variational Design

1. Introduction

Tremendous efforts in recent years were dedicated to the development of automated methods and to increase the degree of intelligence in computer-aided design (CAD) software. Under the umbrella of concurrent engineering, numerous researchers have reported versatile and powerful techniques ranging from the use of parametric technology to artificial intelligence aimed at elevating the automation CAD systems in providing help to the designer.

The field of computer-aided design has seen a revolutionary advancement by the advent of par-

ametric technology. Using this technology, many commercial CAD systems have simplified the iterative design process. In most cases, parametric technology has been implemented characterized by sets of rules associating dimensions by simple equations. For example, the length of two links d_1 and d_2 may be related by a parameter t by in an equation as $d_1 = d_2 t$. This indicates that a change in the length of link 2 will automatically change the length of link 1. Similarly, a change in any geometry causes a change in another geometric property.

The process of design still largely depends on the experience and intuition of engineers. In a typical design process, the initial design data are specified. Design parameters are modified to improve the performance. Based on experience and analytical formulations, a design may undergo a number of iterations before reaching an optimum situation. Computer programs have used this concept to automatically alter designs are those reported by Imamura 1994⁽¹⁶⁾, Burke et al. 1994⁽⁶⁾, and Cutosky and Tenenbaum 1990⁽⁷⁾. This type of software facilitates constraint-based modeling and in many cases provides inferenc-

* Received 2nd November, 1998

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ing via constraint propagation. Some methods specify a parametric dependency between different parts. A design change in the geometry of a part propagates through the complete design. Other efforts at CAD/CAM automation include reasoning schemes that use topological relationships between features (McMahon, et al. 1997⁽²⁰⁾). More recent efforts to automate the mechanical design process in the manufacturing field were demonstrated by Abdel-Malek and Maropis (1998)⁽¹⁾.

For completeness (and to differentiate between our work and that in the main stream of geometric constraint solving), we review works in that field. There are a number of schemes for constraint solving (a classification can be found in Hoffmann and Vermeer 1995⁽¹³⁾), of which we will identify those that are relevant, and delineate the differences.

(a) Instance solvers where constraints are converted to a set of equations to be simultaneously solved using numerical algorithms. While this approach is valuable in treating over-constrained and inconsistent problems, an initial solution must be determined prior to initiating a solution (see a listing of the shortcomings of this approach in Kramer 1992⁽¹⁷⁾ and Grimson 1990). In our proposed approach, the mechanical part is converted to an assembled mechanism, whereby any further changes are inherent in the design of the mechanism.

(b) Generic solvers where sequential constraints are used to investigate first whether the constraints can be placed, then determines the placement (Bouma, et al. 1995⁽⁵⁾, Eggli, et al. 1997⁽⁹⁾, Fudos and Hoffmann 1996⁽¹⁰⁾, Gao, et al. 1988⁽¹¹⁾, Sunde 1988⁽²¹⁾; 1990⁽²²⁾, and Suzuki, et al. 1990⁽²³⁾).

(c) Symbolic solver where manipulation of the algebraic equations defining the constraints is performed using a symbolic manipulator to facilitate the determination of a solution (Anantha, et al. 1996⁽³⁾). Those methods are also sometime called hybrid (Hsu and Bruederlin 1997⁽¹⁵⁾). We do not use symbolic manipulators, however, the algebraic equations are variational in form. Indeed, we will maintain some of kinematic parameters as variables in order to study the effect of changing one parameter on the assembly (analogous to design sensitivity analysis).

(d) Rule-based solver where constraints are expressed by rules and predicates and are solved using logical rules (Arbab and Wang 1990⁽⁴⁾, Sunde 1990⁽²²⁾, and Wetkamp 1992⁽²⁶⁾). Graph representation was also used by Lee and Kim (1996⁽¹⁸⁾; 1998⁽¹⁹⁾), Verroust, et al. (1993)⁽²⁵⁾, Todd, 1989⁽²⁴⁾, and Hoffmann, et al. (1997)⁽¹⁴⁾.

The work is based on recent rigorous mathematical formulations by this group (Zou, et al. 1996⁽²⁷⁾)

which was extended to the automatic computer aided design idea in (Zou, et al. 1997a⁽²⁸⁾; 1997b⁽²⁹⁾). It is based on a computational method for studying the kinematics of mechanical systems as first used multibody dynamics formulations by Haug (1989)⁽¹²⁾ and were implemented into commercial code (DADS 1998)⁽⁸⁾. This paper is a further expansion of the formulation with the ultimate goal of developing an automated CAD environment.

The proposed method of modeling of a mechanical part into an assembly of links and joints will first be presented in Section 2. An assembly will be transformed into a connected graph and some joints will be cut in order to obtain a spanning tree. The mathematical formulation will then be introduced in Section 3. A state vector representation will be formed in Section 4 where the Jacobian will be transformed from joint to state vector notation. A solution to the general problem of computing an assembled configuration will be introduced by forming a Moore-Penrose inverse of the Jacobian. Six variational methods for computing an updated part will be presented and a number of examples will be illustrated.

The significance of the formulation is evident in a CAD environment where a part can be automatically designed because of the inherent formulation therein. A more intelligent system can be developed that is poised to make significant contributions to the computer-assisted mechanical design field. This paper is the basis for such a system applied to 3D-parts.

2. Modeling and Graph Theory

Consider a mechanical part in an assembly shown in Fig. 1(a). Part Γ_1 is connected to part Γ_2 . At some instance during the design process, it is necessary to change the dimension n of part Γ_1 to $n + \delta n$. This change will influence the design of the mechanical part Γ_2 . However, the change in part Γ_2 is subject to some constraints imposed by either its connectivity or functionality. It is proposed to model the constraints using links connected via joints. For example, the dimensions l and k are modeled as solid links (rigid bodies) of the same length, while the design variation is due to the changing link $n + \delta n$. It is also desired to

Fig. 1 (a) Two connected machine parts, (b) The model representation of part Γ_1

Fig. 2 (a) A mechanical part, (b) Modeling of the part

Fig. 3 (a) A mechanical part, (b) Its model with links and joints

absorb the design variation by the upper dimension, namely m . Therefore, this dimension is modeled as a sliding joint (J_3) connecting two revolute joints (J_2 and J_4) is given the generalized coordinate q_3 . It is also necessary to introduce joints J_1 and J_5 to allow for the movement of J_2 and J_4 . Note that by specifying the zero link as ground, a constraint has been imposed on the orientation and length of link k . In addition, the angles q_1 , q_2 , q_4 , and q_5 are maintained as variables subject to change. In fact, the mechanical part is now completely modeled by five links and five joints. The mechanism shown in Fig. 1(b) is a model of the mechanical part (Γ).

As a second example, consider the mechanical part shown in Fig. 2(a) which was addressed by Zou et al.⁽²⁷⁾. In this case, consider another type of variation in the dimension l to $l + \partial l$, but with the constraint that the angle q_1 remains constant, i.e., $q_1 = c$. In this case, the length k is allowed to absorb any design variation with some angle changes in joints (J_2 and J_3) and the mechanism is shown in Fig. 2(b).

To extend this theory to 3D objects, consider the geometry of the mechanical part shown in Fig. 3(a). Due to a design variation in a connecting part, a designer allows this change to be absorbed by specifying two sliding joints (J_2 and J_3), a revolute joint J_4 , and spherical joints (J_1 , J_5 , J_6 , J_7 , and J_8). The modeled part is shown in Fig. 3(b) where the proposed links are numbered 0 through 6. Using graph theory, the mechanism is represented as shown in Fig. 4(a). Design changes are absorbed by variations in the joints and a new part is computed as in Fig. 4(b). Upon modeling the mechanical part as a graph, the joint with the highest number of degrees of freedom is cut in order to reduce the number of equations in the

Fig. 4 (a) Graph representation, (b) The final shape due to a design change

Fig. 5 A pair of bodies are disconnected after cutting joints

analysis. Often, the resulting spanning tree has a pair of bodies that is disconnected after cutting joints. For this case, the tree graph representation is illustrated in Fig. 5, where the ground or chassis shall be called the base body. A joint that connects bodies i and j is cut and body p is the junction node of the two chains that contain bodies i and j , respectively. In this case, it will be necessary to represent the state variation of body j in terms of the state variation of body i although no relative coordinates exist between bodies i and j .

3. Mathematical Formulation

In this section, a constraint formulation for a joint that has no connecting length will be developed. The variation of this constraint will be presented before generalizing the case in the following section. In order to facilitate the forthcoming discussion, Fig. 6 is introduced to define the terminology. Let body i be defined as the inboard of body j . The vectors $\mathbf{r}_i$ and $\mathbf{r}_j$ denote the global position vectors extending from the origin of the global reference frame xyz to the origin of the body reference frame $x'y'z'$. The joint reference frame will be denoted by $x''y''z''$. The vectors $\mathbf{s}_{ij}$ are the joint-position vectors in the body reference frame.

For a pair of connected bodies, the reference frame is defined at the joint connected to the inboard body; thus, the vector $\mathbf{s}_{ji}$ vanishes. A vector $\mathbf{d}_{ij}$ is defined to determine the joint connection points from

Fig. 6 Identification of coordinate systems of a pair of bodies disconnected after cutting joints

the origin of the joint reference frame O''_{ij} on body i to the origin of the joint reference frame O''_{ji} on body j , which is zero for a spherical joint.

In deriving the variational constraints, it is necessary to keep the joint-position vectors and the orientation matrices as variables in order to obtain their differentials. The derivation of basic constraints was presented in our first report and will not be repeated here. In general, the constraint equation is a function of the following parameters

$$\Phi(r_i, A_i, s_{ij}, C_{ij}, r_j, A_j, s_{ji}, C_{ji}) = 0 \quad (1)$$

where A_i is a transformation matrix from the body coordinate system $x'_iy'_jz'_i$ to the global coordinate system xyz , and C_{ij} is the transformation matrix from joint reference frame $x''y''z''$ to body reference frame $x'_iy'_jz'_i$. It is noted that s and C will be considered as variables throughout the forthcoming discussion, therefore, their variation does not vanish.

The variation of a constraint that has no connecting length (such as the spherical constraint) is

$$\delta\Phi = \delta r_j + \delta s_{ji} - \delta r_i - \delta s_{ij} \quad (2)$$

The joint-position vectors can be resolved in the body reference frame as

$$s_{ij} = A_i s'_{ij} \quad (3)$$

Similarly, $s_{ji} = A_j s'_{ji}$. Taking the variation of both sides of Eq. (3) yields

$$\delta s_{ij} = \delta A_i s'_{ij} + A_i \delta s'_{ij} \quad (4)$$

Define the virtual rotation matrix $\delta\tilde{\pi}$ (a skew-symmetric matrix) as

$$\delta\tilde{\pi} = \delta A A^T \quad (5)$$

where the tilde operator (the symbol $\sim$) is used to denote a skew-symmetric matrix generated by the associated vector (e.g., $\tilde{a}$ is the skew symmetric matrix resulting from the elements of the vector $a = [a_x \ a_y \ a_z]^T$). Similar to the virtual rotation matrix $\delta\tilde{\pi}$ resulting from A , define a virtual rotation matrix $\delta\tilde{\xi}$ resulting from the transformation matrix C_{ij} such

that

$$\delta\tilde{\xi}'_{ij} = \delta C_{ij} C_{ij}^T \quad (6)$$

To simplify the computation of virtual rotations, transform using the virtual Euler parameters such that

$$\delta\tilde{\xi}_{ij} = 2A_i E_j \delta p_j \quad (7)$$

where $p_j = [e_0 \ e_1 \ e_2 \ e_3]^T$ is the vector of Euler parameters (e_i) and E_j is the Euler semi-rotation matrix defined as

$$E_j = \begin{bmatrix} -e_1 & e_0 & -e_3 & e_2 \\ -e_2 & e_3 & e_0 & -e_1 \\ -e_3 & -e_2 & e_1 & e_0 \end{bmatrix} \quad (8)$$

Multiplying Eq. (5) from the right by A yields another property $\delta\tilde{\pi}A = \delta A$. Substituting this property into Eq. (4) yields

$$\delta s_{ij} = \delta\tilde{\pi} A_i s'_{ij} + A_i \delta s'_{ij} \quad (9)$$

Using the property above and the property $\tilde{a}b = -\tilde{b}a$, the first term of the right hand side of Eq. (9) can be further simplified as

$$\delta s_{ij} = -\tilde{s}_{ij} \delta\pi_i + A_i \delta s'_{ij} \quad (10)$$

Substituting Eq. (10) and a similar expression for δs_{ji} into Eq. (2) yields the variation as

$$\delta\Phi = \delta r_j - \delta r_i - \tilde{s}_{ji} \delta\pi_j + A_j \delta s'_{ji} + \tilde{s}_{ij} \delta\pi_i - A_i \delta s'_{ij} \quad (11)$$

Note that joint-attachment vectors in Eq. (11) have been maintained as variables. This variation will be later written in terms of a Jacobian and solved for a new set of generalized coordinates.

4. Transformation to a State Vector Variation

In this section, a recursive formulation will be developed to compute the state variation of one link with respect to another. The variation of a constraint that has no connecting length given by Eq. (11) can be written in general terms as

$$\delta\Phi = \Phi_r \delta r_i + \Phi_\pi \delta\pi_i + \Phi_r \delta r_j + \Phi_\pi \delta\pi_j + \Phi_{s_{ij}} \delta s'_{ij} + \Phi_{s_{ji}} \delta s'_{ji} + \Phi_{\xi_{ij}} \delta \xi'_{ij} + \Phi_{\xi_{ji}} \delta \xi'_{ji} = 0 \quad (12)$$

Furthermore, the linearized constraint of Eq. (12) can be written as

$$\delta\Phi = \Phi_z \delta z_i + \Phi_z \delta z_j + \Phi_{\xi_{ij}} \delta \xi^{ij} + \Phi_{s^{ij}} \delta s^{ij} = 0 \quad (13)$$

where the extended vector of the virtual joint-position and virtual rotation vectors are

$$\delta s^{ij} = [\delta s'_{ij} \ \delta s'_{ji}]^T \text{ and } \delta \xi^{ij} = [\delta \xi'_{ij} \ \delta \xi'_{ji}]^T \quad (14)$$

the coefficients of virtual displacement are

$$\Phi_z = [\Phi_r \ \Phi_\pi] \text{ and } \Phi_z = [\Phi_r \ \Phi_\pi] \quad (15)$$

the coefficients of virtual joint-position and rotation are

$$\Phi_{s_{ij}} = [\Phi_{s'_{ij}} \ \Phi_{s'_{ji}}] \text{ and } \Phi_{\xi_{ij}} = [\Phi_{\xi'_{ij}} \ \Phi_{\xi'_{ji}}] \quad (16)$$

and the virtual displacement for bodies i and j are

$$\delta z_i = [\delta r_i \ \delta\pi_i]^T \text{ and } \delta z_j = [\delta r_j \ \delta\pi_j]^T \quad (17)$$

Therefore, the linearized equation can also be written in terms of state vectors ($\tilde{z}$) as

$$\delta\Phi = \Phi_z \delta \tilde{z}_i + \Phi_z \delta \tilde{z}_j + \Phi_{\xi_{ij}} \delta \xi^{ij} + \Phi_{s^{ij}} \delta s^{ij} = 0 \quad (18)$$

Because the connecting joint between bodies i and j is

cut, the state variations of bodies i and j must be written in terms of the state variation of their common junction node p in Fig. 6 as

$$\delta \mathbf{z}_i = \delta \mathbf{z}_{i-1} + \mathbf{B}_i \delta \mathbf{q}_i + \mathbf{M}_{i-1} \delta \mathbf{s}'_{i-1,j} + \mathbf{N}_i \delta \mathbf{\xi}_{i-1,j} \quad (19)$$

where $\mathbf{B}_i$ is the velocity transformation matrix between bodies i and j , defined by

$$\mathbf{B}_i = \begin{bmatrix} \frac{\partial \mathbf{d}_{(i-1),j}}{\partial \mathbf{q}_i} + \tilde{\mathbf{r}}_i \mathbf{H}_i \\ \mathbf{H}_i \end{bmatrix} \quad (20)$$

and the matrices $\mathbf{M}_{i-1}$ and $\mathbf{N}_i$ are expressed as $\mathbf{M}_{i-1} = \begin{bmatrix} \mathbf{A}_{i-1} \\ \mathbf{0} \end{bmatrix}$ and $\mathbf{N}_i = \begin{bmatrix} \tilde{\mathbf{r}}_i - \tilde{\mathbf{d}}_{(i-1),j} \\ \mathbf{I} \end{bmatrix}$.

Equation (19) can be recursively used to obtain the state variation of body i in terms of the state variation of its junction p and of the relative coordinates along the chain as

$$\begin{aligned} \delta \mathbf{z}_i &= \delta \mathbf{z}_{i-2} + \mathbf{B}_{i-1} \delta \mathbf{q}_{i-1} + \mathbf{M}_{i-2} \delta \mathbf{s}'_{i-2,j-1} \\ &\quad + \mathbf{N}_{i-1} \delta \mathbf{\xi}_{i-2,j-1} + \mathbf{B}_i \delta \mathbf{q}_i + \mathbf{M}_{i-1} \delta \mathbf{s}'_{i-1,j} + \mathbf{N}_i \delta \mathbf{\xi}_{i-1,j} \end{aligned} \quad (21)$$

Simplifying and writing in terms of a series yields

$$\delta \mathbf{z}_i = \delta \mathbf{z}_p + \sum_{k=m}^i (\mathbf{B}_k \delta \mathbf{q}_k + \mathbf{M}_{k-1} \delta \mathbf{s}'_{k-1,k} + \mathbf{N}_k \delta \mathbf{\xi}_{k-1,k}) \quad (22)$$

Similarly, the state variation of body j can be written in terms of a series as

$$\delta \mathbf{z}_j = \delta \mathbf{z}_p + \sum_{k=n}^j (\mathbf{B}_k \delta \mathbf{q}_k + \mathbf{M}_{k-1} \delta \mathbf{s}'_{k-1,k} + \mathbf{N}_k \delta \mathbf{\xi}_{k-1,k}) \quad (23)$$

Substituting Eqs. (22) and (23) into Eq. (18) yields

$$\begin{aligned} \delta \Phi &= \Phi_z \left[\delta \mathbf{z}_p + \sum_{k=m}^i (\mathbf{B}_k \delta \mathbf{q}_k + \mathbf{M}_{k-1} \delta \mathbf{s}'_{k-1,k} + \mathbf{N}_k \delta \mathbf{\xi}_{k-1,k}) \right. \\ &\quad \left. + \Phi_{s'} \delta \mathbf{s}^{ij} + \Phi_{\xi'} \delta \mathbf{\xi}^{ij} \right] \\ &\quad + \Phi_z \left[\delta \mathbf{z}_p + \sum_{k=n}^j (\mathbf{B}_k \delta \mathbf{q}_k + \mathbf{M}_{k-1} \delta \mathbf{s}'_{k-1,k} + \mathbf{N}_k \delta \mathbf{\xi}_{k-1,k}) \right] \\ &\quad + \Phi_{s'} \delta \mathbf{s}^{ij} + \Phi_{\xi'} \delta \mathbf{\xi}^{ij} \end{aligned} \quad (24)$$

Collecting similar terms and rearranging yields

$$\begin{aligned} \delta \Phi &= \Phi_z \sum_{k=m}^i (\mathbf{B}_k \delta \mathbf{q}_k + \mathbf{M}_{k-1} \delta \mathbf{s}'_{k-1,k} + \mathbf{N}_k \delta \mathbf{\xi}_{k-1,k}) \\ &\quad + \Phi_z \sum_{k=n}^j (\mathbf{B}_k \delta \mathbf{q}_k + \mathbf{M}_{k-1} \delta \mathbf{s}'_{k-1,k} + \mathbf{N}_k \delta \mathbf{\xi}_{k-1,k}) \\ &\quad + \Phi_{s'} \delta \mathbf{s}^{ij} + \Phi_{\xi'} \delta \mathbf{\xi}^{ij} \end{aligned} \quad (25)$$

where $\Phi_z + \Phi_z = \mathbf{0}$ was used to eliminate the $\delta \mathbf{z}_p$ term in Eq. (25). The constraint variation, written in terms of the Jacobian is

$$\delta \Phi = \Phi_q \delta \mathbf{q} \quad (26)$$

where

$$\begin{aligned} \Phi_q &= [\Phi_z, \mathbf{B}_m \cdots \Phi_z, \mathbf{B}_i | \Phi_z, \mathbf{M}_{m-1} \cdots \Phi_z, \mathbf{M}_{i-1} | \Phi_z, \mathbf{N}_m \\ &\quad \cdots \Phi_z, \mathbf{N}_i | \Phi_z, \mathbf{B}_m \cdots \Phi_z, \mathbf{B}_j | \Phi_z, \mathbf{M}_{n-1} \\ &\quad \cdots \Phi_z, \mathbf{M}_{j-1} | \Phi_z, \mathbf{N}_n \cdots \Phi_z, \mathbf{N}_j | \Phi_{s'} | \Phi_{\xi'}] \end{aligned} \quad (27)$$

and

$$\begin{aligned} \delta \mathbf{q} &= [\delta \mathbf{q}_m \cdots \delta \mathbf{q}_i | \delta \mathbf{s}'_{m-1,m} \\ &\quad \cdots \delta \mathbf{s}'_{i-1,i} | \delta \mathbf{\xi}_{m-1,m} \cdots \delta \mathbf{\xi}_{i-1,i} | \delta \mathbf{q}_n \cdots \delta \mathbf{q}_j | \delta \mathbf{s}'_{n-1,n} \\ &\quad \cdots \delta \mathbf{s}'_{j-1,j} | \delta \mathbf{\xi}_{n-1,n} \cdots \delta \mathbf{\xi}_{j-1,j} | \delta \mathbf{s}^{ij} | \delta \mathbf{\xi}^{ij}]^T \end{aligned} \quad (28)$$

The goal is to develop an automated design environment. The constraint violations due to a design

Fig. 7 (a) A four-bar linkage, (b) The linkage after a design variation and allowing only joint variables to change

variation can be automatically compensated for using one of six methods.

(1) Change in the generalized coordinates $\mathbf{q}$

The aim of this method is to develop a formulation that will allow a design variation to be absorbed by a change in the generalized coordinates. Therefore, an assembled configuration of the mechanism is determined by seeking solutions to the following equation. Note, however, that a discussion for determining such solutions will be addressed in the following section.

$$\Phi_{z_1}^{cut} \mathbf{B}_1 \delta \mathbf{q}_1 + \Phi_{z_2}^{cut} \mathbf{B}_2 \delta \mathbf{q}_2 + \Phi_{z_3}^{cut} \mathbf{B}_3 \delta \mathbf{q}_3 = -\Phi^{cut} \quad (29)$$

This case can be illustrated with the example in Fig. 7(a). Consider the design of a mechanical part modeled as a four bar linkage with sides k , l , m , and n , and angles q_1 , q_2 , q_3 , and q_4 . A design variation is specified to change length of link 1 from l to $l + \delta l$. Using Eq. (29), a computed assembled configuration is determined as shown in Fig. 7(b). Note that the original part has maintained the other three lengths, but has only changed the angles between the links.

(2) Change in the joint-position vectors $\mathbf{s}_{ij}$. In this case, the iterative computation of the following equation will compute an updated set of joint-position vectors. The variation with respect to the $\mathbf{s}$ -vectors is

$$\begin{aligned} \Phi_{z_1}^{cut} \mathbf{M}_0 \delta \mathbf{s}'_{01} + \Phi_{z_2}^{cut} \mathbf{M}_0 \delta \mathbf{s}'_{03} + \Phi_{z_3}^{cut} \mathbf{M}_3 \delta \mathbf{s}'_{32} \\ + \Phi_{s'_{12}}^{cut} \delta \mathbf{s}'_{12} + \Phi_{s'_{21}}^{cut} \delta \mathbf{s}'_{21} = -\Phi^{cut} \end{aligned} \quad (30)$$

where

$$\mathbf{M}_0 = [\mathbf{A}_0 \ \mathbf{0}]^T \text{ and } \mathbf{M}_3 = [\mathbf{A}_3 \ \mathbf{0}]^T.$$

(3) Change in both the generalized coordinates $\mathbf{q}$ and joint-position vectors $\mathbf{s}_{ij}$ by iteratively computing

$$\begin{aligned} \Phi_{z_1}^{cut} \mathbf{B}_1 \delta \mathbf{q}_1 + \Phi_{z_1}^{cut} \mathbf{M}_0 \delta \mathbf{s}'_{01} + \Phi_{z_2}^{cut} \mathbf{B}_2 \delta \mathbf{q}_2 + \Phi_{z_2}^{cut} \mathbf{M}_0 \delta \mathbf{s}'_{03} + \Phi_{z_3}^{cut} \mathbf{M}_3 \delta \mathbf{s}'_{32} + \Phi_{s'_{12}}^{cut} \delta \mathbf{s}'_{12} \\ + \Phi_{s'_{21}}^{cut} \delta \mathbf{s}'_{21} = -\Phi^{cut} \end{aligned} \quad (31)$$

Similarly, the analysis could be stated due to a variation in the orientation of the links as opposed to their length. Consider a design variation in the form of a change in the vectors of Euler parameters ($\mathbf{p}_{01}$, $\mathbf{p}_{03}$, and $\mathbf{p}_{32}$), which leads to three other methods for automatic design, namely,

(4) Change in the generalized coordinates $\mathbf{q}$ (Eq.

Fig. 8 Block diagram of the proposed method

29).

(5) Change in the generalized coordinates $\mathbf{q}$ and joint-position vectors $\mathbf{s}'_{ij}$ (Eq. 30).

(6) Change in the generalized coordinates $\mathbf{q}$, joint-position vectors $\mathbf{s}'_{ij}$, and the remaining vectors of Euler parameters $\mathbf{p}_{ij}$, such that

$$\begin{aligned} & \Phi_{z_1}^{cut} \mathbf{B}_1 \delta \mathbf{q}_1 + \Phi_{z_1}^{cut} \mathbf{M}_0 \delta \mathbf{s}'_{01} + \Phi_{z_1}^{cut} \mathbf{N}_1 (2 \mathbf{A}_0 \mathbf{E}_1 \delta \mathbf{p}_{01}) \\ & + \Phi_{z_2}^{cut} \mathbf{B}_2 \delta \mathbf{q}_2 + \Phi_{z_2}^{cut} \mathbf{B}_3 \delta \mathbf{q}_3 + \Phi_{z_2}^{cut} \mathbf{M}_0 \delta \mathbf{s}'_{03} \\ & + \Phi_{z_3}^{cut} \mathbf{M}_3 \delta \mathbf{s}'_{32} + \Phi_{z_3}^{cut} \mathbf{N}_3 (2 \mathbf{A}_0 \mathbf{E}_3 \delta \mathbf{p}_{03}) \\ & + \Phi_{z_3}^{cut} \mathbf{N}_2 (2 \mathbf{A}_3 \mathbf{E}_2 \delta \mathbf{p}_{32}) + \Phi_{s'_{12}}^{cut} \delta \mathbf{s}'_{12} + \Phi_{s'_{21}}^{cut} \delta \mathbf{s}'_{21} \\ & = -\Phi^{cut} \end{aligned} \quad (32)$$

where $\mathbf{N}_1 = \begin{bmatrix} \tilde{\mathbf{r}}_1 \\ \mathbf{I} \end{bmatrix}$, $\mathbf{N}_2 = \begin{bmatrix} \tilde{\mathbf{r}}_2 \\ \mathbf{I} \end{bmatrix}$, $\mathbf{N}_3 = \begin{bmatrix} \tilde{\mathbf{r}}_3 - \tilde{\mathbf{d}}_{03} \\ \mathbf{I} \end{bmatrix}$ and $\mathbf{d}_{03} = \mathbf{A}_0 \mathbf{C}_{03} [0 \ 0 \ 1]^T \mathbf{q}_4$. In this case, a complete redesign of the part can be expected. A block diagram illustrating the procedure for implementing the proposed method of automatic design in a CAD environment is shown in Fig. 8. It is important to note that the identification of constraints and the choice of variation method are currently specified by the designer but can be automated in future work.

5. Moore-Penrose Solution

For a given initial estimate that does not satisfy the constraint function Φ (Eqs. (29)–(32)), the Moore-Penrose pseudo inverse is used to obtain an initial assembly, i.e., the set of linear equations written as

$$\Phi_{\mathbf{q}}(\mathbf{q}^{(k)}) \delta \mathbf{q}^{(k)} = -\Phi(\mathbf{q}^{(k)}) \quad (33)$$

are solved simultaneously for a correction $\delta \mathbf{q}^{(k)}$. Since the Jacobian of $\Phi(\mathbf{q})$ is not square, the problem of obtaining an assembled configuration can be solved using the Moore-Penrose pseudo inverse in an iterative algorithm (Abdel-Malek and Yeh 1996⁽²⁾). Starting with an initial guess $\mathbf{q}^1$, the new generalized coordinates are calculated by evaluating

$$\Delta \mathbf{q} = \Phi_{\mathbf{q}}^* (-\Phi) \quad (34)$$

where $\Phi_{\mathbf{q}}^*$ is the Moore-Penrose pseudo inverse of the Jacobian $\Phi_{\mathbf{q}} = [\partial \Phi_i / \partial q_j]$, defined by

$$\Phi_{\mathbf{q}}^* = \Phi_{\mathbf{q}} (\Phi_{\mathbf{q}} \Phi_{\mathbf{q}}^*)^{-1} \quad (35)$$

The new set of generalized coordinates is computed as $\mathbf{q}^{(k+1)} = \mathbf{q}^k + \Delta \mathbf{q}^k$. This method converges to an assembled configuration $\mathbf{q}^*$ within a few iterations because of its quadratic rate of convergence.

Fig. 9 (a) A mechanical part, (b) Modeling of the part

6. A Spatial 4-Bar Linkage Example

As a first example, consider the mechanical part shown in Fig. 9(a). A design variation in the length connecting points A and C will be introduced, and accordingly, some constraints will be specified. The requirements are to maintain the perpendicularity of the planes defined by ABFE and BCGF and to maintain the lengths EA, AC, and CG as constants, therefore, these lengths are modeled as links. In order to maintain movement of points A and C in their planes, revolute joints (J_1 and J_4) are introduced at points G and E, respectively. At points A and C, either universal or spherical joints are specified to insure the free movement of the two points. A universal joint J_2 is specified at C and a spherical joint J_3 is specified at A. The constraints are modeled in Fig. 9(b). The length AC, is modeled as link 2 and will undergo a design variation from length n to $n + \partial n$. As a result of this variation, warping in the upper face will take effect.

To form a spanning tree, J_3 is cut yielding two bodies that do not remain connected after cutting a joint. Since the spherical joint is cut, a spherical constraint equation is introduced

$$\Phi^{sph} = \mathbf{r}_3 + \mathbf{s}_{32} - \mathbf{r}_2 - \mathbf{s}_{23} = 0 \quad (36)$$

where the system has four generalized coordinates and three equations. The Jacobian matrix $\Phi_{\mathbf{q}}^{sph}$ in joint coordinate space transformed from the Cartesian space is

$$\Phi_{\mathbf{q}}^{sph} = [\Phi_{z_2}^{sph} \mathbf{B}_1 \ \Phi_{z_2}^{sph} \mathbf{B}_2 \ \Phi_{z_2}^{sph} \mathbf{B}_3] \quad (37)$$

The Jacobian matrix in the Cartesian space for the spherical joint, is given as

$$\Phi_{z_2}^{sph} = [-\mathbf{I} \ \tilde{\mathbf{r}}_2 + \tilde{\mathbf{s}}_{23}] \quad (38)$$

and

$$\Phi_{z_2}^{sph} = -\Phi_{z_2}^{sph} \quad (39)$$

where, $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$ are velocity transformation matrices for the revolute joint, universal joint, and revolute joint, respectively, such that

$$\mathbf{B}_1 = \begin{bmatrix} \tilde{\mathbf{r}}_1 \mathbf{h}_{41} \\ \mathbf{h}_{41} \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} \tilde{\mathbf{r}}_2 \mathbf{h}_{12} & \tilde{\mathbf{r}}_2 \mathbf{g}_{12} \\ \mathbf{h}_{12} & \mathbf{g}_{12} \end{bmatrix} \quad \mathbf{B}_3 = \begin{bmatrix} \tilde{\mathbf{r}}_3 \mathbf{h}_{43} \\ \mathbf{h}_{43} \end{bmatrix} \quad (40)$$

where and $\mathbf{h}_{41}$ are $\mathbf{h}_{43}$ unit vectors along the rotational joint axes in the global reference frame which can be

Table 1 Simulation results due to changing s'_{23}

1 variables	2 initial assembly	3 q	4 s'_{ij}	5 q, s'_{ij}
q_1	0.0764	28.9745		15.3243
q_2	179.8572	142.6831		160.2042
q_3	227.6018	227.4333		227.5725
q_4	208.1882	175.4225		190.5807
s'_{41}			$\begin{bmatrix} -0.3379 \\ -0.3831 \\ 0.2039 \end{bmatrix}$	$\begin{bmatrix} -0.1102 \\ -0.1948 \\ 0.1181 \end{bmatrix}$
s'_{43}			$\begin{bmatrix} -3.6621 \\ -8.1169 \\ -0.2039 \end{bmatrix}$	$\begin{bmatrix} -3.8898 \\ -8.3052 \\ -0.1181 \end{bmatrix}$
s'_{12}			$\begin{bmatrix} 2.2044 \\ 0.3828 \\ -0.3379 \end{bmatrix}$	$\begin{bmatrix} 2.1175 \\ 0.1947 \\ -0.1102 \end{bmatrix}$
s'_{23}		$[0 \ -10 \ 0]^T$	$[0 \ -10 \ 0]^T$	$[0 \ -10 \ 0]^T$
s'_{32}			$\begin{bmatrix} -0.2015 \\ -7.0607 \\ -0.3831 \end{bmatrix}$	$\begin{bmatrix} -0.0416 \\ -7.2443 \\ -0.1948 \end{bmatrix}$
P_1				
P_3				
P_2				
Φ^{sph} norm	1.2547e-05	7.3779e-05	2.1756e-15	6.1485e-07
Iteration number	1	4	2	4

expressed by $h_{41} = A_4 C_{41} [0 \ 0 \ 1]^T$ and $h_{43} = A_d C_{43} [0 \ 0 \ 1]^T$, where

$$C_{41} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } C_{43} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

where h_{12} and g_{12} are two unit vectors along the rotational axes z'_{12} and y'_{12} in the global reference frame defined as $h_{12} = A_1 C_{12} [0 \ 0 \ 1]^T$, where

$$C_{12} = \begin{bmatrix} 0.9293 & 0.3693 & 0 \\ -0.2768 & 0.6967 & 0.6618 \\ 0.2444 & -0.6150 & 0.7497 \end{bmatrix}$$

$$g_{12} = A_1 C_{12} A''_{121}(q_2) [0 \ 1 \ 0]^T$$

and the rotation matrix A''_{121} is written as

$$A''_{121}(q_2) = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the variational generalized coordinates are $\delta q = [\delta q_1 \ \delta q_2 \ \delta q_3 \ \delta q_4]^T$. For an initial setup (column 1 of Table 1), and for a design variation in the joint-position vectors (s'_{ij} , assembled configurations can be computed according to methods 1 to 3 above and entered into columns 3 through 5 of Table 1.

An illustration of the change in variables is shown in Fig. 10 where the part is updated and shown. Note that a deformation in the upper plane of the object has been effected as a result of the motion caused by the two spatial joints (J_2 and J_3).

7. A Spatial Slider-Crank Example

Consider the mechanical part shown in Fig. 11 (a). A design change is to occur in the vector of Euler parameters p_{01} associated with the transformation matrix C_{01} to illustrate the variational methods 4 through 6 above. The part is modeled by introducing a sliding joint J_1 , a universal joint J_2 , a spherical joint

Fig. 10 Updated part due to a design variation and allowing the generalized variables to change

Fig. 11 (a) A mechanical part, (b) Kinematic modeling of the part

J_3 , and a revolute joint J_4 as shown in Fig. 11(b).

Since the spherical joint was cut, a constraint function is introduced such that

$$\Phi^{sph} = r_2 + s_{21} - r_1 - s_{12} = 0 \quad (41)$$

where s_{12} is the spherical joint-position vector in the reference frame of link 1; and s_{21} is the spherical joint-position vector in the reference frame of link 2. The Jacobian matrix of the spherical joint in the reference frame of link 1, written in Cartesian coordinate space is

$$\Phi^{sph}_{z_1} = [-I \ \tilde{r}_1 + \tilde{s}_{12}] \quad (42)$$

In order to transform the Jacobian matrix from Cartesian coordinate space to joint coordinate space, the velocity transformation matrices are used such that

$$\Phi^{sph}_q = [\Phi^{sph}_{z_1} B_{rev} \ \Phi^{sph}_{z_2} B_{std} \ \Phi^{sph}_{z_3} B_{uni}] \quad (43)$$

where the velocity transformation matrix of the revolute joint is $B_{rev} = [\tilde{r}_1 h_{01} \ h_{01}]^T$, and the unit vector h_{01} along the rotational joint axis z'_{01} in the global reference frame is $h_{01} = A_0 C_{01} [0 \ 0 \ 1]^T$.

Using the methods 4 through 6 above, the iterative algorithm is employed until constraint violations are satisfied and the results are entered into columns 2 through 4 of Table 2, where p_{03} is the vector of Euler parameters associated with C_{03} (transformation matrix from translational joint coordinates to ground reference frame); p_{32} is the vector of Euler parameters associated with C_{32} (transformation matrix from universal joint coordinates to the reference frame of

Table 2 Simulation results due to changing p_{01}

Variable	2	3	4
s	q	q, s_e	q, s_e and p_y
s_{01}		$[-0.107402 \ 0.232435 \ -0.083248]^T$	$[-0.0492192 \ 0.235848 \ -0.108398]^T$
s_{02}		$[0.107402 \ 3.63126 \ 0.083248]^T$	$[0.0492192 \ 3.62785 \ 0.108398]^T$
s_{11}		$[0.083248 \ 0.107402 \ -0.232435]^T$	$[0.117225 \ 0.0352158 \ -0.23789]^T$
s_{12}		$[1.86492 \ -0.0174832 \ 0.232441]^T$	$[1.88786 \ 0.0452454 \ 0.235835]^T$
s_{21}		$[-3.07082 \ -0.0408527 \ 0.099306]^T$	$[-3.09467 \ -0.00849357 \ 0.0165346]^T$
q_1	67.089	42.943	50.304
q_{12}, q_1	-21.167, 40.654	-23.451, 15.872	-28.673, 2.560
q_4	-1.86406	-0.232435	-0.23789
P_{01}			$[0.5 \ -0.5 \ -0.5 \ 0.5]^T$
P_{02}			$[0.582164 \ -0.458567 \ -0.497332 \ -0.451069]^T$
P_{12}			$[0.581125 \ -0.459798 \ -0.458758 \ -0.490327]^T$
Φ	1.0419e-06	0.000602553	3.4653e-05
Iteration	3	2	3

link 3).

To illustrate the numerical results of methods 4, an assembled mechanism is shown in Fig. 12(a) and the updated part is shown in Fig. 12(b). Indeed, Fig. 12(b) depicts the original position of the links (i.e., initial configuration of the mechanism) and the final configuration where the deformation has changed the part while maintaining the integrity of the mechanism. The concept of automated design by converting the machine part to a mechanism has been validated. In principle, we believe this approach to design will lead to improved intelligence of modern CAD systems because of the flexibility inherent in the system. Moreover, the field of kinematics is well established and therefore, algorithms needed to represent any spatial mechanism can be developed.

8. Conclusions

An analytical formulation for automated design of mechanical parts and assemblies in a computer-aided design environment has been presented. The formulation presented in this paper and illustrated by numerical examples demonstrates the validity of a general-purpose formulation and experimental computer code for the computation of design parameters after introducing a variation in the original design. Although in its introductory stages, the method and experimental code are the basis of a new generation of CAD systems poised to achieve the goal of automatic design.

It was shown that this formulation is derived with respect to design parameters to obtain design propagations. These constraints are derived keeping the joint-position vectors and orientation matrices (represented by Euler parameters) as variables. Variations of these vectors and matrices were also developed. It was shown that the Jacobian matrix in Cartesian space is transformed to joint coordinate space. The use of relative generalized coordinates in a recursive formulation has simplified obtaining the

Fig. 12 (e) The mechanism after a design variation, (f) The new computed part

differential variations. Because there are more variables than equations, the Moore-Penrose pseudo inverse method is implemented to compute an assembled configuration, hence redesigning the part.

Results of our numerical algorithms have been presented using two spatial examples. While the concept presented herein presents the first treatment of a systematic approach to automated CAD, we state some of the difficulties encountered during the development of this method. In order to have a masked operation of the code invisible to the user, it is important to develop a user interface that converts CAD data to kinematic constraints. While this aspect of the research is clear and can be readily developed, this team of researchers has not yet developed such an interface. Nevertheless, we have a manual systematic approach to converting 3D objects into a mechanism whereby constraints are located and links are defined. Another difficulty that is currently under investigation is the capability to solve the complete set of kinematic constraints simultaneously. While we have introduced the Moore-Penrose inverse as an efficient method, the robustness of the method has not yet been compared with other solvers. However, this does not hinder any of our rigorous formulation for obtaining a suitable solution to the problem.

The method has been shown to be applicable for solids represented in a 3-dimensional CAD environment.

Acknowledgments

This research was funded by the US Army Tank Automotive Research Center (TACOM) through the Automotive Research Center (Department of Defense contract number DAAE07-94-C-R094).

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