Multiobjective Optimal Design of Cylindrical Gear Pairs for the Reduction of Gear Size and Meshing Vibration*

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In the design of gear pairs, the designer has to make design trade-off between the geometrical volume and the meshing vibration. It is generally known from experience that the two objectives show conflicting tendency. In this paper, the authors demonstrate the analysis results of the relation between the geometrical volume and the vibration of a gear pair, and propose a design method for cylindrical gear pairs to balance the conflicting objectives by using a goal programming formulation. The design method reduces both the geometrical volume and the meshing vibration of cylindrical gear pairs while satisfying strength and geometric constraints. The validity of the method will be shown through the design results of the four cases of design speeds and the redesign of a helical gear pair currently being used in elevator reduction drive.

Key Words: Gear, Cylindrical Gear Design, Gear Size (Geometrical Volume), Meshing Vibration, Multiobjective Optimization, Goal Programming Method

1. Introduction

Recently, there have been many researches on the design of gear pairs for small size and low vibration^{(1),(2)}. Most of them, however, have treated gear design problems as single objective optimization to reduce the size (geometrical volume) or the vibration (noise) alone, of course, under the constraint condition of keeping gear strength and/or durability. While these studies have provided us an effective methodology for designing gear pairs, they do not provide for balancing the conflicting tendency among the various performance factors. Typically, the vibration level of a gear pair increases with decreasing gear size, and vice versa. The optimal design cannot be ensured if only a single performance factor, such as the size of a gear pair, is considered as an objective

while others - vibration, in this case - are not considered or simply included as constraints. In case that we want to design a gear pair for reducing both the size and the vibration, it is still very difficult to find the appropriate values for design variables compromising the objectives. This causes from the complicated nature of the effect of the design variables on the size and the vibration. Moreover, the correlation between them is not apparent to the designer. Multiobjective optimization techniques are effective in solving problems involving the conflicting objectives as described above⁽³⁾⁻⁽⁷⁾.

In this paper, the authors demonstrate the conflicting tendency between the size (geometrical volume) and the vibration of a gear pair, and propose a goal programming formulation for cylindrical gear pairs to effectively balance the conflicting objectives. The goal programming method introduced in this paper is one of the most popular and widely used multiobjective optimization techniques^{(6),(7)}. The geometrical volume and the vibrational excitation force have been taken as the objective functions for single objective optimization in the goal programming procedure with geometric and strength constraints. The

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optimal solutions of the single objective optimization is used as the target values for the goal programming method.

In order to validate the proposed design method, the design results are obtained for the four cases of design speeds of the gear pair and the redesign of a helical gear pair currently being used in elevator reduction drive.

Nomenclature

- a : center distance, mm
 b : face width, mm
 C_H : hardness ratio factor for pitting resistance
 C_L : life factor for pitting resistance
 C_R : reliability factor for pitting resistance
 C_T : temperature factor for pitting resistance
 d_a : tip circle diameter, mm
 d_b : base circle diameter, mm
 E_V : total vibrational excitation force, N/mm
 J : geometry factor for bending strength
 K_L : life factor for bending strength
 K_T : temperature factor for bending strength
 K_R : reliability factor for bending strength
 m_n : normal module, mm
 m_t : transverse module, mm
 r_a : tip circle radius, mm
 r_b : base circle radius, mm
 r_w : operating pitch radius, mm
 t_a : circular tooth thickness at tooth tip, mm
 T_f : flash temperature, °C
 T_{lim} : allowable flash temperature, °C
 u : gear ratio
 V : volume of virtual cylinder, mm³
 x_a : addendum modification coefficient
 y : center distance modification coefficient
 z : number of teeth
 α_n : normal pressure angle, deg.
 α_t : transverse pressure angle, deg.
 α_{wt} : operating transverse pressure angle, deg.
 β : helix angle, deg.
 ε_α : transverse contact ratio
 ε_β : overlap contact ratio
 σ_F : bending stress, MPa
 $\sigma_{F\ lim}$: allowable bending stress, MPa
 σ_H : contact stress, MPa
 $\sigma_{H\ lim}$: allowable contact stress, MPa
 EG: existing gear pair
 VEFM: vibrational excitation force minimization
 VM: volume minimization
 GPM: VM & VEFM using goal programming method
 Subscripts
 1: pinion
 2: gear

2. Design by Goal Programming Method

Conventional single objective optimization techniques do not provide for multiobjective conditions and have been used to optimize only a single objective subject to a set of constraints. The first work on the formulation of the multiobjective optimization problem is that of Kuhn and Tucker⁽³⁾. Since then, various works and researches on the multiobjective optimization have been carried out⁽⁴⁾⁻⁽⁷⁾. However, its application to engineering problems is relatively recent, especially in the field of mechanical design. The goal programming method is one of the most popular multiobjective optimization techniques for solving engineering problems. The details of the method can be found in many papers and texts, such as that of Ignizio⁽⁶⁾. The solution procedure of the goal programming method can be summarized as follows;

1) Construct single objective optimization formulas using objective functions ($f_i(\mathbf{X})$) and constraints ($g_j(\mathbf{X}), h_k(\mathbf{X})$) for design variable $\mathbf{X}$, as shown in Eq.(1).

$$\left. \begin{array}{l} \text{Find } \mathbf{X}=(x_1, x_2, \dots, x_{nv}) \\ \text{to minimize } f_i(\mathbf{X}) \\ \text{subject to } g_j(\mathbf{X}) \leq 0, \quad j=1, 2, \dots, l \\ h_k(\mathbf{X})=0, \quad k=1, 2, \dots, m \end{array} \right\} \quad (1)$$

where, nv , l and m are the numbers of design variables, inequality, and equality constraints, respectively.

2) Determine optimum values of respective objective functions (goals, f_i^*) and the corresponding values of design variables using Eq.(1).

3) Construct a new single objective function ($F(\mathbf{X})$) to include the objective functions using respective goals and constraints, as shown in Eq.(2).

$$\left. \begin{array}{l} \text{Minimize } F(\mathbf{X})=\left[\sum_{i=1}^{no} \left(\frac{f_i(\mathbf{X})-f_i^*}{f_i^*} \right)^p \right]^{\frac{1}{p}} \\ \text{subject to } g_j(\mathbf{X}) \leq 0, \quad j=1, 2, \dots, l \\ h_k(\mathbf{X})=0, \quad k=1, 2, \dots, m \end{array} \right\} \quad (2)$$

where, no is the number of objective functions and p is an arbitrary index which is generally 1 or 2.

4) Perform single objective optimization to obtain a compromise solution using Eq.(2).

3. Design Formulation for Cylindrical Gear Pairs

The design formulation of the objective functions has been presented for reducing the size (geometrical volume) and the meshing vibration of cylindrical gear pairs (spur and helical gears). Generally, gear vibration can be decreased with reducing vibrational excitation force. Thus, the size and the total vibrational excitation force have been taken as objective functions.

Fig. 1 Virtual cylinder for size optimization of a gear pair

Fig. 2 Vibration analysis model of a gear pair

3.1 Objective functions

3.1.1 Virtual cylinder for size optimization

A virtual cylinder has been introduced in order to minimize the size of a cylindrical gear pair. The volume V of the cylinder is simply represented as Eq. (3) with its diameter being center distance a and its height being face width b , as shown in Fig. 1.

$$V = \pi \frac{a^2 b}{4} = f_1(X) \quad (3)$$

where, the center distance $a = \left(\frac{z_1 + z_2}{2 \cos \beta} + y \right) m_n$.

There would be several types of objective functions for the minimization of a gear pair, and Eq. (3) uses the center distance to include addendum modification coefficients in the design variables set.

3.1.2 Total vibrational excitation force for vibration optimization Figure 2 shows the vibration analysis model of a gear pair.

Considering the motion along the line of action of a gear pair, the rotational vibration can be analyzed from the vibration system of single degree of freedom⁽⁸⁾⁻⁽¹⁰⁾. The vibration analysis model is shown in Eq. (4).

$$M\ddot{\delta} + 2\zeta\sqrt{Mk_i}\dot{\delta} + \sum k_i\delta = W + \sum k_ie_i \quad (4)$$

In Eq. (4) and Fig. 2, $M = M_1M_2/(M_1 + M_2)$ is the equivalent mass of a gear pair converted along the line of action, where $M_1 = I_1/r_{b1}^2$, $M_2 = I_2/r_{b2}^2$, and I is the mass moment of inertia. The relative displacement δ is given as $\delta = \delta_1 - \delta_2$, where $\delta_1 = r_{b1}\theta_1$, $\delta_2 = r_{b2}\theta_2$ with the rotational angle θ . W is the transmitted

normal load and ζ is the damping coefficient. The spring constant (tooth stiffness) and the composite error of the i th tooth pair are k_i and e_i , respectively.

If we let K as the average value of $\sum k_i$ for time t , and $\delta_s = W/K$, $\xi = \delta/\delta_s$, $\omega_e^2 = K/M$, $\tau = t\omega_e$, $\chi(\tau) = \sum k_i/K$, $\phi(\tau) = \sum k_ie_i/W$, $\chi^* = \chi(\tau) - 1$, $\phi^* = \phi(\tau) - \bar{\phi}$, $\xi^* = \xi - 1 - \bar{\phi}$, and $\bar{\phi}$ as the average value of $\phi(\tau)$ for time, then Eq. (4) can be converted into Eq. (5).

$$\ddot{\xi}^* + 2\zeta\sqrt{\chi(\tau)}\dot{\xi}^* + \chi(\tau)\xi^* = \phi^* - \chi^*(1 + \bar{\phi}) \quad (5)$$

Since the right-hand side of Eq. (5) is a periodic function with the average value of zero and $\chi(\tau)$ has the average value of unity, the solution of Eq. (5) can be approximated to that of Eq. (6)^{(8),(10)}.

$$\ddot{\xi}^* + 2\zeta\dot{\xi}^* + \xi^* = \phi^* - \chi^*(1 + \bar{\phi}) \quad (6)$$

The right-hand side of Eq. (6) represents the vibrational excitation force by the change of tooth stiffness due to gear error and tooth meshing. Thus, the total vibrational excitation force E_v in one period can be given in Eq. (7).

$$E_v = (W/b)(K_0/K)\{\phi^* - \chi^*(1 + \bar{\phi})\} = f_2(X) \quad (7)$$

where, K and K_0 are the average values of the total composite tooth stiffness in transverse plane of the gear pair with and without error, respectively. In this paper, the gear teeth are considered to have ideal tooth profile with no manufacturing or alignment error, i.e. $K_0/K = 1$. Vibrational excitation force is proportional to the vibration acceleration, and minimizing the vibrational excitation force can reduce vibration level of a gear pair^{(8),(10)}. Thus, the total vibrational excitation force E_v in Eq. (7) has been considered as an objective function to minimize the vibration of a gear pair.

3.2 Formulation of optimization problem

The optimization problem has been formulated as Eq. (8).

$$\begin{aligned} \text{Minimize } F(X) &= \left[\left(\frac{f_1(X) - f_1^*}{f_1^*} \right)^2 + \left(\frac{f_2(X) - f_2^*}{f_2^*} \right)^2 \right]^{\frac{1}{2}} \\ \text{subject to } g_1: \sigma_F &\leq \sigma_{F \lim} \frac{K_L}{K_T K_R} \\ g_2: \sigma_H &\leq \sigma_{H \lim} \frac{C_L C_H}{C_T C_R} \\ g_3: T_f &\leq T_{\lim} \\ g_4: t_{a1} &\geq 0.4 m_n \\ g_5: t_{a2} &\geq 0.4 m_n \\ g_6: 1.2 &\leq \varepsilon_a \leq 2.0 \\ g_7: 0.0 &\leq \varepsilon_\beta \leq 3.0 \\ g_8: r_{a1}^2 &\leq r_{b1}^2 + (r_{w1} + r_{w2})^2 \sin^2 \alpha_{wt} \\ g_9: r_{a2}^2 &\leq r_{b2}^2 + (r_{w1} + r_{w2})^2 \sin^2 \alpha_{wt} \\ h_1: J_1/J_2 &= \sigma_{F1}/\sigma_{F2} \\ X_{\text{lower}} &\leq X \leq X_{\text{upper}} \end{aligned} \quad (8)$$

where, the objective functions f_1 and f_2 are the volume V and the total vibrational excitation force E_v , respectively. A value of 2 has been used for the index

p in Eq. (2). The constraints $g_1 \sim g_8$ have been included to assure strength and durability of a gear; g_1 and g_2 are rating formulas for bending strength and pitting resistance by AGMA standards⁽¹¹⁾, respectively. The constraint g_3 has been used to avoid scoring of gear teeth by using Dudley's flash temperature formula⁽¹¹⁾. The constraints g_4 and g_5 are for avoiding pointed tooth tip of pinion and gear, respectively. The constraints g_6 and g_7 have been used for upper and lower bounds of transverse and overlap contact ratios, respectively. The constraints g_8 and g_9 have been included to avoid interference between mating teeth of pinion and gear. The equality constraint h_1 is that for determining addendum modification coefficients of pinion and gear by balancing bending strength of mating teeth. The addendum modification coefficients are determined automatically by equalizing the ratios of bending strengths and geometry factors between pinion and gear.

4. Design Results and Discussions

4.1 Analysis of conflicting tendency between the objective functions

In order to confirm the fact that the objective functions conflict with each other, the values of the geometrical volume and the total vibrational excitation force of a gear pair have been analyzed with the change of design variables. Table 1 shows the input specifications of a helical gear pair for the conflict analysis.

The two of the analysis results are shown in Figs. 3(a) and (b) for the design variables of helix angle β , and the sum of addendum modification coefficients of pinion and gear $x_{a1} + x_{a2}$. In the analysis, other design variables except the selected variable (β or $x_{a1} + x_{a2}$) are fixed to constant values, and the constraints in Eq. (8) are not considered.

In Fig. 3, the design variables are plotted along in the horizontal axes, and the objective functions E_v and V are presented in the left and the right side of the axes of the ordinates, respectively. In Fig. 3(a), the increase of helix angle β reduces the total vibrational excitation force E_v , but apparently increases the volume V . This tendency can also be shown in Fig. 3(b). E_v decreases, but V increases in keeping with the increase of the sum of addendum modification coefficients $x_{a1} + x_{a2}$. From these results,

Table 1 Specifications for conflict analysis

Transmitted power	[kW]	30.0
Gear ratio		2.0
Operating speed (input shaft)	[rpm]	2000.0

it can be clearly confirmed that the size (geometrical volume) and the vibrational performance have the conflicting tendency.

4.2 Compromise solutions of optimization

The vibration acceleration of a gear pair increases at the resonance speed and at the $1/2, 1/3, \dots$, of it. In some cases, the acceleration increases at the $3/4$ according to the dimension of a gear pair. The designers are also concerned about the vibrational performance at the operating speed of the gear pair. Thus, design practices have been carried out at the resonance speed, $3/4$ and $1/2$ of the resonance speed, and the operating speed (2000 rpm). The compromise solutions will be presented later in this section.

The resonance speed is calculated automatically in the optimization process. Pinion torque can be calculated from the transmitted power and the operating speed. Then, meshing stiffness and natural frequency are calculated using the pinion torque and the initial values of other design dimensions by the meshing analysis. The resonance speed is calculated from these values. During design iterations, the resonance speed is recalculated with the change of the gear dimensions supposing that pinion torque in every iteration of an optimization process has the same value ($143.32 \times 10^3 \text{ N}\cdot\text{mm}$). The design variables $\mathbf{X} = (x_1, x_2, \dots, x_{nv})$

include normal module m_n , number of teeth in pinion z_1 , face width b , pressure angle α_n , helix angle β , and addendum modification coefficients x_{a1} and x_{a2} .

Figure 4 shows the design optimization procedure including the goal programming optimization. In Fig. 4 and the following tables and figures, VM and VEFM represent the results of the single objective optimization practices, i.e. the volume minimization and the vibrational excitation force minimization, respectively. GPM represents the results of the multiobjective optimization by the goal programming method, i.e. the minimization of both the volume and the vibrational excitation force (VM and VEFM). As described in

(a) When helix angles are changed (b) When the sum of addendum modification coefficients are changed

Fig. 3 Conflict analysis between the volume $V [\text{mm}^3 \times 10^5]$ and the total vibrational excitation force $E_v [\text{N/mm}]$

Fig. 4 Solution procedure of the proposed design algorithm

Section 2, the single objective optimizations find the optima $f_1^*(X_{VM})$ and $f_2^*(X_{VEFM})$, which are used as the target values in the goal programming optimization. The resultant values of X_{VM} and X_{VEFM} by the optimization are arbitrary real values, which are not usable in practical design. Thus the values are changed to the usable ones X_{VM_design} and X_{VEFM_design} by using the branch and bound method. The integer and the discrete variables such as number of teeth, normal module and pressure angle are modified into the values applicable to the practical design. Normal module has a standard value recommended by the ISO⁽¹⁴⁾. Pressure angle has a value of 14.5°, 20°, 22.5° or 25°, which are generally used in practical design. Helix angle has been rounded off to the first decimal place for the sake of convenience, although it is often rounded off to the third or fourth decimal places in practical design. In consequence, the values of the objective functions $f_1^*(X_{VM})$ and $f_2^*(X_{VEFM})$ will show small difference with the values of those using X_{VM_design} and X_{VEFM_design} . However, the tendency of the design results does not change, and the consistency of the design algorithm can be affirmed. Similarly, the goal programming method finds an optimum $F^*(X_{GPM})$, and the values of the design variables X_{GPM} are changed to the usable values X_{GPM_design} .

In the VM, VEFM, and GPM, we have used sequential quadratic programming (SQP), which deals well with equality constraints as well as inequality

Table 2 Design results ; $[rpm]_{design} = 1 \times [rpm]_{resonance}$

X_{design}	VM	VEFM	GPM
m_n [mm]	1.5	1.5	1.5
z_1	23	28	25
α_n [deg.]	25.0	20.0	22.5
β [deg.]	16.0	25.0	20.0
b [mm]	26.0	33.3	28.2
x_{a1}	0.2926	0.2336	0.2480
x_{a2}	-0.1624	-0.2014	-0.2470
ε_α	1.3658	1.4565	1.4233
ε_β	1.5208	2.9864	2.0467
a [mm]	54.029	69.561	59.861
	$f_1^*(X_{VM})$	$f_2^*(X_{VEFM})$	$F^*(X_{GPM})$
V [mm ³]	59448.3	126388	79447.7
E_V [N/mm]	8.7848	0.4564	1.2291

Table 3 Design results ; $[rpm]_{design} = 3/4 \times [rpm]_{resonance}$

X_{design}	VM	VEFM	GPM
m_n [mm]	1.5	1.5	1.5
z_1	24	28	26
α_n [deg.]	25.0	20.0	25.0
β [deg.]	14.0	23.0	21.0
b [mm]	26.3	36.0	28.0
x_{a1}	0.2506	0.2469	0.2457
x_{a2}	-0.2486	-0.2462	-0.2456
ε_α	1.4040	1.4902	1.3429
ε_β	1.3502	2.9850	2.1293
a [mm]	55.656	68.442	62.662
	$f_1^*(X_{VM})$	$f_2^*(X_{VEFM})$	$F^*(X_{GPM})$
V [mm ³]	63973.6	132336	85619.8
E_V [N/mm]	8.9958	0.3227	1.2626

constraints. With the SQP, a search direction is created first and then a one-dimensional search is performed to improve the design as much as possible in this direction. The method of feasible direction has been used for finding the search direction, and the golden section method has been used for one-dimensional search⁽¹³⁾.

The design results at the resonance speed, 3/4, 1/2 of the resonance speed, and the operating speed are presented in Table 2 – Table 5, respectively. The design solutions of the volume V by VM and the total vibrational excitation force E_V by VEFM are used in GPM as the target values (goals) f_1^* and f_2^* . As mentioned earlier, $f_1^*(X_{VM})$, $f_2^*(X_{VEFM})$ and $F^*(X_{GPM})$ in the tables are the values of the objective functions calculated from the optimized design variables, while X_{design} are the modified values for practical design by using the branch and bound method.

The results of objective functions in Table 2 – Table 5 have been plotted in Fig. 5 – Fig. 8, which show the significant change of the values of objective

Table 4 Design results ; $[\text{rpm}]_{\text{design}} = 1/2 \times [\text{rpm}]_{\text{resonance}}$

X_{design}	VM	VEFM	GPM
m_n [mm]	2.0	2.0	2.0
z_1	26	32	28
α_n [deg.]	25.0	20.0	22.5
β [deg.]	15.0	20.0	20.0
b [mm]	27.5	40.2	34.2
x_{a1}	0.2196	0.2449	0.2499
x_{a2}	-0.2161	-0.2449	-0.2498
ε_α	1.4085	1.5559	1.4386
ε_β	1.1328	2.1883	1.8616
a [mm]	80.759	102.161	89.391
	$f_1^*(X_{\text{VM}})$	$f_2^*(X_{\text{VEFM}})$	$F^*(X_{\text{GPM}})$
V [mm^3]	138105	324771	214630
E_v [N/mm]	3.8206	0.5602	1.2727

Table 5 Design results ; $[\text{rpm}]_{\text{design}} = [\text{rpm}]_{\text{operating}}$

X_{design}	VM	VEFM	GPM
m_n [mm]	2.5	2.5	2.5
z_1	23	27	25
α_n [deg.]	25.0	20.0	22.5
β [deg.]	18.0	28.0	25.0
b [mm]	28.8	49.8	38.2
x_{a1}	0.0100	0.582	0.0082
x_{a2}	0.7496	0.75	0.75
ε_α	1.2949	1.2211	1.2872
ε_β	1.1331	2.9768	2.0555
a [mm]	92.515	117.774	105.264
	$f_1^*(X_{\text{VM}})$	$f_2^*(X_{\text{VEFM}})$	$F^*(X_{\text{GPM}})$
V [mm^3]	185976	527017	319475
E_v [N/mm]	2.5779	0.1081	0.4379

functions. The axes of abscissa of the figures represent the design methods, and the axes of the ordinates represent the objective functions. The values of the total vibrational excitation force E_v are shown along the left side of the axes of the ordinates, and the volume V along the right side.

Figures 5 - 8 demonstrate that, in case of the VEFM, the values of the total vibrational excitation force ($-\square-$) are very small but the volumes ($-\bullet-$) have considerably large values. On the other hand, the volumes decreased but the vibrational excitation force increased in case of the VM. However, the compromise solutions by using the GPM considerably decreased in V in comparison with those of the VEFM, although slightly increased in E_v . On the other hand, the solutions considerably decreased in E_v in comparison with those of the VM, although slightly increased in V . From these results, we may conclude that the GPM will be very useful to design gear pairs with conflicting objectives of minimizing size and vibration.

Fig. 5 Comparison of objective functions in Table 2

Fig. 6 Comparison of objective functions in Table 3

Fig. 7 Comparison of objective functions in Table 4

Fig. 8 Comparison of objective functions in Table 5

In the above results, the optimized gear dimensions are different with each other. The differences are occurred basically from the change of the rotational speed of gears used in the design procedure. In general, when the design procedure is carried out for constant transmitting power, transmitted torque and tooth load decrease with increasing rotational speed. The procedure to keep the gear strength optimum results the smaller gear dimensions, when higher

Table 6 Design specification of the existing helical gear pair

Transmitted power	[kW]	7.5
Gear ratio		6.7826
Operating speed (input shaft)	[rpm]	1130

Table 7 Design results of the newly designed gear pair

X_{design}	EG	VM	VEFM	GPM
m_n [mm]	2.0	2.0	2.0	2.0
z_1	23	21	23	22
α_n [deg.]	20.0	20.0	20.0	20.0
β [deg.]	25.0	18.0	30.0	26.0
b [mm]	35.0	22.0	35.3	28.8
x_{a1}	0.0	0.4140	0.4461	0.4053
x_{a2}	0.25	-0.3289	-0.3571	-0.3203
ε_α	1.4924	1.4992	1.3265	1.3980
ε_β	2.3542	1.0820	2.8091	2.0093
a [mm]	198.001	171.558	206.869	190.424
		$f_1^*(X_{\text{VM}})$	$f_2^*(X_{\text{VEFM}})$	$F^*(X_{\text{GPM}})$
V [mm ³]	1077685	413545	1067460	665873
E_v [N/mm]	0.4316	1.4537	0.0793	0.1708

rotational speed is specified by designing. In Tables 2 and 5, the level of the resonance speed is very larger than that of the operating speed, and thus the gear dimensions in Table 2 are smaller than those in Table 5. This tendency can also be recognized from the gear dimensions at the 3/4 and 1/2 of the resonance speed, which are shown in Table 3 and Table 4.

4.3 Redesign of the existing gear pair

The GPM has been applied to the redesign of a helical gear pair currently being used in an elevator reduction drive in order to obtain a compromise solution improving both the objectives. Table 6 shows the design specifications of the helical gear pair.

Table 7 and Fig. 9 show the design results by the single objectives (VM and VEFM) and the multiobjective optimizations (GPM). The EG of the design results represents the volume and the total vibrational excitation force of the existing helical gear pair for comparison with the redesign solutions.

In Fig. 9, it is clear that the VEFM has reduced the vibrational excitation force considerably, but the volume does not have a noticeable change compared to the result of the EG. Similarly, the VM reduced the volume, but the vibrational excitation force increased significantly than the result of the EG. In other words, the VEFM and the VM can reduce the respective single objectives obviously, but may give bad performance indexes concerning the other objectives. However, the GPM reduced both the volume and the total vibrational excitation force in comparison with the

Fig. 9 Comparison of objective functions in Table 7

result of the EG. As the results of Section 4.2, the volume slightly increased than that of the VM, but decreased appreciably than that of the VEFM. The total vibrational excitation force is almost the same as that of VEFM. The results of the design variables X_{design} are the modified values for practical design by using the branch and bound method.

5. Conclusions

The authors have proposed a multiobjective optimization formulation for cylindrical gear pairs using goal programming method to minimize both the gear size (geometrical volume) and the meshing vibration. The conclusions of this paper can be summarized as follows:

(1) It has been generally known from experience that the gear pairs with light weight may have noise and/or vibration problems. The authors have made it clear for the first time that there is a conflicting tendency between the vibrational excitation force by the tooth meshing and the gear size or the geometrical volume of a gear pair by analytic method.

(2) The design results of the single objective optimizations can minimize single objective function concerned — gear size or meshing vibration —, but may bring about bad influences on another performance factor having the conflicting tendency with the objective function. This fact confirms that the single objective optimization may not be suitable for the design of cylindrical gear pairs having multiple design objectives.

(3) The goal programming method has optimized both the gear size and the total vibrational excitation force considerably. Thus, the proposed method can be effectively used to balance the design objectives of the size and the vibration performance.

(4) The proposed method is quite general, and is expected to effectively solve many practical design problems that are required to obtain compromise solutions.

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