Design and Experimental Evaluation of a Self-Tuning Controller Supplementing a Robust PID Controller*

Ryosuke NAMBA**, Toru YAMAMOTO***
and Masahiro KANEDA****

PID controllers have been widely used in many chemical processes. Because they have only three control parameters and their physical meanings can be easily grasped. However, it is difficult to tune those parameters practically, since the process dynamics often change due to operating conditions or various disturbances. For this problem, a design method of robust PID controllers has been already proposed by the authors in order to guarantee the stability of the control system. However, as that control method is conservative, the desirable setpoint response can not be always obtained. In this paper, a design scheme of a self-tuning pre-filter is proposed to supplement the robust PID controller based on the two-degree-of-freedom control scheme. According to the proposed scheme, the transient property for the setpoint response can be improved keeping the robust stability. Finally, the proposed scheme is experimentally evaluated on an air pressure control system.

Key Words: PID Control, Robust Control, Self-Tuning Control, Two-Degree-of-Freedom Control, Process Control

1. Introduction

In recent years, by dint of surprising developments of computers, many advanced control theories which have complicated control structures, for example, the adaptive control theory⁽¹⁾⁻⁽⁴⁾, H_∞ control theory⁽⁵⁾⁻⁽⁸⁾ and so on, have been studied. On the other hand, PID control algorithms⁽⁹⁾⁻⁽¹²⁾ still continue to be widely used for most industrial control systems, particularly in the chemical process industry. It seems to be the reason that the PID controller is a ready made simple controller left only tuning its three parameters,

i.e., proportional, integral and derivative gains, which are familiar with most of control engineers and operators. Well, in most of real processes, it is difficult to grasp their dynamics accurately, and also they often change due to operating conditions or various disturbances. For this problem, a design scheme of discrete-time robust PID controllers⁽¹³⁾ has been proposed, whose PID parameters were tuned from the viewpoint of the robust stability⁽¹⁴⁾. According to that scheme, though the robust stability of the control system is indeed guaranteed, the setpoint response is not always desirable because the scheme lays emphasis on the only stability.

In the industrial process control systems, many control loops are usually performed as a so-called distributed control system (DCS). DCS is constructed by a lot of PID type lowest-level controllers and some computers which supply setpoint values to the lowest-level controllers. Therefore, though the controller directly working on the controlled object should be PID type, the setpoint value is allowed to be made up by fairly complex calculations.

In this paper, a design method of a self-tuning controller supplementing a robust PID controller is

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** Department of Mechanical Engineering, Graduate School of Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8527, Japan

*** Department of Technology and Information Education, Faculty of Education, Hiroshima University, 1-1-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8524, Japan

**** Department of Communication Engineering, Faculty of Computer Science and Systems Engineering, Okayama Prefectural University, 111 Kuboki, Soja, Okayama 719-1197, Japan

proposed, which yields both the robust stability and the good control performance. This paper is organized as follows. In chapter 2, the controlled object model and the PID control law are formulated. The discussions in this paper start using these formulae. In chapter 3, at first, a design scheme of the robust PID controller which has been already proposed is briefly reviewed. Then, a design scheme of a self-tuning pre-filter is proposed in order to supplement the robust PID controller based on the two-degree-of-freedom control scheme. In chapter 4, the proposed control scheme is experimentally evaluated on an air pressure control system.

2. Problem Formulation

In this paper, let the controlled object be described as follows:

$$A(z^{-1})y(t) = z^{-(k+1)}B(z^{-1})u(t) \quad (1)$$

where $A(z^{-1})$ and $B(z^{-1})$ are the following polynomials consist of system parameters:

$$\left. \begin{aligned} A(z^{-1}) &= 1 + a_1z^{-1} + a_2z^{-2} \\ B(z^{-1}) &= b_0 + b_1z^{-1} + \dots + b_mz^{-m} \end{aligned} \right\} \quad (2)$$

$u(t)$ and $y(t)$ in Eq. (1) are the control input and the corresponding system output, respectively. Furthermore, z^{-1} is a backward shift operator means $z^{-1}y(t) = y(t-1)$, and k denotes the dead time of the system. Here, since process control systems can be usually described in less than or equal to second-order, $A(z^{-1})$ is restricted to a second-order polynomial. High-order components of $A(z^{-1})$ which cannot be approximated by the second-order polynomial are made up by increasing the degree of $B(z^{-1})$.

Next, for the system described by Eq. (1), the following velocity-type PID control law is employed:

$$\Delta u(t) = k_c \left(\Delta + \frac{T_s}{T_I} + \frac{T_D}{T_s} \Delta^2 \right) e(t), \quad (3)$$

where Δ and $e(t)$ denote a differencing operator and the control error signal, that is, they are defined as follows, respectively:

$$\Delta := 1 - z^{-1} \quad (4)$$

$$e(t) := w(t) - y(t). \quad (5)$$

Furthermore, k_c , T_I , T_D denote the proportional gain, the reset time and the derivative time, respectively, and also T_s denotes the sampling interval. In addition, $w(t)$ denotes the setpoint given by piecewise constants.

Here, for simplicity, the PID control law given by Eq. (3) is rewritten as the following equation:

$$C(z^{-1})y(t) + \Delta u(t) - C(z^{-1})w(t) = 0, \quad (6)$$

where $C(z^{-1})$ is defined as follows:

$$\begin{aligned} C(z^{-1}) &:= c_0 + c_1z^{-1} + c_2z^{-2} \\ &= k_c \left(1 + \frac{T_s}{T_I} + \frac{T_D}{T_s} \right) - k_c \left(1 + \frac{2T_D}{T_s} \right) z^{-1} \end{aligned}$$

Fig. 1 Block diagram of the proposed two-degree-of-freedom control system

$$+ \frac{k_c T_D}{T_s} z^{-2}. \quad (7)$$

In the PID control scheme, it is important how to tune PID parameters. A tuning method of PID parameters based on the robust stability has been proposed. That is, the robust PID controller was constructed. However, by using that controller, it is pointed out that the desired setpoint response can not be always obtained because it lays emphasis on the only stability of the control system. Thus, in this paper, a self-tuning pre-filter is inserted in order to supplement the robust PID control system based on the two-degree-of-freedom control scheme. The block diagram of the proposed control system is shown in Fig. 1. In the following section, the design methods of a robust PID controller and a self-tuning pre-filter are discussed.

3. Controller Design

In this chapter, at first, a design scheme of the robust PID controller⁽¹³⁾ is briefly reviewed. Then, a design method of a self-tuning pre-filter is proposed in order to supplement that robust PID controller based on the two-degree-of-freedom control scheme.

3.1 Robust PID controller

3.1.1 PID tuning formula The PID tuning algorithm is derived based on the relationship between the PID control law and the generalized minimum variance control (GMVC) law⁽¹⁸⁾. Consider the following cost function of the GMVC:

$$J = \{y(t+k+1) + \lambda \Delta u(t) - R(z^{-1})w(t)\}^2 \quad (8)$$

where λ is a user-specified parameter which means the weighting factor for the control input.

The control law minimizing the cost function Eq. (8) is given as the following equation:

$$\begin{aligned} F(z^{-1})y(t) + \{E(z^{-1})B(z^{-1}) + \lambda\}\Delta u(t) \\ - R(z^{-1})w(t) = 0 \end{aligned} \quad (9)$$

where $E(z^{-1})$ and $F(z^{-1})$ are obtained by solving the following Diophantine equation:

$$1 = \Delta A(z^{-1})E(z^{-1}) + z^{-(k+1)}F(z^{-1}) \quad (10)$$

$$\left. \begin{aligned} E(z^{-1}) &= 1 + e_1z^{-1} + \dots + e_kz^{-k} \\ F(z^{-1}) &= f_0 + f_1z^{-1} + f_2z^{-2} \end{aligned} \right\} \quad (11)$$

Here in real processes, the estimates of the second term $E(z^{-1})B(z^{-1})$ in Eq. (9) includes uncer-

tainties. That is to say, since the estimate of the dead time is very ambiguous, in that scheme, the dead time k is estimated less, and the degree m of $B(z^{-1})$ is designed bigger than that of the real process. Therefore, the lower degree or/and the higher degree coefficients of z^{-1} in $B(z^{-1})$ are close to zeros, and the relative error of their estimated parameters are considered to become extremely large. Therefore, though the cost function of Eq.(8) is not strictly minimized, those parameters are abandoned to be utilized positively, and $E(z^{-1})B(z^{-1})$ is replaced by the steady state gain $E(1)B(1)$. Thus, the following equation is obtained:

$$\frac{F(z^{-1})}{v}y(t) + \Delta u(t) - \frac{R(z^{-1})}{v}w(t) = 0 \quad (12)$$

where v is defined as

$$v := E(1)B(1) + \lambda. \quad (13)$$

Now, comparing Eq.(6) with Eq.(12), and setting $R(z^{-1})$ and $C(z^{-1})$ as

$$R(z^{-1}) = F(z^{-1}) \quad (14)$$

$$C(z^{-1}) = \frac{F(z^{-1})}{v}, \quad (15)$$

Eq.(12) and Eq.(6) can be regarded as the same control law. Therefore, PID parameters can be tuned based on the relationship between GMVC and PID control laws. That is, PID parameters can be calculated as follows:

$$\left. \begin{aligned} k_c &= -\frac{1}{v}(f_1 + 2f_2) \\ T_i &= -\frac{f_1 + 2f_2}{f_0 + f_1 + f_2} T_s \\ T_d &= -\frac{f_2}{f_1 + 2f_2} T_s \end{aligned} \right\} \quad (16)$$

From Eq.(13) and Eq.(16), note that λ is a parameter which mainly influences the stability of the closed-loop control system, because only proportional gain k_c is changed by the user-specified parameter λ . By the way, the dynamics of the controlled object is often perturbed due to changes of the operating condition or various disturbances. In such a case, the closed-loop control system may become unstable as it is designed based on the only nominal model. Therefore, in the next section, how to determine λ is shown from the viewpoint of the robust stability.

3.1.2 Robust stability condition The model of the controlled object has some uncertainties due to the modeling error, the change of the operating condition, and so on. Now, let $P(j\omega)$ be the frequency transfer function of the nominal model of the controlled object, and $\tilde{P}(j\omega)$ be that of the real controlled object. Then, let $\tilde{P}(j\omega)$ be described as follows:

$$\tilde{P}(j\omega) = \{1 + h(j\omega)\}P(j\omega) \quad (17)$$

where $h(j\omega)$ denotes the multiplicative uncertainties,

and let $h_m(\omega)$ exist, which satisfies the following relationship:

$$|h(j\omega)| \leq h_m(\omega). \quad (18)$$

Furthermore, let $W(j\omega)$ be the transfer function of the closed-loop system, then the necessary and sufficient condition for the closed-loop system to guarantee the robust stability can be expressed as follows⁽¹³⁾:

$$|W(j\omega)|h_m(\omega) < 1. \quad (19)$$

3.1.3 Robust tuning of PID parameters

Now, the robust tuning algorithm of PID parameters is derived based on the robust stability condition reviewed briefly in subsection 3.1.2.

Suppose the nominal model of the controlled object to be described by Eqs.(1) and (2), and similarly, the perturbed model which is perturbed from the nominal state by some causes, to be expressed as follows:

$$\tilde{A}(z^{-1})y(t) = z^{-(k+1)}\tilde{B}(z^{-1})u(t). \quad (20)$$

Let the transfer functions of the nominal model and the perturbed model be $P(z^{-1})$ and $\tilde{P}(z^{-1})$. Then, they are described using Eqs.(1) and (20) as follows, respectively:

$$\left. \begin{aligned} P(z^{-1}) &:= z^{-(k+1)}B(z^{-1})/A(z^{-1}) \\ \tilde{P}(z^{-1}) &:= z^{-(k+1)}\tilde{B}(z^{-1})/\tilde{A}(z^{-1}) \end{aligned} \right\} \quad (21)$$

Applying Eq.(21) to Eq.(17), the following equation concerning the multiplicative uncertainty can be obtained:

$$\begin{aligned} h(z^{-1}) &= \frac{\tilde{P}(z^{-1}) - P(z^{-1})}{P(z^{-1})} \\ &= \frac{A(z^{-1})\tilde{B}(z^{-1}) - \tilde{A}(z^{-1})B(z^{-1})}{\tilde{A}(z^{-1})B(z^{-1})}. \end{aligned} \quad (22)$$

In Fig. 1, the closed-loop transfer function except for the pre-filter, $W(z^{-1})$, is expressed as follows:

$$W(z^{-1}) = \frac{z^{-(k+1)}B(z^{-1})C(z^{-1})}{\Delta A(z^{-1}) + z^{-(k+1)}\tilde{B}(z^{-1})C(z^{-1})}. \quad (23)$$

$h(j\omega)$ in Eq.(17) and $W(j\omega)$ in Eq.(19) are obtained from Eqs.(22) and (23) by using the relation $z = \exp(j\omega T_s)$, respectively. Now, in practice, the estimation of $h_m(\omega)$ in Eq.(18) is thought to be very difficult, but important. Here, it is supposed that $h_m(\omega)$ is given as the absolute value of $h(j\omega)$ calculated from the perturbed model, which is assumed to be most extremely perturbed from the nominal state to the worst direction.

By the way, from Eqs.(13) and (16), it is found that the gain property of the closed-loop transfer function, $|W(j\omega)|$, can be effectively and easily changed using the user-specified parameter λ , because the only gain k_c of the controller $C(z^{-1})$ is changed by v , that is, λ . Therefore, the PID control system taking the robust stability into account, i.e., the robust PID control system can be constructed, where λ is designed so that the closed-loop system may satisfy Eq.(19).

The design procedure can be summarized as fol-

lows:

Robust PID tuning algorithm

[Step 1] Estimate $P(z^{-1})$ and investigate $h_m(\omega)$.

[Step 2] Obtain $E(z^{-1})$ and $F(z^{-1})$ by solving the Diophantine equation (10).

[Step 3] Choose a suitable value of λ by testing the gain property of the left hand side of Eq. (19) due to changing λ .

[Step 4] Calculate PID parameters from Eqs. (13) and (16).

3.2 Two-degree-of-freedom control system

By using the only robust PID controller, it is pointed out that the desirable tracking property in the transient state can not be obtained. Thus, such as Fig. 1, the robust PID control system is equipped with a pre-filter $G_f(z^{-1})$. The feature of the newly proposed control scheme is that the desirable pole-assignment can be realized due to being carried out the pole-zero cancellation between the numerator of $G_f(z^{-1})$ and the denominator of $W(z^{-1})$ in Eq. (23). In practice, parameters of the pre-filter is calculated based on the on-line estimates. In the next section, how to design the self-tuning pre-filter is explained.

3.2.1 Self-tuning pre-filter The pre-filter is designed so that $G_f(z^{-1})W(z^{-1})$ may be identical to $G_m(z^{-1})$ as follows:

$$G_m(z^{-1}) = \frac{z^{-(k+1)}B(z^{-1})C(z^{-1})\beta}{\alpha(z^{-1})}. \quad (24)$$

That is, $G_f(z^{-1})$ is given by

$$G_f(z^{-1}) = \frac{\Delta A(z^{-1}) + z^{-(k+1)}B(z^{-1})C(z^{-1})}{\alpha(z^{-1})} \cdot \beta \quad (25)$$

where the constant β is defined as the following equation so as not to yield the steady state error:

$$\beta := \frac{\alpha(1)}{B(1)C(1)}. \quad (26)$$

Designing $G_f(z^{-1})$ like this, the property of $W(z^{-1})$ in Eq. (23) depends on the pole of $G_f(z^{-1})$, that is, $\alpha(z^{-1})$. $\alpha(z^{-1})$ is designed based on the following two features.

- (i) the rise-time
- (ii) the damping property

Thus, let $\alpha(z^{-1})$ be the denominator part of the discrete-time version of the following desirable second order continuous-time transfer function $G_m(z^{-1})^{(18)}$:

$$G_m(s) = \frac{1}{1 + \sigma s + \mu(\sigma s)^2} \quad (27)$$

That is, $\alpha(z^{-1})$ is defined as follows:

$$\alpha(z^{-1}) = 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} \quad (28)$$

$$\alpha_1 = -2 \exp\left(-\frac{\rho}{2\mu}\right) \cdot \cos\left(\frac{\sqrt{4\mu-1}}{2\mu} \rho\right) \quad (29)$$

$$\alpha_2 = \exp\left(-\frac{\rho}{\mu}\right) \quad (30)$$

$$\rho := T_s/\sigma \quad (31)$$

$$\mu := 0.25(1-\delta) + 0.51\delta \quad (32)$$

where σ and μ denote the rise time designed between 1/2 and 1/3 of the time constant and the damping

coefficient usually designed as 0.0 thru 1.0, respectively. In the case of $\delta=0$ and $\delta=1.0$, the step response of $G_m(s)$ shows the Binomial model response and the Butterworth model response, respectively.

The system parameters a_i ($i=1, 2$) and b_j ($j=0, 1, \dots, m$) are estimated by using the following recursive least squares algorithm⁽¹⁷⁾:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\Gamma(t-1)\phi(t-1)}{1 + \phi^T(t-1)\Gamma(t-1)\phi(t-1)} \varepsilon(t) \quad (33)$$

$$\Gamma(t) = \Gamma(t-1) - \frac{\Gamma(t-1)\phi(t-1)\phi^T(t-1)\Gamma(t-1)}{1 + \phi^T(t-1)\Gamma(t-1)\phi(t-1)} \quad (34)$$

$$\varepsilon(t) = y(t) - \hat{\theta}^T(t-1)\phi(t-1), \quad (35)$$

where $\varepsilon(t)$, $\hat{\theta}(t)$ and $\phi(t-1)$ denote the predictive error, the unknown parameter vector and the data vector as follows, respectively:

$$\hat{\theta}(t) = [\hat{a}_1(t), \hat{a}_2(t), \hat{b}_0(t), \hat{b}_1(t), \dots, \hat{b}_m(t)] \quad (36)$$

$$\phi(t-1) = [-y(t-1), -y(t-2), u(t-k-1), \dots, u(t-k-m-1)]^T. \quad (37)$$

Thus, parameters of the pre-filter are calculated based on on-line estimates.

The design procedure can be summarized as follows:

PID tuning algorithm based on two-degree-of-freedom scheme

[Step 1] Estimate $P(z^{-1})$ and investigate $h_m(\omega)$.

[Step 2] Obtain $E(z^{-1})$ and $F(z^{-1})$ by solving the Diophantine equation (10).

[Step 3] Choose a suitable value of λ by testing the gain property of the left hand side of Eq. (19) due to changing λ .

[Step 4] Obtain $\alpha(z^{-1})$ based on σ and δ by considering the rise time and the damping property of the nominal model.

[Step 5] Calculate PID parameters from Eqs. (13) and (6).

3.2.2 Discussion for control system stability

In this section, the stability of the proposed two-degree-of-freedom control system is discussed.

In the one-degree-of-freedom self-tuning control system, in order to guarantee the stability, it is necessary that $\hat{\theta}$ converges to θ . That is, there must exist a positive definite matrix R defined as

$$R := \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N \phi(t-1)\phi^T(t-1), \quad (38)$$

what is called, the persistently spanning condition must be held.

On the other hand, the whole transfer function of the proposed control system is described as follows:

$$G(z^{-1}) = \frac{z^{-(k+1)}B(z^{-1})C(z^{-1})\hat{T}(z^{-1})\cdot\hat{\beta}}{\alpha(z^{-1})\hat{T}(z^{-1})} \quad (39)$$

where $\hat{\beta}$ is defined as follows:

$$\hat{\beta} := \frac{\alpha(1)}{\hat{B}(1)C(1)}. \quad (40)$$

The polynomial $T(z^{-1})$ constructed the true system parameters and $\hat{T}(z^{-1})$ constructed the estimated system parameters are defined as follows:

$$T(z^{-1}) := \Delta A(z^{-1}) + z^{-(k+1)} B(z^{-1}) C(z^{-1}) \quad (41)$$

$$\hat{T}(z^{-1}) := \Delta \hat{A}(z^{-1}) + z^{-(k+1)} \hat{B}(z^{-1}) C(z^{-1}). \quad (42)$$

In the proposed control system, it is not necessary that $\hat{\theta}$ converges to θ with respect to the stability of the closed-loop system. That is, since $\alpha(z^{-1})$ is a user-specified polynomial and can be designed with considering the asymptotic stability, the characteristic polynomial $\alpha(z^{-1})T(z^{-1})$ in Eq.(39) is stable even if $\hat{T}(z^{-1})$ does not converge to $T(z^{-1})$. Thus, it is found that the stability of the proposed control system is always guaranteed from the above discussion. In addition, the proposed method has a feature that the poles of the closed-loop system goes to the desirable poles and the good control performance can be obtained, when $\hat{\theta}(t)$ converges to the true value θ .

4. Experimental evaluation

The effectiveness of the proposed control scheme is illustrated by applying to an air pressure control system.

4.1 Experimental system

The schematic figure of the experimental air pressure control system is shown in Fig. 2. The control objective is to regulate the air pressure y to any desired values by manipulating the control valve position u . Here, all units of the values, such as the pressure and the valve position, are transformed into voltage. Furthermore, let $u(t)$ be limited as $1.2 \text{ [V]} \leq u(t) \leq 5.0 \text{ [V]}$ to hold the linearity. From Fig. 2, it can be seen that for the water level d to be changed has

Fig. 2 Schematic figure of an experimental plant

influence on the dynamics of the air pressure control system as a disturbance.

4.2 Modeling

Several mathematical models of the controlled object were identified from the input-output data when the water level in the tank changes from 30 [cm] to 50 [cm] in increments 10 [cm] by using the function 'varx' in *Matlab*. Those models made by the above procedure are shown as follows:

$$y(t) = a \cdot y(t-1) + b \cdot u(t-2) \quad (43)$$

$$30 \text{ [cm]} \quad a=0.9794, b=0.0289 \quad (44)$$

$$40 \text{ [cm]} \quad a=0.9775, b=0.0316 \quad (45)$$

$$50 \text{ [cm]} \quad a=0.9747, b=0.0356. \quad (46)$$

Here, suppose that the model in the case of 40 [cm] is the nominal model. Moreover, the degree of the system is considered as one degree, since the degree of the system is often thought to be one degree in the process control.

Next, as the PID control system is constructed for the nominal model, each Bode diagram is drawn, where λ is changed from 0.02 to 0.1 in increments 0.02 as shown in Fig. 3. From Fig. 3, it is found that a stable control system can be constructed in the case of $\lambda \geq 0.04$.

4.3 Analysis of robust stability

The function $h_m(\omega)$ bounding the worst perturbation of the controlled object should be determined in order to apply the robust stability condition Eq.(19) to the robust PID tuning scheme. To consider this matter, it is examined how great effect the uncertainty due to the perturbation of the controlled object has on the stability of the closed-loop system, when

Fig. 3 Bode diagram of the PID control system constructed for the nominal model

the water level increases, and decreases, respectively.

First, let the perturbed model $\tilde{P}(z^{-1})$ corresponding to the case of 50 [cm] be the worst perturbed model when the water level increases from the nominal state (the case of 40 [cm]). In this case, the frequency uncertainty function $h(j\omega)$ is calculated from $h(z^{-1})$ of Eq.(22) using the above $\tilde{P}(z^{-1})$ and the nominal model $P(z^{-1})$. Here, let the absolute value $|h(j\omega)|$ be $h_1(\omega)$. Then, regarding $h_1(\omega)$ as $h_m(\omega)$ in Eq.(19), the gain property of the left hand side of Eq.(19), $|W(j\omega)h_m(\omega)|$ was tested for some values of λ as shown in Fig. 4. From Fig. 4, in the case of $\lambda=0.04$, since the gain plot is over 0 [dB] line, it can be guessed that the robust stability condition (19) is not satisfied. And also, it can be guessed that the closed-loop system will be stable in the case of $\lambda \geq 0.08$.

Fig. 4 Gain plots of $|W(j\omega)h_m(\omega)|$ to indicate effect of λ , where the water level 50 [cm] is considered as the worst case

Fig. 5 Gain plots of $|W(j\omega)h_m(\omega)|$ to indicate effect of λ , where the water level 30 [cm] is considered as the worst case

On the other hand, Fig. 5 shows the gain property of the left hand side of Eq.(19) similar to Fig. 4, regarding the case of 30 [cm] as the worst perturbed model, when the water level decreases from the nominal state (the case of 40 [cm]) to the case of 30 [cm]. From Fig. 5, it is clear that for the water level decreasing, the control is quite easy, i.e., is stable in the case of $\lambda \geq 0.08$. Therefore, regarding the previous $h_1(\omega)$ as $h_m(\omega)$, it can be seen that the user-specified parameter λ so as to satisfy the robust stability condition Eq.(19) should be determined so that λ is greater than or equal to 0.08 in the case where the water level is changed from 30 [cm] through 50 [cm].

4.4 Control results

Some control results are shown so as to confirm the above discussion. At first, as a comparative experiment, the control result using Ziegler & Nichols tuning method⁽¹²⁾ is shown in Fig. 6, where the sampling interval is 1 [sec]. Furthermore, the reference signals 3.0 [V] and 3.5 [V] are nearly equal to 0.7 [kgf/cm²] and 0.85 [kgf/cm²], respectively. From Fig. 6, it can be seen that in the worst perturbed state, the response becomes very oscillatory, i.e., the system is unstable. Here, the dash-dot line denotes the change-

Fig. 6 Control result using Ziegler & Nichols tuning method

Fig. 7 Control result using the one-degree-of-freedom self-tuning control scheme

Fig. 8 Control result using the only robust PID control scheme

Fig. 9 Control result using the proposed two-degree-of-freedom control scheme

ing behavior of the water level.

Next, the result using the one-degree-of-freedom self-tuning control scheme is shown in Fig. 7. In this experiment, λ was chosen as 0.04 from Fig. 3. From Fig. 7, where the one-degree-of-freedom self-tuning control scheme is adopted, it is found that even if the nominal stability is guaranteed, the stability of the control system with uncertainties can not be always done.

The control result using the only robust PID control scheme mentioned in section 3.1 is shown in Fig. 8. In this experiment, λ was chosen as 0.08 from Fig. 4.

On the other hand, Fig. 9 shows the control result using the newly proposed two-degree-of-freedom control scheme mentioned in section 3.2. Here, the parameters σ and δ in Eqs. (31) and (32) are set as 16 [sec] and 0.0, respectively. The value of σ is determined based on the time constant of the nominal model, Eq. (45).

Comparing Fig. 8 with Fig. 9, it can be seen that the control result using the proposed two-degree-of-freedom control scheme is superior to that of using the only robust PID control scheme on the setpoint

response. Therefore, it is clear that the proposed two-degree-of-freedom control scheme works well if the frequency uncertainty function $h(j\omega)$ in designing the robust controller is identified, and the parameter estimation in the self-tuning scheme is performed, adequately.

5. Conclusions

In this paper, a design scheme of a self-tuning pre-filter was proposed based on the two-degree-of-freedom control scheme. When the only robust PID controller is adopted, the desirable setpoint response can not be always obtained. Whereas, using the proposed two-degree-of-freedom control scheme, the desirable setpoint response can be expected due to the self-tuning pre-filter supplementing the robust PID controller. The effectiveness of the proposed scheme was experimentally evaluated on an air pressure control system. The control results mean that a unified approach of the robust PID controller and the self-tuning controller was successfully illustrated in this paper.

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