Design of Robust Tracking Controllers Using Sliding Mode Technique*

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A simple methodology of designing robust tracking controllers using sliding mode technique is proposed in this paper for a class of nonlinear dynamic systems. This control scheme contains a simple perturbation estimation process embedded in the controller, so that the robustness of system's stability can be maintained without knowing the upper bound information of perturbation itself and the estimation error of it. The perturbation estimator can be implemented by using a simple high pass filter. The tracking accuracy of control can be adjusted through two stages. The first stage is done by refining the perturbation estimation process. The second stage is done by adjusting the parameter of controller. Chattering phenomenon will never occur under this control scheme. The comparison between the proposed control scheme and the traditional sliding mode control in the frequency domain is given. A numerical example and a practical experiment are demonstrated for showing the applicability of the proposed control scheme.

Key Words : Robust Tracking, Sliding Mode, Perturbation Estimation

1. Introduction

Sliding mode control (SMC) has been traditionally recognized as a high gain control technique with outstanding robustness features for both system stabilization and tracking problem⁽¹⁾. The main feature of SMC is that the system's state trajectories are driven toward a switching surface, or sliding surface, chosen by the designer, to attain the design object. Then a discontinuously high-speed switching action maintains the state on this surface once the system enters the sliding surface. This method offers an advantage of insensitivity to all parameter variations and external disturbances, but the knowledge of the upper bound of perturbation is often required⁽²⁾⁻⁽¹⁰⁾. Another disadvantage of using discontinuous control in traditional SMC control is the chattering phenomenon^{(5),(7),(9),(11),(12)}, which may excite the unmodeled high frequency dynamics of the system⁽⁴⁾. In order to eliminate the chattering phenomenon, sliding

layer or saturation function is often used to replace the discontinuous control^{(4),(6),(8)}. However, it is usually found that the tracking error is then induced, and the chattering phenomenon can only be alleviated, but not totally eliminated when larger switching gain is used to overcome uncertainties and finite switching action can only be obtained in practice. Elmali and Olgac⁽¹¹⁾ proposed a control scheme in which a perturbation estimation process is embedded in the traditional SMC so that the knowledge of the upper bound of perturbation is not needed. However, the knowledge of the upper bound of the perturbation estimation error is still required. Besides, the chattering phenomenon still can not be avoided in practice due to the utilization of signum or saturation function.

In this paper we use a simple perturbation estimation process embedded in the controller, so that the knowledge of upper bound of the perturbation is not required. Another feature of the proposed control scheme is that, although the sliding mode control technique is utilized, we do not use the signum or saturation function in designing the controller in order to avoid chattering phenomenon, and the robustness of system's stability still can be maintained at the

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same time. The desired tracking accuracy can in general be achieved by adjusting the parameter of controller or refining the perturbation estimation process. We also show the superiority of proposed control scheme over traditional SMC for attenuating the perturbation. Finally the feasibility of proposed control scheme is demonstrated by using a numerical example and Ball and Beam experimental system.

2. System Descriptions and Assumptions

Consider a class of MIMO nonlinear systems described by the following differential equation

$$\begin{aligned} \dot{\zeta}_i^{(n_i)} &= f_i(\mathbf{z}_1, \dots, \mathbf{z}_m) + \Delta f_i(\mathbf{z}_1, \dots, \mathbf{z}_m) \\ &+ \sum_{j=1}^m [g_{ij}(\mathbf{z}_1, \dots, \mathbf{z}_m) + \Delta g_{ij}(\mathbf{z}_1, \dots, \mathbf{z}_m)] u_j + d_i(t), \\ i &= 1, 2, \dots, m, \end{aligned} \quad (1)$$

where $\mathbf{z}_i \triangleq [\zeta_i \dot{\zeta}_i \dots \zeta_i^{(n_i-1)}]^T \in R^{n_i}$ are state sub-vectors. The vectors $\mathbf{f} \triangleq [f_1 f_2 \dots f_m]^T \in R^m$ and $\Delta \mathbf{f} \triangleq [\Delta f_1 \Delta f_2 \dots \Delta f_m]^T \in R^m$ are nonlinear driving terms and their parameter variations respectively. The matrices $\mathbf{G} \triangleq [\mathbf{g}_1 \dots \mathbf{g}_m]^T \in R^{m \times m}$, $\mathbf{g}_i = [g_{i1} \dots g_{im}]^T \in R^m$ and $\Delta \mathbf{G} \triangleq [\Delta \mathbf{g}_1 \dots \Delta \mathbf{g}_m]^T \in R^{m \times m}$, $\Delta \mathbf{g}_i = [\Delta g_{i1} \dots \Delta g_{im}]^T \in R^m$ are input gains and their uncertainties respectively. Define $\mathbf{d} \triangleq [d_1 d_2 \dots d_m]^T \in R^m$ as the external disturbance and $\mathbf{u} = [u_1 u_2 \dots u_m]^T \in R^m$ as the control input. Note that $\sum_{i=1}^m n_i = n$ and the parameter variations and disturbances $\Delta \mathbf{f}$, $\Delta \mathbf{g}$ and $\mathbf{d}$ are all bounded by unknown constants.

If $\mathbf{G}$ is diagonal, it is assumed that each diagonal term $g_{ii} \neq 0$. If $\mathbf{G}$ is not diagonal, we assume that $\mathbf{G}^{-1}$ exists for all time. Besides, the state variables of the system are all measurable. The objective of the control is to track the desired signals $x_{id}(t)$, $i=1, 2, \dots, m$, in spite of the existence of parameter variations, model uncertainties and/or disturbances.

3. Design of Robust Tracking Controllers

It is noted that Eq.(1) is a system with m coupling subsystems. Each subsystem can be represented in controllable canonical form as

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= x_{i3} \\ &\vdots \\ \dot{x}_{in_i} &= f_i(\mathbf{x}_1, \dots, \mathbf{x}_m) + \Delta f_i(\mathbf{x}_1, \dots, \mathbf{x}_m) \\ &+ [\mathbf{g}_i^T(\mathbf{x}_1, \dots, \mathbf{x}_m) + \Delta \mathbf{g}_i^T(\mathbf{x}_1, \dots, \mathbf{x}_m)] \mathbf{u} + d_i(t), \end{aligned} \quad (2)$$

where $x_{ij} \triangleq \zeta_i^{(j-1)}$, $j=1, 2, \dots, n_i$, $i=1, 2, \dots, m$, are state variables of each subsystem. The following is the design procedure of the proposed control scheme.

Step 1: Sliding surface design

In order to achieve the objective of tracking any desired signal, the sliding function is designed as

$$\begin{aligned} \sigma_i &\triangleq \left(\frac{d}{dt} + \lambda_{i,1} \right) \left(\frac{d}{dt} + \lambda_{i,2} \right) \dots \left(\frac{d}{dt} + \lambda_{i,n_i-1} \right) e_i(t) \\ &= \sum_{k=0}^{n_i-1} c_{i,k+1} e_i^{(k)}, \quad i=1, \dots, m, \end{aligned} \quad (3)$$

where $c_{i,n_i}=1$, $e_i \triangleq x_{i1} - x_{id}$ is the tracking error, and the set of real coefficients $\{c_{i,1}, \dots, c_{i,n_i-1}\}$ are chosen such that $\text{Re}(\lambda_{i,j}) > 0$, $i=1, \dots, m$, $j=1, \dots, n_i-1$. The reason can be easily seen that once the sliding condition is satisfied, i.e., $\sigma_i \dot{\sigma}_i < 0$, σ_i will approach to zero in a finite time, and the tracking error e_i will also approach to zero as $t \rightarrow \infty$.

Step 2: Controller design

The control input of the system is designed as

$$\mathbf{u} = \mathbf{G}^{-1} [-\mathbf{f} - \boldsymbol{\phi} - \mathbf{K}_p \boldsymbol{\sigma} - \Delta \mathbf{p}_{est}], \quad (4)$$

where $\mathbf{K}_p = \text{diag}(k_{p,i}) \in R^{m \times m}$, $k_{p,i}$ is a designed coefficient which has relationship with tracking accuracy (the reason will be seen later) and is a positive number. $\boldsymbol{\phi}$ is a $m \times 1$ vector, and its components are defined by

$$\begin{aligned} \phi_i &\triangleq c_{i,1}(x_{i2} - \dot{x}_{id}) + \dots + c_{i,n_i-1}(x_{i,n_i} - \dot{x}_{id}^{(n_i-1)}) \\ &- \dot{x}_{id}^{(n_i)}, \quad i=1, 2, \dots, m. \end{aligned}$$

Because the desired tracking signals may be piecewise continuous in t or they are not smooth functions, for example, they may experience step changes with time, $\boldsymbol{\phi}$ may be piecewise continuous. It implies that $\mathbf{u}$ may be piecewise continuous. The vector $\Delta \mathbf{p}_{est} = [\Delta p_{1,est} \dots \Delta p_{m,est}]^T$ denotes the estimation of system's perturbation $\Delta \mathbf{p} \triangleq [\Delta p_1 \Delta p_2 \dots \Delta p_m]^T$, $\Delta p_i = \Delta f_i + \Delta \mathbf{g}_i^T \mathbf{u} + d_i$, $i=1, 2, \dots, m$. Note that the structure of controller (4) is different from the controllers proposed in Refs.(11), (12), in which sign function (or boundary layer) is used in the design of controllers. Note that there is no artificially discontinuous function in the proposed control input Eq.(4) as those in the traditional SMC^{(1),(4)-(7),(9),(11),(12)}, therefore, the chattering phenomenon caused by the proposed controller will not happen. The other advantage of Eq.(4) is that the knowledge of upper bound of the perturbation estimation error is not required. If $\mathbf{G}$ is diagonal, then the controller becomes

$$\begin{aligned} u_i &= [-f_i - \phi_i - k_{p,i} \sigma_i - \Delta p_{i,est}] / g_{ii}, \\ i &= 1, 2, \dots, m. \end{aligned}$$

Differentiating Eq.(3) and using Eq.(2), one has

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{\phi} + \mathbf{f} + \mathbf{G} \mathbf{u} + \Delta \mathbf{p}. \quad (5)$$

Substituting Eq.(4) into Eq.(5) yields

$$\dot{\boldsymbol{\sigma}} = -\mathbf{K}_p \boldsymbol{\sigma} + \Delta \mathbf{p} - \Delta \mathbf{p}_{est}. \quad (6)$$

It is easy to verify from Eq.(6) that the sliding condition $\boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} < 0$ is satisfied if $\Delta p_{i,est}$ is exactly equal to Δp_i .

Step 3: Estimation of perturbations

In order to estimate the perturbation Δp_i , we first define a calculable state variable $x_{in_i,cal}(t)$, and an auxiliary input $\bar{\mathbf{u}}(t)$ as

$$\begin{aligned}
 x_{in_i,cal}(t) &\triangleq \int_{t_0}^t [f_i + g_i^T \bar{u}] d\tau + x_{in_i,cal}(t_0), \\
 \bar{u} &\triangleq G^{-1}[-f - \phi - K_p \bar{\sigma} - \Delta p_{est}], \\
 \text{where } \bar{\sigma} &\triangleq [\bar{\sigma}_1 \ \bar{\sigma}_2 \ \dots \ \bar{\sigma}_m]^T \in R^{m \times 1}, \text{ and} \\
 \bar{\sigma}_i &\triangleq \sum_{k=0}^{n_i-2} C_{i,k+1} e_i^{(k)} + C_{i,n_i}(x_{in_i,cal} - x_{id}^{(n_i-1)}), \\
 i &= 1, \dots, m.
 \end{aligned} \quad (8)$$

Remark : In fact Eq. (7) can be written in the form of differential equation

$$\dot{x}_{in_i,cal}(t) = f_i + g_i^T \bar{u} \quad (9)$$

with initial condition $x_{in_i,cal}(t_0)$. It can be implemented by hardware or numerical integral algorithm with a digital computer.

On the other hand, from Eq. (2) it is known that

$$x_{in_i}(t) = \int_{t_0}^t [f_i + g_i^T u + \Delta p_i] d\tau + x_{in_i}(t_0). \quad (10)$$

Let $\Delta I_i(t)$ be the difference between measurable state $x_{in_i}(t)$ and calculable state $x_{in_i,cal}(t)$. From Eqs. (3), (7), (8) and (10), one can obtain

$$\begin{aligned}
 \Delta I_i(t) &\triangleq x_{in_i}(t) - x_{in_i,cal}(t) \\
 &= \int_{t_0}^t [-k_{p,i}(\sigma_i - \bar{\sigma}_i) + \Delta p_i] d\tau \\
 &\quad + x_{in_i}(t_0) - x_{in_i,cal}(t_0) \\
 &= \int_{t_0}^t [-k_{p,i} \Delta I_i(\tau) + \Delta p_i(\tau)] d\tau \\
 &\quad + x_{in_i}(t_0) - x_{in_i,cal}(t_0).
 \end{aligned}$$

The preceding equation implies that

$$\Delta p_i(t) = \left(\frac{d}{dt} + k_{p,i} \right) \Delta I_i(t), \quad i = 1, 2, \dots, m. \quad (12)$$

So the quantity of each perturbations can be obtained by computing the derivative of $\Delta I_i(t)$. However, due to the enhancement of noise, it is in general not desired to use a pure differentiator to estimate perturbations in practice. In order to estimate the perturbations practically, we use a high-pass filter with transfer function $s/(1 + \epsilon_i s)$ to approximate that of a differentiator^{(8),(15)}; which means that

$$\Delta p_{i,est}(s) = \left(\frac{s}{1 + \epsilon_i s} + k_{p,i} \right) \Delta I_i(s), \quad (13)$$

where ϵ_i is a small positive constant.

Step 4: The determination of $\lambda_{i,j}$ and ϵ_i

From Eqs. (3), (6), (12) and (13) the transfer function between perturbations $\Delta p_i(s)$ and tracking error $e_i(s)$ is

$$\begin{aligned}
 \frac{e_i(s)}{\Delta p_i(s)} &= \frac{\epsilon_i s^2}{(1 + \epsilon_i s)(s + k_{p,i})^2 (s + \lambda_{i,1})(s + \lambda_{i,2}) \dots (s + \lambda_{i,n_i-1})}.
 \end{aligned} \quad (14)$$

Figure 1 depicts the block diagram of Eq. (14), and its Bode plot is shown in Fig. 2 by assuming $k_{p,i}$, $Re(\lambda_{i,j}) > 1$ and $\lambda_{i,j} \neq \lambda_{i,k}$, $j \neq k$, $\lambda_{i,1} = \lambda_{i,min}$. Figure 2 and Eq. (14) clearly show that, in order to get higher the attenuation of the magnitude of Eq. (14) within the perturbation spectrum and the faster speed the system can enter the sliding surface, $\lambda_{i,j}$ can be chosen such

Fig. 1 Block diagram between perturbation and tracking error of proposed control strategy

Fig. 2 Bode plot of proposed control technique and traditional SMC

that $Re(\lambda_{i,j}) > 1$, and the parameter ϵ_i should be chosen as small as possible, i.e., $0 < \epsilon_i \ll 1$. In fact the smaller the ϵ_i , the higher the attenuation of perturbation. From the practical point of view, however, ϵ_i can not be too small if there is high frequency noise contained in the operating frequency range. On the other hand, hardware constraints of the control system may also limit the choice of ϵ_i . Note that although the choice of ϵ_i will affect the accuracy of perturbation estimation and tracking, there are still some parameter which can be adjusted to improve the tracking accuracy, this is discussed in the following section.

The design procedure of the proposed control algorithm is summarized as follows:

Step 1: Design the sliding function by using Eq. (3). For practical implementation, Eq. (3) can be realized as

$$\begin{aligned}
 \sigma_i &= C_{i,1}(x_{i1} - x_{id}) + C_{i,2}(x_{i2} - x_{id}^{(1)}) + \dots \\
 &\quad + C_{i,n_i}(x_{in_i} - x_{id}^{(n_i-1)}).
 \end{aligned}$$

Note that all state variables are available.

Step 2: Calculate $\Delta I_i(t)$ by using Eqs. (11) and (9).

Step 3: Estimate Δp_i by using Eq. (13).

Step 4: Implement control effort u by using Eq. (4).

4. Robustness of System's Stability and Adjustment of Tracking Accuracy

Estimation errors usually do exist in practice. According to Eq. (6), the sliding condition may not

hold if estimation errors exist. The following theorem shows that each sliding variable σ_i will eventually be bounded by a small value which is proportional to the bound of estimation error and inverse proportional to the designed coefficient $k_{p,i}$.

Theorem 1: Consider the nonlinear dynamic system (2) with sliding function (3) and controller (4). If the perturbation Δp_i is bounded, then the sliding variable will be driven to a band of width $2\beta_i/k_{p,i}$, where $|\Delta p_i - \Delta p_{i,est}| \leq \beta_i$, i.e., $|\sigma_i| \leq \beta_i/k_{p,i}$, $i=1, \dots, m$. $\square$

Proof: It can be verified from Fig. 1 that the relationship between perturbation estimation error and perturbation is $\Delta p_i - \Delta p_{i,est} = \frac{\epsilon_i s^2}{(1 + \epsilon_i s)(s + k_{p,i})} \Delta p_i$.

Therefore, $\Delta p_i - \Delta p_{i,est}$ is bounded if the perturbation Δp_i is bounded and $0 < \epsilon_i < 1$. Now consider a Lyapunov function $V = \sigma_i^2/2$. Differentiating this Lyapunov function with respect to time and using Eq. (6) yields

$$\begin{aligned} \dot{V} &= \sigma_i \dot{\sigma}_i = -k_{p,i} \sigma_i^2 + \sigma_i (\Delta p_i - \Delta p_{i,est}) \\ &\leq -k_{p,i} |\sigma_i|^2 + \beta_i |\sigma_i| = -|\sigma_i| (k_{p,i} |\sigma_i| - \beta_i) \end{aligned}$$

Therefore, if $|\sigma_i| > \beta_i/k_{p,i}$, it will result in $\sigma_i \dot{\sigma}_i < 0$, which also implies that each sliding variable will eventually be driven by the controller (4) into a band of width $2\beta_i/k_{p,i}$ and stay thereafter. $\triangle$

The preceding theorem indicates that, as long as the perturbation is bounded, not only the robustness of stability of proposed control scheme is guaranteed but also the perturbation estimation error is bounded. The reason why we used an auxiliary control $\bar{u}(t)$ is to ensure the calculable state variable $x_{in,cal}(t)$ is bounded. This is proved in the following theorem:

Theorem 2: Given the dynamic system (2) with sliding function (3) and controller (4). If the perturbation Δp_i is bounded, then the calculable state variable $x_{in,cal}(t)$ is also bounded. $\square$

Proof: From Theorem 1 it is known that under the proposed control scheme, $|\Delta p_i - \Delta p_{i,est}|$ is bounded if Δp_i is bounded. This indicates perturbation estimation $\Delta p_{i,est}$ is also bounded. From Eq. (13) it implies $\Delta I_i(t)$ is bounded function, and from Eq. (11) one can see $x_{in,cal}(t)$ is bounded. $\triangle$

Theorem 1 also indicates that, although each sliding variable will not be zero as $t \rightarrow \infty$ if perturbation estimation error exists, its bound can be reduced by decreasing β_i or increasing $k_{p,i}$. It is interesting to find out the least upper bound of tracking error after the system enters the band of width $2\beta_i/k_{p,i}$ of σ_i . Figure 1 shows that the tracking error e_i is obtained from σ_i through a sequence of first-order low-pass filters. Let w_i be the output of the first low-pass filter, then from the Theorem we have

Fig. 3 Block diagram between perturbation and tracking error of traditional SMC inside the boundary layer

$$\begin{aligned} |w_i(t)| &= \left| \int_0^t e^{-\lambda_{i,1}(t-\tau)} \sigma_i(\tau) d\tau \right| \\ &\leq \frac{\beta_i}{k_{p,i}} \int_0^t e^{-\lambda_{i,1}(t-\tau)} d\tau \leq \frac{\beta_i}{k_{p,i} \lambda_{i,1}}. \end{aligned}$$

Applying the same method to outputs of the rest of the filters, we then obtain

$$|e_i(t)| \leq \frac{\beta_i}{k_{p,i} \lambda_{i,1} \lambda_{i,2} \cdots \lambda_{i,n_i-1}}.$$

The preceding equation clearly indicates that there are two ways to improve the tracking accuracy. One is to reduce the value of β_i , this can be done by reducing the value of ϵ_i . Since ϵ_i can not be arbitrarily small in practice, this method of improving tracking accuracy can be treated as “rough tuning”. The other way to improving tracking accuracy is to increase the value of designed parameter $k_{p,i}$ of controller or the locations of root $\lambda_{i,j}$. The desired tracking accuracy in general can be easily achieved in this stage. This method of improving tracking accuracy can be treated as “fine tuning”.

Figure 3 shows the block diagram between perturbation and tracking error of traditional SMC technique when system's dynamics are inside a boundary layer⁽¹³⁾. The definition of sliding surface function σ and the discontinuity gain $k_i^{(13)}$ are the same as those of proposed control scheme for fair comparison. The Bode plot of Fig. 3 is also shown in Fig. 2. It clearly indicates that the proposed control scheme outperform the traditional SMC in all frequency ranges since the proposed control scheme yields higher attenuation of perturbation. The minimum discrepancy is $-20 \log \frac{\epsilon_i}{k_{p,i}}$. It also implies that the traditional SMC technique can not filter out DC offsets in perturbation signal, whereas the proposed control scheme can have infinite attenuation at zero frequency; which also means that the traditional SMC can not remove the influence of DC offsets completely on the tracking error as proposed controller can do.

5. Example

Consider the following nonlinear dynamic equations

$$\begin{aligned} \dot{x}_{11} &= x_{12} \\ \dot{x}_{12} &= x_{12}x_{21} - 0.3x_{11}x_{22} + 0.5x_{21} - 2x_{22} + x_{33} \sin(x_{12}) \\ &\quad + (3.76 + 0.5x_{22})u_1 + 0.2 \cos(2t) \\ \dot{x}_{21} &= x_{22} \\ \dot{x}_{22} &= x_{23} \end{aligned}$$

$$\dot{x}_{23} = x_{11} + x_{12}x_{22} + 0.2x_{22}^2 - 2x_{23} + (2.5 + 0.5x_{12})u_2 + 0.5\sin(t).$$

Note that we define $\Delta p_1 \triangleq -0.3x_{11}x_{22} + 0.5x_{21} + 0.2\cos(2t)$ and $\Delta p_2 \triangleq 0.2x_{22}^2 + 0.5\sin(t)$. The desired trajectories are $x_{1d} = 2\sin(2t)$ and $x_{2d} = 2\cos(t)$. The constants (c_{11}, c_{21}, c_{22}) and $(k_{p,1}, k_{p,2}, \epsilon_1, \epsilon_2)$ are chosen to be $(2, 2, 2)$ and $(5, 10, 0.001, 0.001)$ respectively. Note that

$$G = \begin{bmatrix} 3.76 + 0.5x_{22} & 0 \\ 0 & 2.5 + 0.5x_{12} \end{bmatrix},$$

and $x_{22} \in R - \left\{-\frac{3.76}{0.5}\right\}$, $x_{12} \in R - \left\{-\frac{2.5}{0.5}\right\}$ for ensuring the existence of G^{-1} for all time. The simulation results are shown from Fig. 4 to Fig. 6. Figure 4

Fig. 4 Control inputs (example)

Fig. 5 Tracking error e_1 with different $k_{p,i}$ (example)

Fig. 6 Tracking errors e_1, e_2 and $e'_1(2000\%\Delta p)$, $e'_2(2000\%\Delta p)$ (example)

shows the trajectories of control inputs, which are all continuous functions. Let (e'_1, e'_2) be the tracking errors when Δp_i , $i=1, 2$, are twenty-fold increase; e'_1 be the tracking error when $(k_{p,1}, k_{p,2})$ is reduced to $(1, 2)$. Figure 5 is the comparison of tracking error e_1 between the case when $k_{p,1}=1$, $k_{p,2}=2$ and $k_{p,1}=5$, $k_{p,2}=10$. One can see that increasing the accuracy coefficients $k_{p,i}$ will indeed reduce tracking errors. e_2 has the similar result. Figure 6 shows that the system is very robust against the variations of perturbation, since it is difficult to tell the difference between e_1 and e'_1 , e_2 and e'_2 when the perturbations Δp_i , $i=1, 2$, are twenty-fold increase. The reason is that the proposed controllers have the ability of perturbation estimation in order to cope with the change of perturbation.

6. Ball and Beam Experiment

In this section the proposed control scheme is applied to control “Ball and Beam” experimental system⁽¹⁴⁾. The objective of this experiment is to control the position of an iron ball by manipulating the oblique angle of level arm driven by a servo motor. The dynamic equations of the ball and beam system, written in state space form, is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f(x) + b(x)V_{in}, \end{aligned}$$

where

Fig. 7 Control input (simulation)

Fig. 8 Tracking error (simulation)

Fig. 9 Control input (experiment)

Fig. 10 Tracking error (experiment)

$$f(x) = \frac{-49x_3x_4^2}{\left[5g \cos\left(\sin^{-1}\left(\frac{-7x_3}{5g}\right)\right)\right]^2} - \frac{k_m^2 k_g^2}{J_{eq} R_m} x_4,$$

$$b(x) = -\frac{5}{7} g \frac{r}{L} \frac{k_m k_g}{J_{eq} R_m} \cos\left[\sin^{-1}\left(\frac{-7x_3}{5g}\right)\right],$$

and $(k_m, k_g, J_{eq}, R_m, r, L) = (0.0767 \text{ V} \times \text{sec} / \text{rad}, 70, 0.0029 \text{ kgm}^2, 2.6 \Omega, 2.54 \text{ cm}, 43.18 \text{ cm})^{(14)}$. The desired position of the ball is chosen to be $x_d = 11 \text{ cm}$. The constants (c_1, c_2, c_3) and (k_p, ϵ) are selected as $(5.5, 5.5, 5.5)$ and $(5.5, 0.001)$ respectively. It can be shown that the inverse of $b(x)$ exists for all time in the region $\{x | x_1, x_2 \in R, x_3 \in R - \left\{\pm \frac{5g}{7}\right\}, x_4 \in R\}$. The simulation results are shown in Figs. 7 and 8. According to Eq. (4), the controller is designed without any knowledge of perturbations in this experiment. Figures 9 and 10 are the experimental results of using C language to implement the controller to actually control the "Ball and Beam" system. From these results one can see this experiment is successful, and the applicability of the proposed control scheme is verified.

7. Conclusions

The robust tracking controller with perturbation estimation scheme is successfully proposed for a class of nonlinear dynamic system in this paper. Since the proposed control input function is piecewise continu-

ous, there is no chattering phenomenon at all. The tracking accuracy can be adjusted through two stages. The first stage is to reduce the perturbation estimation errors and it is done by choosing a smaller value of ϵ_i . The second stage is done by increasing the value of coefficient $k_{p,i}$ or the value of roots of sliding functions. In addition, due to the estimation process embedded in the controllers, the controllers have the ability of self-adjustment to cope with large variations of perturbation. This is demonstrated in the example that the tracking accuracy as well as robustness of stability still can be maintained at the same time even the perturbation becomes very large compared to its original value. In summary, the proposed control scheme has the following advantages: 1. The chattering phenomenon caused by the proposed controllers will not happen. 2. The knowledge of least upper bound of perturbations is not required. 3. The estimation process is simple. 4. The tracking accuracy is adjustable. 5. The proposed control scheme can yield higher attenuation of perturbation than the traditional SMC does, and it has the ability to filter out DC offset.

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