Modeling and Control for an Asymmetric Hydraulic Active Suspension System*

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In this paper we present a model for an automotive active suspension system which includes the dynamics of an asymmetric hydraulic actuator. In this model the force exerted by a single-rod cylinder is regarded as an internal state, and the sum of the oil flow rates through the orifice of a servo valve as the control input. We obtain a linear time invariant (LTI) state state equation and propose a force-tracking-free one-step control method which can accept various linear control techniques. An optimal state feedback control is applied as an example. Quarter car test rig experiment results show the effectiveness of the proposed approach in modeling and control.

Key Words: Active Suspension, Asymmetric Hydraulic Actuator, LTI Model, Force Control, Linear Control

1. Introduction

Over the last three decades, extensive research in the field of automotive active suspension design has been carried out to improve suspension quality which is mainly defined by ride comfort and road holding ability. Sky-hook control is simple but effective, especially in ride comfort improvement⁽¹⁾ and has been adopted in a real car applications⁽²⁾. Many other control methods like LQG, adaptive and H_∞ control were tried to meet such needs as the balance between ride comfort and road holding, robustness against model uncertainty⁽³⁾⁻⁽⁵⁾.

The above methods basically assume that the actuator dynamics is ideal or so fast that any reference force command can be generated exactly by the actuator. And in the design of a control system, two separate steps are usually followed: design of the

reference force command, which neglects the actuator dynamics, and design of the force controller, which takes the actuator dynamics into account. A proportional (P) type force control, however, requires an impractical enormous feedback gain⁽⁶⁾. The limitation of a proportional-integral-derivative (PID) force controller is analyzed in terms of the slightly damped open loop zeros inherent from the system-actuator configuration⁽⁷⁾. In order to track a relatively simple sky-hook force command, a complicated adaptive sliding mode control is used⁽⁸⁾. Despite the significant research efforts, the overall performance of an active suspension remains quite dependent on the specific force controllers in the conventional two-step approach.

Limited package space makes most active suspension systems accept single-rod hydraulic cylinders combined with control valves which we call *asymmetric hydraulic actuators*, yet the dynamic model for the actuators to date is based on the symmetric ones^{(6),(8)}. However, the behaviour of the asymmetric actuators is very different and attention needs to be paid to the derivation of model equations⁽⁹⁾. Hence, this paper seeks to solve this problem as well.

In this paper, we propose a model for a quarter car active suspension system which includes an asymmetric hydraulic actuator. The force exerted by the

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actuator is regarded as one of internal states rather than a virtual input, and the total oil flow rates through a servo valve as the control input. By approximating a nonlinear equation at a nominal operating point, we obtain an LTI dynamic equation in state space.

Next, utilizing the structure of the proposed model, we propose a one-step control which is free of the force tracking loop. Here, neither the generation of the reference force nor tracking of that force command is necessary any longer. As a consequence, many powerful control algorithms for LTI systems can be applied with no difficulty and the linear quadratic regulator (LQR) technique is used as an example. The validity and effectiveness of the proposed modeling and control method are justified through the quarter car test rig shaking experiment at the control laboratory of POSTECH.

The rest of the paper is organized as follows. After the modeling of a quarter car with an asymmetric hydraulic actuator is presented in Section 2, a force-tracking-free one-step control is given and the LQR method is applied in Section 3. Test rig experiment results are illustrated in Section 4. Concluding remarks follow in Section 5.

Nomenclature

- M_s : sprung mass = 280 (kg)
 M_u : unsprung mass = 50 (kg)
 K_t : tire stiffness = 1.79×10^5 (N/m)
 B_t : tire damping coefficient = 895 (N/(m·s))
 K_s : passive spring constant = 2.35×10^4 (N/m)
 B_s : passive damping coefficient = 1500 (N/(m·s))
 K_{eq} : gain of servo valve = 3.56 (m^{3.5}/kg^{0.5}/V)
 z_s : sprung vertical displacement (m)
 z_u : unsprung vertical displacement (m)
 z_r : road vertical displacement (m)
 z_{su} : cylinder stroke, i.e., $z_s - z_u = -0.08 \sim 0.08$ (m)
 v : input voltage to servo valve (V)
 q_1, q_2 : oil flow rates through servo valve (m³/s)
 q_s : sum of oil flow rate, i.e., $q_1 + q_2$ (m³/s)
 p_1, p_2 : pressures in cylinder chamber (Pa)
 $\Delta p_1, \Delta p_2$: pressure drops across orifice (Pa)
 P_s : supply pressure = 85×10^5 (Pa)
 A_1, A_2 : piston areas = $1.96 \times 10^{-3}, 1.35 \times 10^{-3}$ (m²)
 B : bulk modulus of oil = 1.4×10^9 (Pa)
 V_1, V_2 : oil volumes in chamber (m³)
 V_{10}, V_{20} : neutral volume when $z_{su} = 0$ = $1.96 \times 10^{-4}, 1.35 \times 10^{-4}$ (m³)
 L : length of piston rod = 0.16 (m)
 l_1, l_2 : lengths of piping line = 0.02 (m)
 f_a : actuating force exerted by cylinder (N)

Fig. 1 Configuration of quarter car active suspension system

2. Modeling of a Quarter Car

In this section, we derive equations for an asymmetric hydraulic actuator and obtain a state space model for a quarter car active suspension system.

2.1 Asymmetric hydraulic actuator

The actuator considered in this paper is a combined single-rod cylinder and a 4-way servo valve (Fig. 1). Both the oil flow rates into and out of the cylinder are controlled by a single servo valve's spool motion, which is controlled by a solenoid. We make the following assumptions:

- The valve dynamics is fast enough to be neglected, i.e., $x_{sp} = K_v v$.
- Oil leakage and friction are negligible in the cylinder.
- The direction of oil flow is solely determined by the sign of the spool position, i.e., P_s is sufficiently high so that reverse flow does not occur.

The pressure drop across the valve orifice is defined according to the sign of the input voltage as follows.

$$\Delta p_1 = \begin{cases} P_s - p_1 & \text{if } v \geq 0 \\ p_1 & \text{if } v < 0 \end{cases} \quad (1)$$

$$\Delta p_2 = \begin{cases} p_2 & \text{if } v \geq 0 \\ P_s - p_2 & \text{if } v < 0 \end{cases} \quad (2)$$

If the geometry of the valve orifice is symmetric, the oil flow rates are as follows⁽¹⁰⁾.

$$q_i = K_{eq} v \sqrt{\Delta p_i}, \quad i = 1, 2 \quad (3)$$

The sign of q_1 is chosen positive when the oil flows into the cylinder and negative when it does out of the cylinder. The sign convention for q_2 is reversely defined.

From the continuity equation of fluid⁽¹⁰⁾, we have

$$\begin{aligned} q_1 &= A_1 \dot{z}_{su} + (V_1/B) \dot{p}_1 \\ q_2 &= A_2 \dot{z}_{su} - (V_2/B) \dot{p}_2 \end{aligned} \quad (4)$$

where V_1 and V_2 are

$$\begin{aligned} V_1(z_{su}) &= V_{10} + A_1 z_{su} \\ V_2(z_{su}) &= V_{20} - A_2 z_{su}. \end{aligned} \quad (5)$$

The force exerted by the cylinder is defined by

$$f_a = A_1 p_1 - A_2 p_2 \quad (6)$$

Differentiating both sides of Eq. (6) and rearranging Eq. (4), the following cylinder equation is obtained.

$$\dot{f}_a = -B \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2} \right) \dot{z}_{su} + B \left(\frac{A_1}{V_1} q_1 + \frac{A_2}{V_2} q_2 \right) \quad (7)$$

From Eq. (3), q_s is determined by

$$q_s = K_{eq} v (\sqrt{\Delta p_1} + \sqrt{\Delta p_2}). \quad (8)$$

For simplicity, defining

$$r_1 = \frac{\sqrt{\Delta p_1}}{\sqrt{\Delta p_1} + \sqrt{\Delta p_2}} \quad (9)$$

$$r_2 = \frac{\sqrt{\Delta p_2}}{\sqrt{\Delta p_1} + \sqrt{\Delta p_2}} \quad (10)$$

$$\alpha(z_{su}) = -B \left(\frac{A_1^2}{V_1(z_{su})} + \frac{A_2^2}{V_2(z_{su})} \right) \quad (11)$$

$$\beta(z_{su}, r_1, r_2) = B \left(\frac{A_1}{V_1(z_{su})} r_1 + \frac{A_2}{V_2(z_{su})} r_2 \right) \quad (12)$$

and rearranging Eq. (7), we obtain

$$\dot{f}_a = \alpha(z_{su}) \dot{z}_{su} + \beta(z_{su}, r_1, r_2) q_s. \quad (13)$$

Though Eq. (13) is the exact description of the asymmetric hydraulic actuator, the nonlinearity in α and β creates inconvenience in dealing with them. Thus, by defining the nominal values, $\hat{r}_1$, $\hat{r}_2$, $\hat{\alpha}$ and $\hat{\beta}$ for $z_{su} = 0$, $\Delta p_1 = \Delta p_2 = P_s/2$,

$$\hat{r}_1 = \hat{r}_2 = 1/2 \quad (14)$$

$$\hat{\alpha} = -\hat{B} \left(\frac{A_1^2}{V_1(0)} + \frac{A_2^2}{V_2(0)} \right) \quad (15)$$

$$\hat{\beta} = \frac{\hat{B}}{2} \left(\frac{A_1}{V_1(0)} + \frac{A_2}{V_2(0)} \right) \quad (16)$$

we arrive at the final LTI equation from Eq. (13).

$$\dot{f}_a = \hat{\alpha} \dot{z}_{su} + \hat{\beta} q_s. \quad (17)$$

As will be shown through experiment in the later section, the linearization error from Eq. (13) to Eq. (17) exerts a negligible influence on the behaviour of the hydraulic actuator.

We remark that the above equations can be also applied to symmetric actuators. However, the fact that the piston areas are equal in both sides allows more simplification: by noting $P_s = p_1 + p_2$ and defining the load pressure by $p_L = p_1 - p_2$, we will have Eq. (17) without approximation, which is the same as that in the literature^{(6),(8)}.

2.2 State space description of a quarter car

The equations of motion for the quarter car in Fig. 1 are as follow.

$$M_s \ddot{z}_s = f_a \quad (18)$$

$$M_u \ddot{z}_u = -B_t (\dot{z}_u - \dot{z}_r) - K_t (z_u - z_r) - f_a. \quad (19)$$

The tire is modeled as a parallel spring and damper. Note that passive elements (a coil spring and a shock absorber) are not included in this model, because their effects are overwhelmed by the actuating force of the actuator if the supply pressure is sufficiently high.

Finally, the state space model of a quarter suspension system with actuator dynamics is derived by combining Eqs. (17), (18) and (19) as

Table 1 Eigenvalue distribution of A

type	eigenvalues
passive	$-2.15 \pm 8.46i$, $-24.48 \pm 57.84i$
active	0 , $-1.35 \pm 23.22i$, $-7.60 \pm 1.05 \times 10^3 i$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1 w + \mathbf{B}_2 u \quad (20)$$

where the state vector $\mathbf{x}$, the disturbance input w and the control input u are chosen as

$$\mathbf{x} = [z_s - z_u \quad \dot{z}_s \quad z_u - z_r \quad \dot{z}_u \quad f_a]^T,$$

$$w = \dot{z}_r, \quad u = q_s$$

and the system matrices are

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1/M_s \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -K_t/M_u & -B_t/M_u & -1/M_u \\ 0 & \hat{\alpha} & 0 & -\hat{\alpha} & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ B_t/M_u \\ 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \hat{\beta} \end{bmatrix}.$$

Note that the force f_a belongs to the state vector $\mathbf{x}$ and the sum of oil flow rates q_s is the control input. And it is easily verified that $(\mathbf{A}, \mathbf{B}_1)$ is a stabilizable pair and $(\mathbf{A}, \mathbf{B}_2)$ is a controllable one.

The eigenvalues of $\mathbf{A}$ (the open loop poles) are compared in Table 1 with those of the passive system which is equipped only with a spring and damper. Totally different eigenvalue distribution implies that an active suspension system is not merely a supplement of a decoupled actuator parallel to a spring and damper. This also explains why the conventional force control approach is not an easy task.

3. Force-Tracking-Free Controller Design

In this section, we propose a one-step control system which is free of force tracking and compare it with the conventional one containing the force controllers.

First, note that many active suspension control systems to date are configured with an inner and an outer loops as shown in Fig. 2. The controller design procedure for this case takes two successive steps, as follows.

1. (outer loop) Assuming the dynamics of an actuator is ideal or negligible, find a reference force that will best improve the suspension quality.

2. (inner loop) Taking the actuator dynamics into consideration, design a force controller that tracks the above force command.

Although this sequential approach seems logical, the design procedure is complicated: it is difficult to

Fig. 2 Block diagram of the conventional control system

Fig. 3 Block diagram of the proposed control system

optimize both loops simultaneously. A further defect is that the overall performance of a suspension system highly depends on the capability of a specific force controller.

On the other hand, the one-step control system we propose (Fig. 3) is simpler than the conventional one (Fig. 2). We no longer assume that the actuator is ideal or has negligible dynamics. Rather we regard the active suspension system as a generalized plant in which both the vehicle and actuator dynamics are contained. Owing to this structure, the force controller is not required and only one controller is sufficient in order to improve the quality of the suspension system. Thus, many well-established control techniques applicable to LTI systems can be used in this case with no difficulty. We note that, since the limited actuator capacity is fully reflected in the modeling and controller design phase, the controller-dependent performance deviation for the force control approach seldom occurs.

Now, we employ the LQR method as an example, which is useful for balancing conflicting control objectives. It is assumed that the state is measurable for simplicity. Suppose that a linear quadratic cost functional is chosen as

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\rho_1 \dot{z}_s^2 + \rho_2 \dot{z}_{su}^2 + \rho_3 \dot{z}_{ur}^2 + \rho_4 \dot{z}_u^2 + \rho_5 \dot{f}_a^2 + r u^2) dt \quad (21)$$

where scalars $\rho_i \geq 0$ and a scalar $r > 0$. Then the control input which minimizes Eq.(21) is given by a state feedback

$$u = -r^{-1} B_2^T P x \quad (22)$$

where $P \geq 0$ is a stabilizing solution to the following algebraic Riccati equation⁽¹¹⁾

$$A^T P + P A - r^{-1} P B_2 B_2^T P + Q = 0 \quad (23)$$

(a) $\dot{z}_s / \dot{z}_r$ (b) $(\dot{z}_u - \dot{z}_r) / \dot{z}_r$

Fig. 4 Comparison of Bode plots between passive and active systems (dotted line: passive, solid line: active)

where $Q = \text{diag}[\rho_1, \rho_2, \rho_3, \rho_4, \rho_5]$.

The true control input v can be calculated from Eqs.(8) and (22) using the measurement of the pressures p_1 and p_2 as

$$v = u \times [K_{eq}(\sqrt{\Delta p_1} + \sqrt{\Delta p_2})]^{-1}. \quad (24)$$

The above conversion is always possible during the normal operation of the actuator. Possible failure, when the piston reaches the stroke limit, seldom occurs because an extra safety stroke is added in devising a hydraulic system.

4. Experiment Results

In this section we present quarter car test rig experiment results and compare them with the theoretic simulation based on the proposed model.

4.1 Response of the proposed model

We select the design parameters in Eq.(21) as

$$Q = \text{diag}[104.1, 3906.2, 62500, 400, 1.0 \times 10^{-16}], \quad r = 2.25 \times 10^8. \quad (25)$$

The corresponding state feedback gain is

$$K = -[4166 \ 752 \ 8864 \ 1508 \ 2.43] \times 10^{-6}. \quad (26)$$

As shown in Fig. 4, the ride comfort measured by the sprung acceleration and the road holding ability by the tire deflection are more improved near the sprung

(a) Linear case (dashed line : z_s , dotted line : z_u , solid line : z_r)

(d) Oil pressures (dotted line : p_1 , solid line : p_2)

(b) Nonlinear case (dashed line : z_s , dotted line : z_u , solid line : z_r)

(e) Sum of flow rates (dotted line : nonlinear, solid line : linear)

(c) Actuator force (dotted line : nonlinear, solid line : linear)

(f) Voltage input to the servo valve

Fig. 5 Theoretic simulation (time response to a 1 Hz sinusoidal road disturbance)

resonance frequency (1~2 Hz) than those of a conventional passive system.

Figure 5 represents time response when a 1 Hz 0.3 m peak-to-peak sinusoidal road disturbance is applied. While the transmission of the road roughness (z_r) to the sprung mass (z_s) is kept within 35%, which implies good ride comfort, the unsprung mass (z_u) tracks the road profile (z_r) efficiently, which means good road holding (Fig. 5(a)). Also, it is apparent that the linearized LTI model in Eqs. (17) and (20) is a reasonable choice because differences between the

linear and nonlinear cases (Fig. 5(a) and (b), (c), (e)) are rarely found. Figure 5(d) and (f) show the plot of the pressures and the real control input by Eq. (24).

4.2 Test rig shaking experiment

The shaking experiments are performed on the laboratory test rig at POSTECH, which is devised to imitate a quarter portion of a real passenger sedan. The test rig, as shown in Fig. 6, is equipped with two sets of asymmetric hydraulic actuators (one for the excitation of road profiles and the other for the

actuator of active suspension), automotive parts like sprung/unsprung mass, pneumatic tires and mechanical supporting guides. PC-based control system drives the servo valves through Digital-to-Analog converts and acquires the measured signals of the accelerometer, load cell, pressure sensor and liner potentiometer by Analog-to-Digital converters.

Fig. 6 Quarter car test rig at POSTECH

(a) Vertical displacement (dashed line: z_s , dotted line: z_u , solid line: z_r)

(b) Actuator force

(c) Oil pressures (dotted line: p_1 , solid line: p_2)

(d) Sum of flow rates

(e) Voltage input to the servo valve

Fig. 7 Experiment (time response to a 1 Hz sinusoidal road disturbance)

5. Concluding Remarks

In this paper we have proposed a novel model and control for an automotive active suspension system. The dynamics of an asymmetric hydraulic actuator suitable for the automotive application is included in the model. By considering the force as a state and the flow rate as an input, and approximating the nonlinear parameters at a normal operation point, we obtained an LTI state space equation. The merit of this approach has enabled a one-step control method which removed the unnecessary force control loop of the previous approach. Accordingly, many powerful linear control techniques like the LQR method used in this paper can be easily adopted. As a result, the system analysis and controller design can be done in a systematic way and the performance of an active suspension is not influenced by the limited capability of force controllers. We have established the effectiveness of the proposed model and control by demonstrating a reasonable agreement between the theoretic simulation and the actual test rig experiments. Future research will aim at the investigation of the neglected modeling uncertainties and the application of robust control methods.

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