Analytical Prediction of Three Dimensional Chatter Stability in Milling*

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The chip regeneration mechanism during chatter is influenced by vibrations in three directions when milling cutters with ball end, bull nose, or inclined cutting edges are used. A three dimensional chatter stability is modeled analytically in this article. The dynamic milling system is formulated as a function of cutter geometry, the frequency response of the machine tool structure at the cutting zone in three Cartesian directions, cutter engagement conditions and material property. The dynamic milling system with nonlinearities and periodic delayed differential equations is reduced to a three dimensional linear stability problem by approximations based on the physics of milling. The chatter stability lobes are predicted in the frequency domain using the proposed analytical solution, and verified experimentally in milling a Titanium alloy with a face milling cutter having circular inserts.

Key Words : Chatter, Stability, Milling

1. Introduction

Chatter vibrations occur due to relative vibrations between the cutting tool and workpiece, which leads to time-varying chip thickness. Since the cutting force is proportional to the chip thickness, the structural modes in the overall machine tool system are self excited, leading to unstable chatter vibrations. Unless avoided, machining with the presence of chatter leads to poor surface finish, chipping of the tool, and large dynamic loads on the spindle and table structures⁽¹⁾. It is essential to predict the stable cutting conditions in milling, the spindle speeds and depth of cuts, in order to increase the material removal rates.

The fundamental mechanism behind the chatter vibrations are presented by Thusty⁽²⁾, Tobias⁽³⁾, and later by Merritt⁽⁴⁾. Thusty's chatter stability theory relates the frequency response of the structure and material cutting constant directly to the chatter vibration free depth of cut, and has been widely used by researchers and in industry due to its simple and

straightforward formulation. However, the cutting forces have time varying directions and magnitude in milling, hence extrapolation of orthogonal cutting chatter theory may not always led to accurate prediction of chatter stability in milling operations. Opitz⁽⁵⁾ attempted to solve chatter in milling using simulations with analog computers. However, the cutter had ten teeth, the system had a single degree of freedom flexibility in only one direction, and the radial width of cut was almost half immersion center face milling which could have been approximated by Thusty's formulation. However, Sridhar et al.⁽⁶⁾ presented the first basic two dimensional chatter stability formulation in milling using a simple example, although they did not propose a mathematical solution due to complexity of periodic delayed differential equations. Opitz et al.⁽⁵⁾ approximated the three dimensional chatter stability into a one dimensional scalar problem by considering the average resultant dynamic cutting force. They used Thusty's one dimensional chatter stability law to identify the stability lobes. However, this simplification does not represent the general three dimensional eigenvalue system correctly, and hence fails to predict the chatter stability except at conditions where the modes are dominant only in one direction.

Considering the time varying cutting forces and

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chip loads, as well as the nonlinearities in the process due to tool jumping out of cut caused by excessive vibrations, Tlustý and Ismail⁽⁷⁾ presented numerical algorithms in predicting the chatter stability lobes in milling. The numerical method was applied by successive researchers for various applications^{(8)–(10)}.

Minis and Yanushevsky⁽¹²⁾ presented the first analytical prediction of chatter stability by iteratively solving the two dimensional dynamic milling problem. They used Floquet theory in solving the delayed differential equations which describe the dynamic milling process. Altintas and Budak⁽¹³⁾ solved the chatter stability directly without any iteration, and showed that the two dimensional chatter stability in end milling can be predicted as accurately as in time domain solutions, but in a shorter computation time period⁽¹⁴⁾. Their method was later extended to include distributed dynamics along the thin webs⁽¹⁴⁾, variable pitch cutters⁽¹⁵⁾ and face milling cutters with inclination angle⁽¹⁶⁾. Altintas et al.⁽¹⁷⁾ presented analytical prediction of chatter stability lobes for ball end milling operations, but the dynamics in the spindle direction were not considered, and hence the solution was valid only for two dimensional dynamic milling processes.

This paper extends the two dimensional chatter stability theory of Budak and Altintas⁽¹³⁾ to a three dimensional chatter problem, which is experienced when ball ended cutters are used, and the chip thickness regeneration is affected by the structural modes in all three axes of the cut. This problem was experienced during face milling of a helicopter part made of titanium alloy at a low angular speed range on a CNC horizontal machining center. Depending on the speed and depth of cut, the chatter was caused by either the spindle's bending modes in the feed plane, or by the axial flexibility of the machine which caused vibrations in the direction of the spindle axis. There were also cross coupling between modes in all three directions.

The analytical chatter stability model is formulated in the next section of the paper, followed by its application to the machining of Titanium on the mentioned horizontal machining center. The stability law is compared against experimental measurements. The authors show how the material removal rate can be maximized in milling difficult to cut aerospace materials using the proposed three dimensional chatter stability theory.

2. Three Dimensional Chatter Stability in Milling

End mills with ball end and bull nose, high inclination and helix angles, as well as indexed cutters with bull and circular inserts, produce nonuniform chip

Fig. 1 Chip regeneration mechanism in face milling with circular inserts when there are relative vibrations between the tool and workpiece in all three directions

thickness distribution along the cutting edge and work contact zone, creating cutting forces in three Cartesian directions. The cutting forces may excite the vibration modes in all three directions, and therefore lead to modulation of chip load as shown in Fig. 1. The cutting forces excite the structure in the feed (X), normal (Y), and spindle axis (Z) directions, causing dynamic displacements x , y and z , respectively. The dynamic displacements are carried to rotating tooth number (j) in the direction of chip thickness with the coordinate transformation of:

$$v_j = (x \sin \phi_j + y \cos \phi_j) \sin \gamma - z \cos \gamma \quad (1)$$

where ϕ_j is the instantaneous angular immersion of tooth (j) measured clockwise from the normal (Y) axis, and γ is the axial immersion measured from the tool tip in the spindle (Z) axis. γ is equal to the helix angle for end mills, to the inclination or equivalent lead angle for inserted cutters. Since the depth of cut is known prior to the chatter solution, the axial angular immersion can be assumed to be acting in the middle of the maximum axial depth of cut, i.e. $\gamma = \pi/2 + \pi/4$ from the spindle axis, for ball end mills or cutters having circular inserts. If the spindle rotates at an angular speed of Ω (rad/s), the immersion angle varies with time as $\phi_j(t) = \Omega t$, causing three

dimensional modulation of chip load with time. The resulting chip thickness consists of a static part ($c \sin \phi \sin \gamma$), which is due to the rigid body motion of the cutter, and the dynamic component caused by the vibrations at the present and previous tooth periods, where c is the feed per tooth. Since the chip thickness is measured in the radial direction (v_j), the total chip load can be expressed by,

$$h(\phi_j) = [c \sin \phi_j \sin \gamma + (v_{j,0} - v_j)]g(\phi_j) \quad (2)$$

where ($v_{j,0}$, v_j) are the dynamic displacements of the cutter at the previous and present tooth periods, respectively. $g(\phi_j)$ determines whether the tooth is in or out of cut, i.e.,

$$\left. \begin{aligned} g(\phi_j) &= 1 \leftarrow \phi_{st} < \phi_j < \phi_{ex} \\ g(\phi_j) &= 0 \leftarrow \phi_j < \phi_{st} \text{ or } \phi_j > \phi_{ex} \end{aligned} \right\} \quad (3)$$

where ϕ_{st} , ϕ_{ex} are start and exit immersion angles of the cutter to and from the cut, respectively. Henceforth, the static component of the chip thickness ($c \sin \phi_j \sin \gamma$) is dropped from the expressions since it does not contribute to the chatter stability. Substituting v_j into (2) yields,

$$h(\phi_j, \gamma) = [\Delta x \sin \phi_j + \Delta y \cos \phi_j] \sin \gamma - \Delta z \cos \gamma g(\phi_j) \quad (4)$$

where $\Delta x = x(t) - x(t - T)$, $\Delta y = y(t) - y(t - T)$, $\Delta z = z(t) - z(t - T)$, t is the present time and T is the tooth period. ($x(t)$, $y(t)$, $z(t)$) and ($x(t - T)$, $y(t - T)$, $z(t - T)$) represent the relative dynamic displacements between the work and cutter at the present (t) and previous tooth periods ($t - T$), respectively. The tangential (F_t), radial (F_r) and axial (F_a) cutting forces acting on the tooth j is proportional to the axial depth of cut (a) and chip thickness (h)⁽¹⁸⁾,

$$\begin{aligned} \begin{Bmatrix} dF_r \\ dF_t \\ dF_a \end{Bmatrix} &= K_t a \begin{Bmatrix} K_r \\ 1 \\ K_a \end{Bmatrix} \begin{Bmatrix} \Delta x \sin(\phi) \sin(\gamma) \\ \Delta y \cos(\phi) \sin(\gamma) - \Delta z \cos(\gamma) \end{Bmatrix} g(\phi_j) \\ &= K_t a \begin{Bmatrix} K_r \\ 1 \\ K_a \end{Bmatrix} \begin{Bmatrix} \sin(\phi) \sin(\gamma) & \cos(\phi) \sin(\gamma) \\ -\cos(\gamma) \end{Bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} g(\phi_j) \end{aligned} \quad (5)$$

where average cutting coefficients K_t , K_r and K_a are assumed to be constant. Resolving the cutting forces in the x , y , z directions,

$$\begin{aligned} \begin{Bmatrix} dF_x \\ dF_y \\ dF_z \end{Bmatrix} &= \begin{bmatrix} -\sin(\gamma) \cos(\phi) & -\cos(\phi) & -\cos(\gamma) \sin(\phi) \\ -\sin(\gamma) \cos(\phi) & \sin(\phi) & -\cos(\gamma) \cos(\phi) \\ \cos(\gamma) & 0 & -\sin(\gamma) \end{bmatrix} \begin{Bmatrix} dF_r \\ dF_t \\ dF_a \end{Bmatrix} \end{aligned}$$

$$\times \begin{Bmatrix} dF_r \\ dF_t \\ dF_a \end{Bmatrix} \quad (6)$$

and summing the cutting forces contributed by all teeth, the total dynamic milling forces acting on the cutter are found as,

$$F_x = \sum_{j=0}^{N-1} F_{xj}; F_y = \sum_{j=0}^{N-1} F_{yj}; F_z = \sum_{j=0}^{N-1} F_{zj} \quad (7)$$

where $\phi_j = \phi_p + j\phi$, and cutter pitch angle is $\phi_p = 2\pi/N$. Substituting the chip thickness (4) and tooth forces (5) into (6) and (7), and rearranging the resulting expressions in matrix form yields,

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = a K_t [A] \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} \quad (8)$$

where time varying directional dynamic milling force coefficients [$A(3 \times 3)$] are given by,

$$\begin{aligned} a_{xx} &= \sum_{j=0}^{N-1} g_j [-(\sin \phi \sin^2 \gamma \cos \phi) K_r \\ &\quad - \sin \phi \sin \gamma \cos \phi - (\sin^2 \phi \sin \gamma \cos \gamma) K_a] \\ a_{xy} &= \sum_{j=0}^{N-1} g_j [-(\sin^2 \gamma \cos^2 \phi) K_r \\ &\quad - \sin \gamma \cos^2 \phi - (\sin \gamma \cos \phi \cos \gamma \sin \phi) K_a] \\ a_{xz} &= \sum_{j=0}^{N-1} g_j [(\sin \gamma \cos \phi \cos \gamma) K_r \\ &\quad + \cos \gamma \cos \phi + (\cos^2 \gamma \sin \phi) K_a] \\ a_{yx} &= \sum_{j=0}^{N-1} g_j [-(\sin \phi \sin^2 \gamma \cos \phi) K_r \\ &\quad + \sin^2 \phi \sin \gamma - (\sin \gamma \cos \phi \cos \gamma \sin \phi) K_a] \\ a_{yy} &= \sum_{j=0}^{N-1} g_j [-(\sin^2 \gamma \cos^2 \phi) K_r \\ &\quad + \sin \phi \sin \gamma \cos \phi - (\sin \gamma \cos^2 \phi \cos \gamma) K_a] \\ a_{yz} &= \sum_{j=0}^{N-1} g_j [(\sin \gamma \cos \phi \cos \gamma) K_r \\ &\quad - \cos \gamma \sin \phi + (\cos^2 \gamma \cos \phi) K_a] \\ a_{zx} &= \sum_{j=0}^{N-1} g_j [(\sin \phi \sin \gamma \cos \gamma) K_r - (\sin \phi \sin^2 \gamma) K_a] \\ a_{zy} &= \sum_{j=0}^{N-1} g_j [(\sin \gamma \cos \phi \cos \gamma) K_r \\ &\quad - (\sin^2 \gamma \cos \phi) K_a] \\ a_{zz} &= \sum_{j=0}^{N-1} g_j [-(\cos^2 \gamma) K_r + (\cos \gamma \sin \gamma) K_a] \end{aligned}$$

Considering that the angular position of the parameters changes with time and angular velocity, the Eq. (8) can be expressed in time domain in a matrix form as,

$$\{F(t)\} = a K_t [A(t)] \{\Delta(t)\} \quad (9)$$

where $\{\Delta(t)\} = \{\Delta x \ \Delta y \ \Delta z\}^T$. As the cutter rotates, the directional factors vary periodically with time, which is the fundamental difference between milling and operations like turning where the direction of the force is constant. However, similar to two dimensional milling stability as presented earlier⁽¹³⁾, the average component of the Fourier series expansion of [A], can be considered by eliminating the periodic time varying coefficients.

$$[A_0] = -\frac{1}{T} \int_0^T [A(t)] dt$$

$$= \frac{1}{\phi_p} \int_{\phi_{st}}^{\phi_{ex}} [A(\phi)] d\phi = \frac{N}{4\pi} [a]. \quad (10)$$

where $[A_0]$ is valid only between the entry (ϕ_{st}) and exit (ϕ_{ex}) angles of the cutter (i.e. $g_s(\phi_s)=1$), and $\phi_p=2\pi/N$ is cutter pitch angle. Analytically evaluated elements of the directional factor matrix $[a]$ are given as follows:

$$a_{xx} = \left[-K_r(\sin^2 \phi \sin^2 \gamma) - \sin^2 \phi \sin \gamma \right. \\ \left. - K_a \left(-\frac{\sin 2\phi}{2} + \phi \right) \left(\frac{\sin 2\gamma}{2} \right) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{xy} = \left[-K_r \left(\frac{\sin 2\phi}{2} + \phi \right) \sin^2 \gamma \right. \\ \left. - \left(\frac{\sin 2\phi}{2} + \phi \right) \sin \gamma - K_a \frac{\sin 2\gamma}{2} \sin^2 \phi \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{xz} = 2 \left[K_r \left(\frac{\sin 2\gamma}{2} \sin \phi \right) - \cos \gamma \sin \phi \right. \\ \left. - K_a (\cos^2 \gamma \cos \phi) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{yx} = \left[-K_r(\sin^2 \phi \sin^2 \gamma) + \left(-\frac{\sin 2\phi}{2} + \phi \right) \sin \gamma \right. \\ \left. - K_a \left(\frac{\sin 2\gamma}{2} \sin^2 \phi \right) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{yy} = \left[-K_r \left(\frac{\sin 2\phi}{2} + \phi \right) \sin^2 \gamma + \sin^2 \phi \sin \gamma \right. \\ \left. - K_a \left(\frac{\sin 2\phi}{2} + \phi \right) \left(\frac{\sin 2\gamma}{2} \right) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{yz} = 2 \left[K_r \left(\frac{\sin 2\gamma}{2} \sin \phi \right) + \cos \gamma \cos \phi \right. \\ \left. + K_a (\cos^2 \gamma \sin \phi) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{zx} = 2 \left[-K_r \left(\frac{\sin 2\gamma}{2} \cos \phi \right) + K_a (\sin^2 \gamma \cos \phi) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{zy} = 2 \left[K_r \left(\frac{\sin 2\gamma}{2} \sin \phi \right) - K_a (\sin \phi \sin^2 \gamma) \right]_{\phi_{st}}^{\phi_{ex}}$$

$$a_{zz} = 2 \phi \left[-K_r \cos^2 \gamma + K_a \frac{\sin 2\gamma}{2} \right]_{\phi_{st}}^{\phi_{ex}}$$

The dynamic milling expression (9) is therefore reduced to the following

$$\{F(t)\} = \frac{N}{4\pi} a K_t [a] \{\Delta(t)\} \quad (11)$$

where $[a]$ is time invariant but immersion dependent directional cutting coefficient matrix. The regenerative vibration vector is $\{\Delta(t)\} = [1 - e^{-i\omega_c T}] [G(i\omega)] \{F(t)\}$, where the transfer function matrix ($[G(i\omega)]$) identified at the cutter-workpiece contact zone,

$$[G(i\omega)] = \begin{bmatrix} G_{xx}(i\omega) & G_{xy}(i\omega) & G_{xz}(i\omega) \\ G_{yx}(i\omega) & G_{yy}(i\omega) & G_{yz}(i\omega) \\ G_{zx}(i\omega) & G_{zy}(i\omega) & G_{zz}(i\omega) \end{bmatrix} \quad (12)$$

The dynamic cutting force system is reduced to $\{F(t)\} = \frac{N}{4\pi} a K_t (1 - e^{-i\omega_c T}) [a] [G(i\omega)] \{F(t)\}$. The chatter stability of the system is determined by the characteristic equation of the three dimensional dynamic mill-

ing system, and given by

$$\det\{[I] + \Lambda[\Phi]\} = 0. \quad (13)$$

where $[\Phi] = [a][G(i\omega_c)]$ and the complex eigenvalue Λ is,

$$\Lambda = -\frac{N}{4\pi} a K_t (1 - e^{-i\omega_c T}) = \Lambda_R + i\Lambda_I \quad (14)$$

The eigenvalue (Λ) can be solved for a given chatter frequency ω_c , static cutting constants (K_t, K_r, K_a) which can be stored as a material dependent quantity for any milling cutter geometry⁽¹⁸⁾, and radial immersion (ϕ_{st}, ϕ_{ex}) and frequency response function of the structure (12). The characteristic equation becomes a cubic function,

$$a_0 \Lambda^3 + a_1 \Lambda^2 + a_2 \Lambda + 1 = 0 \quad (15)$$

where

$$a_0 = (-\Phi_{11}\Phi_{22}\Phi_{33} + \Phi_{11}\Phi_{23}\Phi_{32} + \Phi_{21}\Phi_{12}\Phi_{33} \\ - \Phi_{21}\Phi_{13}\Phi_{32} - \Phi_{31}\Phi_{12}\Phi_{23} + \Phi_{31}\Phi_{13}\Phi_{22})$$

$$a_1 = (\Phi_{22}\Phi_{33} - \Phi_{23}\Phi_{32} + \Phi_{11}\Phi_{33} + \Phi_{11}\Phi_{22} - \Phi_{21}\Phi_{12} \\ - \Phi_{31}\Phi_{13})$$

$$a_2 = (-\Phi_{33} - \Phi_{22} - \Phi_{11})$$

Since the transfer functions are complex, the eigenvalue has a real and an imaginary part, $\Lambda = \Lambda_R + i\Lambda_I$. Substituting the eigenvalue and $e^{-i\omega_c T} = \cos \omega_c T - i \sin \omega_c T$ in Eq.(14) gives the critical axial depth of cut at chatter frequency ω_c ,

$$a_{lim} = -\frac{2\pi}{NK_t} \left[\frac{\Lambda_R(1 - \cos \omega_c T) + \Lambda_I \sin \omega_c T}{(1 - \cos \omega_c T)} \right. \\ \left. + i \frac{\Lambda_I(1 - \cos \omega_c T) - \Lambda_R \sin \omega_c T}{(1 - \cos \omega_c T)} \right] \quad (16)$$

Since a_{lim} must be a real number, and the imaginary part of the Eq.(16) must vanish,

$$\Lambda_I(1 - \cos \omega_c T) - \Lambda_R \sin \omega_c T = 0 \quad (17)$$

By substituting,

$$\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin \omega_c T}{1 - \cos \omega_c T}. \quad (18)$$

into the real part of the Eq.(16), the final expression for chatter free axial depth of cut is found as:

$$a_{lim} = -\frac{2\pi\Lambda_R}{NK_t} (1 + \kappa^2) \quad (19)$$

Therefore, given the chatter frequency (ω_c), the chatter limit in terms of the axial depth of cut can directly be determined from Eq.(19), which has a similar form with the classical orthogonal cutting stability model proposed by Tlustý⁽²⁾. The corresponding spindle speeds are also found in a manner presented by Tlustý⁽²⁾ and Tobias⁽³⁾. From Eq.(18), the angular distance travelled by the tooth due to chatter frequency ω_c at tooth period T is found as

$$\omega_c T = \cos^{-1} \frac{\kappa^2 - 1}{\kappa^2 + 1} = -\cos^{-1} 2\phi. \quad (20)$$

Note that $\kappa = \Lambda_I/\Lambda_R = \tan \phi$ and ϕ is the phase shift of the eigenvalue. Thus if k is the integer number of full vibration waves (i.e. lobes) imprinted on the cut arc,

$$\omega_c T = \pi - 2\phi + 2k\pi = \epsilon + 2k\pi \quad (21)$$

where $\phi = \tan^{-1} \kappa$ and $\epsilon = \pi - 2\phi$ is the phase shift

Fig. 2 Titanium Ti6Al4V mounted on the angle plates of a Horizontal CNC machining center

between inner and outer modulations (present and previous vibration marks). The spindle speed n (rev/min) is simply calculated by finding the tooth passing period T (s) and corresponding spindle speed n [rev/min]

$$T = \frac{1}{\omega_c} (\epsilon + 2k\pi) \rightarrow n = \frac{60}{NT} \quad (22)$$

Then the stability lobes are calculated as outlined in the previous article given for two dimensional chatter stability⁽¹³⁾.

3. Engineering Application and Experimental Proof

The three dimensional chatter problem was experienced when milling an aerospace part made of Titanium alloy Ti6Al4V on a three axis horizontal CNC machining center, see Fig.2. The direct and cross transfer functions of the machine in the feed (x) and normal (y) directions on the spindle, and axial direction (z) on the angle plate where the work is clamped, are measured. The three most flexible modes are a 44 Hz mode of the angle plate and table in (G_{zz}), and two spindle quill modes at 28 Hz and 55 Hz. The 44 Hz mode in z direction is due to rocking of entire front half of the machine's base. The 28 Hz and 55 Hz modes are cantilevered modes of the quill and tool holder in (x) direction. In order to account for all the modes and cross coupling of the modes, the measured transfer functions are used directly in the solution. However, the basic modal parameters of the machine identified from measurements are given in Table 1 as an approximate guidance for the analysis. In this paper, a face milling cutter with 76.2 mm diameter and four circular inserts

Table 1 Modal parameters of three axis CNC machining center at the cutting point. The (–) signs in the stiffness values represent displacement opposite to the force in cross transfer functions.

G	ω_n (Hz)	ζ	$k \times 10^6$ [N/m]	G	ω_n (Hz)	ζ	$k \times 10^6$ [N/m]
G_{xx}	28	0.17	25.4	G_{zy}	28	0.13	(–) 80.3
	55	0.12	43		54	0.07	324.7
G_{yy}	28	0.10	215	G_{zx}	49	0.096	(–) 1669.34
	55	0.06	618		98	0.043	(–) 1140.0
	109	0.09	173				
G_{zz}	44	0.20	72.8	G_{yz}	42.3	0.076	1290
	84	0.15	197.4		49	0.065	1818
	98	0.06	906.2		57.4	0.06	(–) 1078

Fig. 3 Stability lobes when the measured direct and cross transfer functions of machine tool spindle and angle plate are considered in all three directions. Cutting Conditions: Workpiece Titanium Ti6Al4V; slot milling with $c=0.1$ [mm/tooth] feed rate; cutting force windows are displayed exactly for one revolution of spindle in both speeds.

having 12 degree average rake angle and 25.4 mm diameter is used. The cutting constants $K_t=1800$ [N/mm²], $K_r=0.16$, $K_a=0.45$ for the circular inserts and cutter pocket geometry are evaluated by oblique transformation of orthogonal cutting parameters as presented in Ref.(18) for Ti6Al4V alloy. The proposed three dimensional chatter stability is used in predicting the stability lobes which leads to optimal cutting conditions for maximum material removal rate. Analytically evaluated stability lobes and experimental measurements are shown in Fig. 3 for the slot milling of Ti6Al4V. The chatter vibrations are caused

Fig. 4 Stability lobes when the measured direct and cross transfer functions of machine tool spindle and angle plate are considered only in xy plane. The cutting conditions are same as in Fig. 3.

by 28 Hz at speeds under 150 rev/min where the stability pockets are very narrow. The chatter was detected by checking the surface finish and power spectrum of the microphone and force measurement signals. When the spectrum is dominated by the tooth passing frequency, the process is chatter free and stable. Chatter is present when the spectrum is dominated by the modal frequencies of the structure. It can be seen that the absolute limit depth of cut is about 7.5 [mm], and even higher depth of cut can be removed at speeds about 375 rev/min and 250 rev/min or at tooth passing frequencies of 25 Hz and 16.7 Hz, respectively. Although the lobes are due to the 28 Hz and 55 Hz spindle modes, the lobe pockets are shifted due to the directional orientation of axial modes. The prediction still suffers at 200 rev/min but above 8 mm depth of cut. The predicted lobe, which causes the discrepancy between the experiments, is due to the 28 Hz mode of the spindle. This can be due to difficulties in measuring the direct and cross transfer functions at such a low frequency range, and approximations made in the analytical solution of 3-D chatter stability in milling. However, overall the solution agrees reasonably well with the experimental observations. Sample cutting force measurements for one spindle period and their Fourier Spectrums evaluated from the whole cutting test are shown in Fig. 3. When the spindle speed is 225 rev/min or tooth passing frequency is 15 Hz, the chatter is observed at 55 Hz. The cutting forces are dominated by 55 Hz chatter frequency. When the speed is increased to 375 rev/min, the chatter disappears completely and the cutting forces have quasi static wave forms at 25 Hz tooth passing frequency. The spectrum has almost no strength at the 55 Hz chatter frequency. When the influence of axial modes

are neglected ($G_{xz}=G_{yz}=G_{zz}=0$), the predicted stability lobes were not in agreement with experimental measurements, see Fig. 4. It can be seen from Fig. 3 and 4 that the axial modes change the shape and location of lobes caused by the in plane spindle modes due to orientation factors given in Eq.(10). However, when all the modes and cross transfer functions are included in the 3-D chatter stability solution proposed here, the predicted stability lobes agree reasonably well with the experimental results.

4. Conclusion

A three dimensional chatter stability theory is proposed for cutters having ball end, bull nose and inclined cutting edges, and for machine tool structures which have dynamic flexibility in all three directions. The theory extends the previously developed two dimensional chatter stability law presented by Budak and Altintas^{(13),(14)}. It is shown that it is important to consider the structural modes in all directions, if they contribute to the chip regeneration mechanism. The solution is purely analytical and in the frequency domain, which can be easily implemented in a computer program in analyzing complex chatter vibrations in milling with helical, bull nose, and ball ended cutters. The viability of the proposed method is proven with a number of experiments conducted in low speed milling of Titanium alloy with a face milling cutter having circular inserts.

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