Gear-Cutting Tool for Screw-Compressor Rotors*

(Built-Up Hob)

Katsumi KANEKO**, Kazumasa KAWASAKI***
and Hisashi TAMURA***

A built-up hob for a screw-compressor rotor is proposed. The hob is composed of cutter blades and a hob head. The blade is cut out from a tempered planar plate of high speed tool steel by wire-cut electrodischarge machining (EDM). During the wire cutting process, the wire electrode has a fixed angle to ensure both the side and the end relief angle of the cutting edge, so that the rake face regrinding sharpens the cutting edge without changing the profile of the rake face. In other words, theoretically, the hob has no accuracy deterioration due to rake face regrinding. The formulae of rack tooth profile are derived from the female rotor tooth profile expressed by numerical formulae. Next, the fundamental helicoid of the hob and the profile of the cutting edge of the blade are calculated. The hob was fabricated and the rotor hobbing was carried out.

Key Words: Machine Element, Gear, Tool, Hob, Screw Compressor, Tooth Profile, Wire-Cut EDM, Built-Up Hob, Regrind

1. Introduction

The rotor of a screw compressor can be considered to be a kind of helical gear, in which, the rotor tooth profile is composed of a number of curved lines. It is desired that the rotor is machined as efficiently as a helical gear. To respond to this requirement, rotor machining should be carried out using a hob. This study is related to the fabrication and characterization of such a hob.

The cutting edge profile of the hob consists of a number of curved lines, corresponding to the rotor tooth profile, which is not less than seven⁽¹⁾; furthermore, the pressure angle α at a point on the cutting

edge changes continuously along the edge from nearly 0 to ± 90 degrees. This wide range of change of the pressure angle induces significant difficulties during the process of precision machining of the hob, in particular when machining the clearance surface of the cutting edge. It also induces a rapid deterioration in accuracy due to the rake surface regrinding. Hobs which do not show accuracy deterioration due to regrinding have not yet been fabricated. The conventional involute hob, whose cutting edge is a simple curve, is not an exception⁽²⁾, although the accuracy does not deteriorate so rapidly.

The reason for the accuracy deterioration is as follows. The ideal clearance surface of the cutting edge becomes a very complicated surface because the cutting edge profile must theoretically change according to the decrease of the hob diameter due to regrinding. Therefore, the ideal clearance surface cannot be expressed mathematically at present, and hence cannot be ground precisely by a grinding wheel. As a result, a clearance surface which does not induce accuracy deterioration due to regrinding is difficult to be machined at present.

The accuracy deterioration of the hob induces

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** Graduate School of Science and Technology, Niigata University, 8050 Ikarashi 2-nocho, Niigata 950-2181, Japan

*** Department of Mechanical and Production Engineering, Niigata University, 8050 Ikarashi 2-nocho, Niigata 950-2181, Japan. E-mail: kawasaki@eng.niigata-u.ac.jp

that of the rotor.

To tackle the accuracy deterioration problem, a built-up type hob with no accuracy deterioration and with no decrease of the hob diameter due to regrinding of the rake surface is proposed in this paper.

2. Construction of a Built-Up Hob

The proposed hob is mainly composed of a hob-head and many blades, each of which has a cutting edge of a given profile. The blades are cut out from a planar plate using wire-cut electrodischarge machining (EDM) in a simple manner.

When the cutting edges wear down, the blades are removed from the hob-head and the rake surface of each blade is reground to sharpen the edge using a conventional surface grinding machine. In the proposed hob, the entire contour of the rake surface does not change due to regrinding, theoretically and only the blade thickness decreases. Taking the thickness decrease into consideration, the blades are built into the hob-head in such a manner that the hob diameter does not change. Since neither the blade contour nor the hob diameter change, the proposed hob does not induce any accuracy deterioration due to the rake surface regrinding.

2.1 Blade

The proposed hob is composed of a hob-head, blades, wedges and bolts as shown in Fig. 1, along with screws at both sides of the hob-head. Each blade is clamped to the hob-head by a wedge and bolts.

Figure 2 shows a blade (loophole) for the male rotor and a blade for the female rotor. The calculation method for the cutting edge profile is described below. First, we consider that an infinite number of sheets of infinitesimally thin films having the same contour are piled up, as shown in Fig. 3(a). A part of the blade contour corresponds to the profile of the cutting edge of the hob. Second, we consider that each film is shifted by a fixed phase difference toward the arrow shown in Fig. 3(a), and that the films are

Fig. 1 Built-up hob

bound in one block which constitutes a blade. The shifting of the films induces a clearance (relief) angle to the cutting edge when the upper film surface becomes the rake surface of the blade in the rotor hobbing.

The pressure angle on the cutting edge changes from nearly 0 to ± 90 degrees, thus, two clearance angles must be considered: an end and a side clearance angle. Figure 3(b) shows the inducement of the end clearance angle δ , and Fig. 3(c) depicts the inducement of the side clearance angle Δ . In the figure, the contours of two sheets of film are shown; the upper film contour $ABCD$, and the lower contour $A'B'C'D'$ of a phase difference η per unit thickness of the blade. A part of the lower contour shown by the dashed line $A'-D'$ is recessed with respect to the upper one and will form both the side and the end clearance surfaces, along with $A-D$. The clearance angles δ and Δ are calculated by the following equations:

$$\tan \delta = \eta, \quad \tan \Delta = \tan \delta \tan \alpha \quad (1)$$

A part of the lower contour, the line $B'-C'$, represents an advanced shift from the respective upper line and forms a blade base surface, along with $B-C$. The angle between the base surface and rake surface is 90°

Fig. 2 Blades

Fig. 3 Blade for the built-up hob

$+\delta$. The remaining part of the lower contour lines $A'-B'$ and $C'-D'$ of the zero pressure angle will form each of the side surfaces of the blade, along with $A-B$ and $C-D$, respectively.

The blade explained above can be machined easily using wire-cut EDM, in which, a wire electrode is inclined on the surface $[ABB'A']$ by an angle δ , as shown in Fig. 3(b), and is moved along the contour of the film during the machining of the blade.

When the blades are built up on the hob-head, the cutting edge of each blade should be on the fundamental helicoids⁽³⁾, which is explained below, while the clearance surface should be within the fundamental helicoid. In the proposed hob, the direction of the thread is right handed with a lead angle γ on the hob pitch cylinder of diameter $2H$. The rake surface is a plane containing the hob axis, so that the side clearance angle should be less than $90^\circ - \gamma$ on the right-side cutting edge, and $90^\circ + \gamma$ on the left-side edge. The modification of the side clearance angle corresponding to angle γ can be ensured with additional wire inclination, when machining the clearance surface $[ADD'A']$. The inclination angle on the plane perpendicular to both the side surface $[ABB'A']$ and the rake surface $[ABCD]$ of the blade is γ . The angle γ is generally small, so that Eq.(1) always remains valid.

2.2 Hob-head and clamps

The hob-head should have a special form, corresponding to the blade, as shown in Fig. 1. There are two important planes to be set accurately. One is the plane $[bb']$ (see Fig. 1) which will be in contact with the blade base surface $[BB']$ (see Fig. 3) and the other is the plane $[ab]$ which will be in contact with the blade rake surface $[AB]$. These two planes make an angle of $90^\circ + \delta$ which should be related accurately to the end clearance angle δ . Plane $[ab]$ must contain the hob axis because it lies on the rake surface. Plane $[a'', b'']$, being parallel to the plane $[ab]$, makes a projection on the hob-head and forms a groove together with the back of the forward blade, when the blades are built-up on the hob-head. The blade is clamped by a wedge inserted in the groove and tightened to the hob-head with bolts.

The proposed built-up hob is composed of ten blades of the same type. Each blade should be built up with an axial phase difference P_d :

$$P_d = 2\pi H \tan \gamma / 10. \quad (2)$$

Two kinds of screws are equipped at both sides of the hob-head; one is a fine pitch screw to accurately adjust the axial phase difference of each blade, and the other is needed to prevent slippage of the blade from the hob-head during hobbing.

2.3 Regrindable hob

When the cutting edges wear down, the blades are removed from the hob-head and the rake surfaces are reground to sharpen the cutting edges, as stated above. Rake surface regrinding involves successively tearing off the films forming a contour as shown in Fig. 3, so that the contour of the new rake surface remains unchanged. Only the blade thickness is decreased due to the regrinding. Hence, there is no change in the blade, apart from the thickness change. The position of the wedge can be regarded as a measure of the thickness decrease.

Planes $[ab]$ and $[bb']$ are the planes fixed on the hob-head and determine the blade position accurately. It is to be noted that the contour of the reground blades remains unchanged, hence the diameter and the cutting edge contour of the hob will not change theoretically, when the reground blades are built-up on the hob-head.

These are the basic features of the proposed hob.

3. Derivation of the Rotor Tooth Profile

The proposed hob is designed by first deriving the rotor tooth profile, since the profile of the cutting edge of the blade must be determined according to that of the rotor. In general, the rotor tooth profile is composed of a number of lines which are expressed by formulae or discrete points^{(4),(5)}. In this study, the tooth profile expressed by formulae is treated. The tooth profile is designed by first specifying the radius of curvature of the tip tooth curve of the male rotor. To illustrate the circular arc of the tip, an exaggerated sketch is shown in Fig. 4. Points O' and O_f denote the locations of the male and female rotor axes, respectively. Point Q is a cusp in the rotor casing. The straight line T is perpendicular to the line connecting the cusp Q and the pitch point P . A part of the circle of radius r_1 , which connects smoothly with both the line T at point A_1 and the addendum circle of the male rotor at point A_5 , is designated as the arc of the tip tooth curve of the male rotor. Point O_c is the center of the circle of radius r_1 . Coordinates

Fig. 4 Expression of a circular arc

(X', Y') of an arbitrary point on the arc are expressed in the coordinate system $O-X'Y'$, which is a complex plane⁽⁶⁾ and which rotates with the male rotor, as follows:

$$\begin{aligned} \mathbf{M}_0(\theta) &\equiv X' + iY' = (R_a - r_1)e^{i\psi_m} + r_1e^{i(\psi_m + \psi_1)} \\ \tan\{\psi_1(\theta)\} &= \overline{P_0P'} / \{(\overline{O'A_5} - \overline{O_cA_5}) - \overline{O'P'}\} \\ &= R_m \sin(\psi_m - j\theta) / \{(R_a - r_1) - R_m \cos(\psi_m - j\theta)\}, \end{aligned} \quad (3)$$

Where $i = \sqrt{-1}$, θ is a variable parameter expressing the tip arc and having a physical meaning for the rotation angle of the female rotor, j is the gear ratio, $2R_a, 2R_m$ are the outer and the pitch diameters of the male rotor, respectively and ψ_m is a fixed angle which is determined from points O' and A_5 , as shown in Fig. 4.

The normal of the arc at an arbitrary point $\mathbf{M}_0$ intersects the pitch circle of the male rotor at point P_0 . When point P_0 rotates with the male rotor by an angle $(-j\theta)$ around axis O' , and reaches the pitch point P , the point $\mathbf{M}_0(\theta)$ meshes with the female rotor tooth which is rotated by an angle θ around the axis O_f . At this instant, the coordinate $\mathbf{M}_0(\theta)$ is expressed in the complex coordinate system $O_f-x_fy_f$ which rotates with the female rotor. The coordinate expressed in $O_f-x_fy_f$ represents a part of the female rotor tooth profile $\mathbf{M}_1$ which meshes with $\mathbf{M}_0$ assuming θ to be a variable parameter:

$$\mathbf{M}_1(\theta) = \{\mathbf{M}_0(\theta)e^{i(-j\theta)} - E\}e^{i(-\theta)}, \quad (4)$$

where $E(=\overline{O'O_f})$ is the distance between the rotor centers. The rest of the tooth profiles of the female rotor can be expressed in a similar way. The tooth profile of the male rotor does not need to be expressed, because the rack tooth profile can be determined by the female rotor profile only, as explained below.

4. Cutting Edge Profile

4.1 Rack tooth profile

The rack tooth profile is determined from the profile of the female rotor. $\mathbf{M}(\theta)$ in $O_f-x_fy_f$ stands for the tooth profile of the female rotor. $\mathbf{M}(\theta)$ is used in this section to deduce the rack tooth profile. When the pitch circle of the female rotor rotates through an angle θ around its axis and rolls on the pitch line of the rack, the center of the pitch circle travels a distance of $R_f\theta$ along the pitch line, as shown in Fig. 5, and the point of contact between the circle and the line determines an instantaneous center. Therefore, the foot of the perpendicular to $\mathbf{M}$ from the point of contact becomes the point of meshing with the rack. The point of meshing is also the point on the rack tooth, thus, the coordinate of the meshing point can be expressed in the complex coordinate system $O-xy$ which moves with the rack, and which is denoted by

$\mathbf{R}(\theta)$. Assuming the angle θ as available parameter, the rack tooth profile is expressed by $\mathbf{R}(\theta)$ as follows (see Fig. 5):

$$\mathbf{R}(\theta) \equiv x(\theta) + iy(\theta) = \mathbf{M}(\theta)e^{i(\theta - \pi/2)} + R_f\theta - iR_f, \quad (5)$$

where R_f is the pitch diameter of the female rotor.

A three-dimensional coordinate system $O-xyz$ associated with the rack is introduced to deduce the helical rack tooth surface from the profile $\mathbf{R}(\theta)$, such that the $x-z$ plane coincides with the pitch plane of the rack. The surface is expressed by the row vector $\mathbf{R}(\theta, z)$, as follows:

$$\mathbf{R}(\theta, z) = (x(\theta) - z \tan \beta, y(\theta), z)^T \quad (6)$$

where the coordinate z is assumed to be a variable parameter to express the surface, β is the helix angle on the pitch cylinder of the rotor, and T means the transposition from the column vector to the row vector. The helix angle β is given as a design specification.

4.2 Fundamental helicoid of the hob

The fundamental helicoid⁽³⁾ of the hob corresponds to the tooth surface of the cylindrical worm which should mesh with the rack by a skew angle. Therefore, the cutting edges of the hob must be on the fundamental helicoid when the worm becomes a tool gear, that is, a hob. The cutting edge profile, stated in section 2, is determined from this fundamental helicoid which is generated by the rack tooth surface expressed in Eq. (6).

In Fig. 6, a pitch surface of the rack with pitch t and a hob pitch cylinder of radius H are shown. The pitch t is determined from both the number of teeth and the pitch diameter of the rotor. The lead angle γ of the hob, as stated in section 2, is determined from the hob diameter $2H$, the pitch t and the helix angle β . $O-XYZ$, $O-xyz$ and $O_h-x_hy_hz_h$ are the coordinate systems fixed to the space, rack and hob, respectively. At the instant when the hob has rotated by an angle ϕ [radian] around its axis, the rack has traveled a distance of $(-t\phi/2\pi)$ along the X -axis. The rack tooth surface is expressed in $O-XYZ$ as follows:

Fig. 5 Expression of the rack tooth profile

Fig. 6 Relationship between rack and hob

$$\mathbf{R}(\theta, z; \phi) = (x(\theta) - z \tan \beta - t\phi/2\pi, y(\theta), z)^T \quad (7)$$

The instantaneous contact line between the rack and the fundamental helicoid is called a generatrix, and is determined by the meshing condition⁽⁷⁾. Namely, the relative velocity $\mathbf{W}(\theta, z; \phi)$ of both members is perpendicular to the common surface normal at the point of meshing:

$$\mathbf{N} \cdot \mathbf{W} = 0$$

$$\mathbf{N}(\theta, z; \phi) = \frac{\partial \mathbf{R}(\theta, z; \phi)}{\partial \theta} \times \frac{\partial \mathbf{R}(\theta, z; \phi)}{\partial z} \quad (8)$$

From Eq. (8), $z = z(\theta; \phi)$ is derived. Substituting $z = z(\theta; \phi)$ into Eq. (7), the instantaneous contact line is determined by $\mathbf{R}(\theta, z(\theta; \phi); \phi)$. $\mathbf{R}(\theta; \phi)$ is also a line on the fundamental helicoid, therefore, the line expressed in $O_h - x_h y_h z_h$ represents the fundamental helicoid of the hob, assuming ϕ to be a variable parameter. The surface is denoted by $\mathbf{L}_h(\theta, \phi)$:

$$\mathbf{L}_h(\theta, \phi) = \mathbf{C}(\phi) \{ \mathbf{B}(\lambda) \mathbf{R}(\theta, \phi) + (0, H, 0)^T \} \quad (9)$$

$$\lambda = \pi/2 - (\beta + \gamma)$$

where $\mathbf{B}$ and $\mathbf{C}$ are the coordinate transformation matrices relating to the rotation around the y_h -axis and z_h -axis, respectively:

$$\mathbf{B}(\lambda) = \begin{pmatrix} \cos \lambda & 0 & -\sin \lambda \\ 0 & 1 & 0 \\ \sin \lambda & 0 & \cos \lambda \end{pmatrix},$$

$$\mathbf{C}(\phi) = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10)$$

4.3 Cutting edge profile

Using two parameters u and v , the rake surface of the blade, which includes the hob axis, is expressed by $(0, u, v)^T$. The cutting edge profile of the blade is determined as the line of intersection of the rake surface $(0, u, v)^T$ and the fundamental helicoid

Table 1 Dimensions of screw rotor and hob (mm)

	Male	Female
[Rotor]		
Center distance E	87.5	
Number of teeth	4	6
Outer dia. $2R_o, 2R_f$	113.8	105.0
Lead	152.0	228.0
[Hob]		
Pitch dia. $2H$	250.0	
Lead angle γ	2.28100° (2° 17')	
Number of starts	1	
Number of gashes	10	
Gash lead	∞	
End clearance angle δ	10°

Fig. 7 Trial product of the hob

$$\mathbf{L}_h(\theta, \phi):$$

$$\mathbf{L}_h(\theta, \phi) - (0, u, v)^T = 0 \quad (11)$$

From Eq. (11), each of the terms u , v and ϕ can be expressed as a function of θ , therefore, the cutting edge profile is determined by two parametric expressions, as follows:

$$u = u(\theta), \quad v = v(\theta) \quad (12)$$

5. A Hob Fabricated on Experimental Basis

The dimensions of a screw rotor and a hob fabricated for trial hobbing are listed in the Table 1. Two kinds of blades and wedges were fabricated: one for the male rotor and the other for the female rotor. The hob-head can be used in common. The blades of thickness 10 mm were cut out from a tempered planar plate of high-speed tool steel of hardness $H_v 1000$ using wire-cut EDM in three steps: rough, medium, and finish cut. The radius of the wire electrode was 0.125 mm. The height of the contour and the pitch of the machined blade are measured by Universal Measuring Machine (UMM) in order to confirm the accuracy of machining. The pitch and height errors of a blade were within $\pm 5 \mu\text{m}$. Figure 7 shows the built-up hob fabricated on an experimental basis. The blades for the female rotor are built up on the hob-head and the blades for the male rotor are at both sides of the hob.

The phase adjustment of each blade is carried out carefully when assembling the blades on the hob head.

Fig. 8 Rotor fabricated by hobbing

We had a difficult time adjusting the blades within $\pm 10 \mu\text{m}$ because of the existing slight irregularities in each blade. Figure 8 shows the rotor fabricated by hobbing. The material for the male rotor is the carbon steels for machine structural use S45C (JIS) and that of the female rotor is the spherical graphite cast iron FCD500 (JIS). The rotors were measured by Coordinate Measuring Machine (CMM)⁽⁸⁾. The pitch and profile error are about $\pm 10 \mu\text{m}$ and $\pm 20 \mu\text{m}$ respectively. Inducement of a large pitch error of the rotor will be due to the blade phase-difference error and cutting edge profile error. The performance test of the rotors assembled in a screw compressor has not been carried out yet.

6. Conclusion

In general, the accuracy deterioration of a hob due to rake surface regrinding to sharpen the cutting edges is unavoidable, because the clearance surface of the cutting edge cannot be machined precisely, taking regrinding into consideration. A rapid progress of the deterioration is observed especially for a hob with a complex profile, such as a hob for a screw rotor.

To improve the accuracy deterioration due to rake surface regrinding of the hob, a built-up hob with theoretically no deterioration is proposed. The proposed hob is composed of a hob-head and a number of blades. When the cutting edges wear down, the blades are removed from the hob-head and the rake

surfaces are reground by a conventional surface grinding machine. The contour of the rake surface does not change theoretically. Only the blade thickness is decreased due to the regrinding.

The blades are machined using wire-cut EDM. The machining is very simple, that is, only a wire electrode is inclined at a fixed angle during blade machining.

The method for calculating the cutting edge profile of the hob is also presented in this paper, using both the complex number plane and the kinematics of skew gearing.

A built-up hob was fabricated on an experimental basis according to the proposed method and rotor hobbing was carried out.

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