Mass Transport in Pulsatile Flow through Asymmetric Stenosis*

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The characteristics of a pulsatile flow through stenosis are already well known, however, mass transport around stenosis is not yet well understood. In this study, mass transport in the pulsatile flow between flat plates with asymmetric stenosis is analyzed numerically. The effects of the pulsation and asymmetry of the stenosis on the mass transport are investigated. Flow pattern, concentration pattern and the distribution of the wall concentration are obtained. It is found that an asymmetry of the stenosis increases the wall concentration downstream of the stenosis. In the asymmetric stenosis case, the wall concentration downstream of the stenosis recovers at a lower pulsatile frequency than the symmetric stenosis case.

Key Words: Fluid Dynamics, Bio-Fluid Mechanics, Diffusion, Pulsation, Numerical Analysis

1. Introduction

A pulsatile flow through a stenosis is of interest in many fields, for example, local narrowing in an artery or in a trachea in the bioengineering field, and a valve or a corroded part in a piping system in the engineering field and so on. In these flows, the mass transport around the stenosis sometimes plays an important role. There are many reports about flow through a stenosis, and the flow field has been clarified not only for steady flows but also for a wide range of frequencies of pulsatile flows⁽¹⁾. On the other hand, there are few reports about mass transport around a stenosis. It is, therefore, important to clarify the concentration field around the stenosis.

Schneiderman et al.⁽²⁾ and Back et al.⁽³⁾ reported on the mass transport in steady flows through stenosis. Yamaguchi⁽⁴⁾ and Rappitsch and Perktold^{(5),(6)} studied the mass transport in pulsatile flow through stenosis.

These works pointed out that the mass transport along the downstream wall of a stenosis is very low, however, the effect of the pulsation was not well clarified. We therefore investigated in a previous research the effect of pulsation on the mass transport at an axisymmetric stenosis⁽⁷⁾.

The flow through an asymmetric stenosis has been reported by Liu and Yamaguchi⁽⁸⁾, Pedley et al.⁽⁹⁾⁻⁽¹¹⁾, Tutty⁽¹²⁾ and by the current authors⁽¹³⁾. This research revealed that the vortex downstream of an asymmetric stenosis does not flow away as for an axisymmetric stenosis and that the difference in the flow fields between asymmetric and axisymmetric stenosis is rather large. If the flow fields are different, it is considered that the concentration fields will also be different. It is, therefore, necessary to clarify the concentration field around an asymmetric stenosis.

In this study, the mass transport in the pulsatile flow between flat plates with asymmetric stenosis is analyzed numerically. The effect of asymmetry of the stenosis on the mass transport is discussed.

Nomenclature

c : concentration
 $\mathcal{D}$: diffusivity
 G : wall configuration
 H : distance between the flat plates
 l : stenosis length
 Q : flow rate
 Re : Reynolds number $\equiv \rho \bar{U} H / \mu$

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- Sc : Schmidt number $\equiv \mu/\rho\mathcal{D}$
 St : Stokes number $\equiv \rho\omega H^2/(4\mu)$
 t : time
 $\bar{U}$: mean-time velocity
 x : coordinate axis parallel to the wall
 y : coordinate axis perpendicular to the wall
 δ : stenosis depth
 μ : viscosity
 ρ : density
 Ψ : stream function
 ω : angular velocity of the pulsating flow

Overlines

- $*$: dimensional quantity
 ave : time-averaged quantity
 $lower$: quantity on the lower wall
 $upper$: quantity on the upper wall
 w : quantity on the wall

2. Basic Equations

It is assumed that the concentration is a passive quantity, that is to say, the concentration field has no influence on the flow field. The governing equations for the two-dimensional pulsatile flow of Newtonian fluids are given by the vorticity transport equation and the Poisson equation for the stream function. The nondimensionalized governing equation for the concentration field is given as follows:

$$4St \frac{\partial c}{\partial t} + Re \left(\frac{\partial \Psi}{\partial y} \frac{\partial c}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial c}{\partial y} \right) = \frac{1}{Sc} \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) \quad (1)$$

where the x and y axes are taken in the directions parallel and perpendicular to the wall, respectively. H^* is selected as characteristic length, and ρ^* , $\bar{U}^*$, μ^* and ω^* as characteristic quantities. The concentration is nondimensionalized using $c = c^*/c_{inlet}^*$, where c_{inlet}^* is the constant inlet concentration. In the case that there is a stenosis on the wall, all of the equations are transformed from the physical space to the computational space using a boundary fit method.

3. Numerical Method

3.1 Discretization of equations and computational grid

The governing equations are transformed to the computational space and reduced to second-order accuracy centered finite difference equations, with the exception of the convective term of Eq.(1) which is reduced to third-order accuracy by an up-wind scheme. The Poisson equation is solved using the SOR method, and the vorticity transport equation and Eq.(1) are solved using the explicit Euler method.

A boundary fit method is used to generate the

Fig. 1 Stenosis configuration

Fig. 2 Change of flow rate

computational grid, and the following function is adopted to express the configurations of stenosis (Fig. 1), where $G(x)_{upper}$ and $G(x)_{lower}$ are the upper and lower wall configurations, respectively.

$$G(x)_{upper} = 1 - \delta_{upper} \exp(b_1 x^2), \quad b_1 = \frac{\log 0.5}{l_{upper}^2} \quad (2)$$

$$G(x)_{lower} = \delta_{lower} \exp(b_2 x^2), \quad b_2 = \frac{\log 0.5}{l_{lower}^2} \quad (3)$$

The calculation region is $-15 \leq x \leq 20$, where x is nondimensionalized by H^* . The number of grid points in the x direction is 701, and in the y direction the number depends on the flow condition. In this study, at least three grid points are placed in the Stokes layer of concentration σ_c , which is given by Eq.(4). It was confirmed that the number of grid points does not strongly influence the numerical results.

$$\sigma_c = \frac{\sigma_c^*}{H^*} = \frac{1}{H^*} \sqrt{\frac{2\mathcal{D}^*}{\omega^*}} = \sqrt{\frac{1}{2StSc}} \quad (4)$$

3.2 Boundary conditions for flow field

In order to simulate the pulsatile flow, the sinusoidal inlet flow rate (Fig.2) is given by the following Ψ condition on the wall.

$$\Psi_{upper} = 1 + \cos t, \quad \Psi_{lower} = 0 \quad (5)$$

There is no slip along the wall. The flow at the inlet is assumed to be fully developed, and the vorticity is given by first-order extrapolation at the outlet.

3.3 Boundary conditions for concentration field

The boundary condition on the wall concentration can be given by a constant concentration or a constant permeability rate. The mass flux distribution on the wall, for which the wall boundary condition is given as

constant concentration, and the concentration distribution, for which the wall boundary condition is given as constant permeability rate, show a similar tendency, if the convection dominates the concentration field. In this study, the boundary condition on the wall concentration is, therefore, given as constant permeability rate.

The permeability rate for the wall is given as 0.002 in the range of $-5 \leq x \leq 10$, and in the range of $-15 \leq x < -5$, $10 < x \leq 20$, no mass flux through the wall is assumed. The reason why mass transport through the wall only occurs around the stenosis is to remove the inlet and outlet boundary effects.

At the inlet, a uniform concentration ($c=1$) is assumed. In this calculation range, the fluid particles at $x=-5$ flow upstream up to $x=-6$ during one cycle. Therefore, there is little effect at the inlet ($x=-15$) even though the Schmidt number is low.

At the outlet, the concentration is given as follows in case of positive velocity (fluid particles flow away from the computational region):

$$\frac{\partial^2 c}{\partial z^2} = 0. \quad (6)$$

In the case of negative velocity (fluid particles flow into the computational region), $c=0$ is given. In this calculation range, the fluid particles at the outlet flow upstream up to $x=17$ during one cycle. Therefore, there is little effect on the test section ($-5 < x < 10$) regardless of whether $c=0$ or $c=1$ at the outlet, which is numerically confirmed.

3.4 Parameter ranges

Since in this study laminar flow is assumed, the

ranges of dimensionless parameters are set to $Re=200$ and $St=0 \sim 200$ so that no local disturbance of the flow appears.

Under high Schmidt number conditions, the Stokes layer for the concentration (Eq. (4)) becomes very thin. This very thin layer is peeled from the wall where the flow separates, and it becomes difficult to map a fine computational grid to follow the layer. Therefore, all computations were performed in the range of $St \times Sc \leq 1500$. The high Sc condition is discussed accordingly. The condition $Sc=1 \sim 5$ corresponds to mass and heat transport in a gas. In bioengineering, it corresponds to the transport phenomena in a trachea, and so forth.

4. Results and Discussion

4.1 Basic flow and concentration distribution with $Sc=1$

The flow and the concentration fields are calculated under the conditions $Sc=1$, $Re=200$, $St=100$, and the wall configuration $\delta_{upper} = l_{upper} = 0.5$, $\delta_{lower} = 0$. The contour plots of the concentration and the instantaneous stream lines are shown in Figs. 3 and 4, respectively. By comparing these figures, we find that throughout one cycle, the concentration is strongly influenced by the vortex. This is because the Stokes layer of concentration has a similar thickness to that of viscosity under the condition $Sc=1$, the vortex can therefore strongly influence the concentration field near the wall. In this stenosis configuration, vortices are generated in turns on the upper and the lower walls. Iso-concentration lines are also curled up in

Fig. 3 Contour plots of concentration ($Sc=1$)

Fig. 4 Instantaneous stream lines

Fig. 5 Concentration on the upper wall for the case with $Sc=1$ Fig. 6 Concentration on the lower wall for the case with $Sc=1$

turns and thick and thin boundary layers of concentration appear in turns.

Figures 5 and 6 show the wall concentration distribution on the upper and the lower walls, respectively. In the pulsatile flow through an asymmetric stenosis, the vortex downstream of the stenosis is stronger than that in the axisymmetric stenosis case and only negligibly flows away. The concentration field downstream of the stenosis is, therefore, strongly influenced by the vortex. The peak point in the concentration curve flows downstream with the vortex, and the peak concentration caused by the vortex is higher than the concentration at the stenosis.

4.2 Effect of vortex enhancement with $Sc=1$

Vortex enhancement is the phenomenon that the vortex downstream of the stenosis becomes strongest at a certain pulsation frequency. We found in our

previous research that this phenomenon strongly influences the flow and concentration fields⁽⁷⁾. We then computed the concentration field with $Sc=1$ in the range of $St=0\sim 500$ in order to investigate the effect of vortex enhancement.

Figure 7 shows the mean-time concentration distribution on the upper wall for various St , in which the vortex enhancement occurs at $St=150$. It is found that the peak value of the concentration downstream of the stenosis appears at $St=150$, which is the same frequency as for vortex enhancement. This is because under the condition of $Sc=1$ the influence of the vortex is very strong.

4.3 Effect of stenosis asymmetry with $Sc=1$

The flow and the concentration fields are calculated under the conditions $Sc=1$, $Re=200$, $St=100$, and the symmetric wall configuration $\delta_{upper}=\delta_{lower}=$

0.25, $l_{upper}=l_{lower}=0.5$. Figure 8 shows the instantaneous contour plots of the concentration. We find that the vortex in the symmetric stenosis case flows away

Fig. 7 Effect of St on the mean wall concentration with $Sc=1$

much faster than that in the asymmetric stenosis case.

Figure 9 shows the concentration distribution on the wall. By comparing this figure with Figs. 5 and 6, we find that the concentration downstream of the stenosis and the amplitude of the concentration change are lower than those of the asymmetric stenosis case. We conclude therefore that the influence of the vortex in the asymmetric stenosis case is relatively strong.

The length of the stenosis is fixed at the value of $l_{upper}=l_{lower}=0.5$, and the value of δ is varied as follows: (1) $\delta_{upper}=0.5$, $\delta_{lower}=0$, (2) $\delta_{upper}=0.4$, $\delta_{lower}=0.1$, (3) $\delta_{upper}=0.3$, $\delta_{lower}=0.2$, and (4) $\delta_{upper}=\delta_{lower}=0.25$. Figure 10 shows the mean-time concentration distribution on the upper wall. It is found that the concentration downstream of the stenosis increases with increasing asymmetry of the stenosis,

Fig. 8 Contour plots of concentration (symmetric stenosis)

Fig. 9 Concentration on the wall for the case of symmetric stenosis

Fig. 10 Effect of asymmetry on the mean wall concentration

Fig. 11 Contour plots of concentration ($Sc=5$) Fig. 12 Concentration on the upper wall for the case of $Sc=5$ Fig. 13 Concentration on the lower wall for the case of $Sc=5$

and that the curve has a wavy shape in the case of $\delta_{upper}=0.5$. This is because the vortex becomes stronger with the increasing asymmetry of the stenosis.

4.4 Basic flow and concentration distribution with $Sc=5$

The flow and the concentration fields are calculated under the conditions $Sc=5$, $Re=200$, $St=100$, and the wall configuration $\delta_{upper}=l_{upper}=0.5$, $\delta_{lower}=0$. Figure 11 shows the instantaneous contour plots of the concentration. It is found that the boundary layer of the concentration is thinner than in the $Sc=1$ case, however, the boundary layer curls up with the vortex similar to the $Sc=1$ case.

Figures 12 and 13 show the wall concentration distributions on the upper and lower walls, respectively. The concentration field downstream of the stenosis is strongly influenced by the vortex similar to

the $Sc=1$ case, however, a new concentration peak appears on the upper wall at $x=0.5$. This peak does not appear in the $Sc=1$ case, because the diffusion is too strong to maintain a large concentration difference at this short distance.

4.5 Effect of stenosis asymmetry with $Sc=5$

The flow and the concentration fields are calculated under the conditions $Sc=5$, $Re=200$, $St=100$, and the symmetric wall configuration $\delta_{upper}=\delta_{lower}=0.25$, $l_{upper}=l_{lower}=0.5$. Figure 14 shows the instantaneous contour plots of the concentration. It is found that the concentration distribution near the wall does not change in spite of the flowing away of the vortex. This is also confirmed by the wall concentration distribution shown in Fig. 15. The amplitude of the concentration change is much smaller than that of the asymmetric stenosis case (Fig. 12).

Fig. 14 Contour plots of concentration ($Sc=5$, symmetric stenosis) Fig. 15 Concentration on the wall for the case of symmetric stenosis and $Sc=5$ Fig. 16 Effect of asymmetry on the mean all concentration for the case of $Sc=5$

Next, the length of the stenosis is fixed at a value of $l_{upper}=l_{lower}=0.5$, and the value of δ is varied as in the foregoing four cases mentioned in section 4.3. Figure 16 shows the mean-time concentration distribution on the upper wall. It is found that the concentration downstream of the stenosis increases with increasing asymmetry of the stenosis, similar to the $Sc=1$ case. We conclude therefore that the effect of the vortex manifests itself in the asymmetric stenosis case even for $Sc=5$. In addition, the concentration peak at $x=0.5$ only appears in the asymmetric stenosis case at a high Sc condition.

4.6 Effect of vortex enhancement with $Sc=5$

The concentration field with $Sc=5$ is computed with the range of $St=0\sim 500$ in order to investigate the effect of the vortex enhancement. Two types of wall configurations are employed: (1) symmetric

case with $\delta_{upper}=\delta_{lower}=0.25$, $l_{upper}=l_{lower}=0.5$, and (2) asymmetric case with $\delta_{upper}=l_{upper}=0.5$, $\delta_{lower}=0$. Figures 17 and 18 show the mean-time concentration distributions on the upper wall in the symmetric and asymmetric cases, respectively. The mean-time concentration has a wavy shape in the asymmetric stenosis case. We conclude therefore that the influence of the vortex is much stronger in the asymmetric case. The peak value of the concentration downstream of the stenosis appears at $St=150$, which is the same frequency at which the vortex enhancement occurs. This is because the influence of the vortex is also very strong under the condition $Sc=5$.

In the symmetric stenosis case, the wall concentration dramatically recovers for the range of $St=0\sim 80$. On the other hand, in the asymmetric stenosis case, the wall concentration dramatically recovers for

Fig. 17 Effect of St on the mean wall concentration for the case of symmetric stenosis and $Sc=5$

Fig. 18 Effect of St on the mean wall concentration for the case of asymmetric stenosis and $Sc=5$

the range of $St=0\sim 20$. This is because in the symmetric stenosis case the flowing away of the vortex plays an important role in the recovery of the concentration, however, in the asymmetric stenosis case, the vortex barely flows away. In the asymmetric stenosis case, the formation of the vortex on the upper and lower walls in turn plays an important role, and therefore, the concentration recovers dramatically for the lower range of $St=0\sim 20$. We conclude from these results that the vortex in the asymmetric stenosis case has a strong influence on the wall concentration even for the high Sc case.

5. Conclusion

Mass transport in pulsatile flow between flat plates with stenosis is investigated under the condition

of constant permeability on the wall. The following results are obtained:

(1) Under the condition $Sc=1\sim 5$, the vortex has a strong influence on the concentration field. The wall concentration downstream of the stenosis has its maximum value at the pulsatile frequency in which the vortex enhancement appears.

(2) The wall concentration downstream of the stenosis and the effect of the vortex increase with increasing the asymmetry of the stenosis.

(3) Under the condition $Sc=5$, the concentration has a small peak at $x=0.5$, which does not appear in the $Sc=1$ case.

(4) In the asymmetric stenosis case, the wall concentration downstream of the stenosis recovers for lower pulsatile frequencies compared with the symmetric stenosis case.

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