Micro-Control Actions of Segmented Actuator Patches Laminated on Deep Paraboloidal Shells*

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Deep paraboloidal shells of revolution are common components for horns, nozzles, rocket fairings, etc. This study is to investigate the micro-control actions and distributed control effectiveness of precision deep paraboloidal shells laminated with segmented actuator patches. Mathematical models and governing equations of the paraboloidal shells laminated with distributed actuator layers segmented into patches are presented first, followed by formulations of distributed control forces and micro-control actions including meridional/circumferential membrane and bending control components based on an assumed mode shape function and the Taylor series expansion. Distributed control forces, patch sizes, actuator locations, micro-control actions, and normalized control authorities of a deep paraboloidal shell are analyzed in a case study. Analysis indicates that 1) the control forces and membrane/bending components are mode and location dependent, 2) the meridional/circumferential membrane control actions dominate the overall control effect, 3) there are optimal actuator locations resulting in the maximal control effects at the minimal control cost for each natural mode.

Key Words: Piezoelectric, Distributed Actuator, Modal Control Effects, Control Authorities

1. Introduction

There are many advanced paraboloidal shell structures and components used in civil, mechanical, and aerospace structures and systems, ranging from horns, nozzles, rocket fairings, solar collectors, communication antennas, optical mirrors, etc. Static and dynamic characteristics of paraboloidal shell struc-

tures have been investigated over the years⁽¹⁾⁻⁽⁷⁾. This study is to investigate the distributed control effectiveness and micro-control actions with respect to actuator locations of precision deep paraboloidal shells laminated with segmented actuator patches.

Distributed control of shell structures (or distributed elastic structural systems) based on the "smart structures and structronics" technology has been quickly developing for over a decade. Distributed control of one- and two-dimensional (1D and 2D) flat distributed systems (e.g., beams and plates) has been widely studied, so as rings⁽⁸⁾. Distributed control of 2D curved distributed systems—shells—is more challenging, because of the coupled membrane/bending behavior and the in-plane/out-of-plane motions⁽⁹⁾. Control of cylindrical shells with fully distributed actuator, partially distributed actuators, segmented actuator patches, line actuators, etc. were investigated^{(9),(10)}; micro-electromechanics of segmented actuators was also evaluated⁽¹¹⁾. Acoustics, vibration and distributed actuation of spherical shells were recently investigated⁽¹²⁾⁻⁽¹⁵⁾. Distributed sensing and

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control of conical shells were evaluated⁽¹⁶⁾, so the micro-sensing characteristics of toroidal shells⁽¹⁷⁾. Distributed modal signals of linear and nonlinear paraboloidal shells were recently investigated⁽¹⁸⁾. In this study, mathematical models of the paraboloidal shells laminated with distributed actuator layers segmented into patches are defined first, followed by formulations of distributed control forces and detailed membrane/bending micro-control actions. Distributed actuator control forces and control authorities at various actuator locations on a deep paraboloidal shell are investigated. Distributed modal control effects and micro-meridional/circumferential membrane/bending control actions are evaluated and general design guidelines are proposed.

2. System Model and Distributed Actuation

A generic paraboloidal shell of revolution defined in a tri-orthogonal coordinate system is shown in Fig. 1, where $\alpha_1 = \phi$ and $\alpha_2 = \psi$, a meridional radius of curvature $R_1 = R_\phi = b/\cos^3 \phi$, a circumferential radius of curvature $R_2 = R_\psi = b/\cos \phi$ and the constant $b = a^2/(2c) = 2f$ where “ a ” is the radial distance, “ c ” is the meridian height at the pole, and “ f ” is the focal length. The Lamé parameters are $A_1 = b/(\cos^3 \phi)$ and $A_2 = (b \sin \phi)/(\cos \phi)$. The parabola equation is given as $z = z(r) = c \left[1 - \left(\frac{r}{a} \right)^2 \right]$, where “ z ” is the vertical rise and “ r ” is the horizontal radius.

Assume a thin distributed actuator layer is

Fig 1 A generic paraboloidal shell of revolution and a part of its cross-section

laminated on the paraboloidal shell of revolution, which induces control forces/moments leading to precision control of the paraboloidal shell. Figure 2 illustrates a deep paraboloidal shell laminated with a distributed actuator layer segmented into actuator patches. Since the layer is thin, the mass and stiffness properties can be neglected and only the control actions are considered. Mathematical models are defined first, followed by micro-control actions and their control effectiveness.

Mathematical model of paraboloidal shells with spatially distributed actuators can be simplified from the system equations of a piezothermoelastic shell laminate system (19) and is derived as follows.

$$\frac{1}{\cos^3 \phi} \frac{\partial}{\partial \psi} (N_{\psi\psi}^m - N_{\psi\psi}^e) + \frac{\partial}{\partial \phi} [(N_{\phi\phi}^m - N_{\phi\phi}^e) \tan \phi] - (N_{\psi\psi}^m - N_{\psi\psi}^e) \frac{1}{\cos^2 \phi} + Q_{\phi 3} \tan \phi + \frac{b \sin \phi}{\cos^4 \phi} q_\phi = \frac{b \sin \phi}{\cos^4 \phi} \rho h \frac{\partial^2 u_\phi}{\partial t^2}, \quad (1)$$

$$\frac{1}{\cos^3 \phi} \frac{\partial}{\partial \psi} (N_{\psi\psi}^m - N_{\psi\psi}^e) + \frac{\partial}{\partial \phi} (N_{\phi\phi}^m \tan \phi) + N_{\psi\psi}^m \frac{1}{\cos^2 \phi} + \frac{\sin \phi}{\cos^3 \phi} Q_{\phi 3} + \frac{b \sin \phi}{\cos^4 \phi} q_\psi = \frac{b \sin \phi}{\cos^4 \phi} \rho h \frac{\partial^2 u_\psi}{\partial t^2}, \quad (2)$$

$$\frac{\cos \phi}{b \sin \phi} \frac{\partial}{\partial \psi} \left[\sec^3 \phi \frac{\partial}{\partial \psi} (M_{\psi\psi}^m - M_{\psi\psi}^e) + \frac{\partial}{\partial \phi} (M_{\phi\phi}^m \tan \phi) + M_{\phi\phi}^m \sec^2 \phi \right] + \frac{1}{b} \frac{\partial}{\partial \phi} \left\{ \frac{\partial}{\partial \psi} M_{\psi\psi}^m + \cos^3 \phi \frac{\partial}{\partial \phi} [(M_{\phi\phi}^m - M_{\phi\phi}^e) \tan \phi] - (M_{\psi\psi}^m - M_{\psi\psi}^e) \cos \phi \right\} - (N_{\phi\phi}^m - N_{\phi\phi}^e) \tan \phi - (N_{\psi\psi}^m - N_{\psi\psi}^e) \frac{\sin \phi}{\cos^3 \phi}$$

Fig 2 Segmented distributed actuators laminated on a deep paraboloidal shell

$$+ q_3 \frac{b \sin \phi}{\cos^4 \phi} = \frac{b \sin \phi}{\cos^4 \phi} \rho h \frac{\partial^2 u_3}{\partial t^2}, \quad (3)$$

where N_{ij}^m and M_{ij}^m are the elastic forces and moments; N_{ij}^e and M_{ij}^e are the electric control forces and moments, respectively; $\rho h \cdot \partial^2 u_i / \partial t^2$ denotes the inertia force effect; ρ is the mass density; h is the shell thickness; q_i is the input force; $Q_{\phi 3}$ and $Q_{\psi 3}$ are the transverse shear effects; and the superscripts “ m ” and “ e ” respectively denote the elastic (or mechanical) and electrically induced components (Note that “ e ” is changed to “ a ” when the actuator forces are explicitly defined in later sections.). The elastic force and moment resultants of thin paraboloidal shells following the Love-Kirchoff thin shell assumptions are defined as

$$N_{\phi\phi}^m = \frac{K}{b} \left[\cos^3 \phi \frac{\partial u_\phi}{\partial \phi} + \mu \cot \phi \frac{\partial u_\psi}{\partial \psi} + \mu \frac{\cos^2 \phi}{\sin \phi} u_\phi + (\cos^3 \phi + \mu \cos \phi) u_3 \right], \quad (4)$$

$$N_{\psi\psi}^m = \frac{K}{b} \left[\cot \phi \frac{\partial u_\psi}{\partial \psi} + \frac{\cos^2 \phi}{\sin \phi} u_\phi + \mu \cos^3 \phi \frac{\partial u_\phi}{\partial \phi} + (\cos \phi + \mu \cos^3 \phi) u_3 \right], \quad (5)$$

$$N_{\phi\psi}^m = \frac{K(1-\mu)}{2b} \left[-\frac{\cos^2 \phi}{\sin \phi} u_\psi + \cos^3 \phi \frac{\partial u_\psi}{\partial \phi} + \frac{\cos \phi}{\sin \phi} \frac{\partial u_\phi}{\partial \psi} \right], \quad (6)$$

$$M_{\phi\phi}^m = \frac{D}{b^2} \left\{ \cos^6 \phi \left(\frac{\partial u_\phi}{\partial \phi} - \frac{\partial^2 u_3}{\partial \phi^2} \right) - 3 \cos^5 \phi \sin \phi \left(u_\phi - \frac{\partial u_3}{\partial \phi} \right) + \mu \frac{\cos^2 \phi}{\sin \phi} \left(\frac{\partial u_\psi}{\partial \phi} \right. \right. \quad (7)$$

$$\left. - \frac{1}{\sin \phi} \frac{\partial^2 u_3}{\partial \psi^2} + u_\phi \cos^3 \phi - \cos^3 \phi \frac{\partial u_3}{\partial \phi} \right\},$$

$$M_{\psi\psi}^m = \frac{D}{b^2} \left\{ \frac{\cos^2 \phi}{\sin \phi} \left(\frac{\partial u_\psi}{\partial \psi} - \frac{1}{\sin \phi} \frac{\partial^2 u_3}{\partial \psi^2} \right) + u_\phi \cos^3 \phi - \cos^3 \phi \frac{\partial u_3}{\partial \phi} \right\} + \mu \left[\cos^6 \phi \left(\frac{\partial u_\phi}{\partial \phi} - \frac{\partial^2 u_3}{\partial \phi^2} \right) - 3 \cos^5 \phi \sin \phi \left(u_\phi - \frac{\partial u_3}{\partial \phi} \right) \right], \quad (8)$$

$$M_{\phi\psi}^m = \frac{D(1-\mu)}{2} \left(-\frac{\cos^5 \phi + 2 \cos^3 \phi \sin^2 \phi}{b^2 \sin \phi} u_\psi - \frac{2 \cos^3 \phi}{b^2 \sin^2 \phi} \frac{\partial u_3}{\partial \psi} - \frac{2 \cos^4 \phi}{b^2 \sin \phi} \frac{\partial^2 u_3}{\partial \psi \partial \phi} + \frac{\cos^4 \phi}{b^2} \frac{\partial u_\psi}{\partial \phi} + \frac{\cos^4 \phi}{b^2 \sin \phi} \frac{\partial u_\phi}{\partial \psi} \right) \quad (9)$$

The membrane stiffness $K = Yh/(1-\mu)^2$ and the bending stiffness $D = Yh^3/[12(1-\mu^2)]$, where Y is the modulus of elasticity and μ is Poisson's ratio of the shell. The control forces/moments induced by the applied control signal ϕ^a in the spatially distributed actuator layer are

$$N_{\phi\phi}^e = N_{\phi\phi}^a = d_{31} Y_p \phi^a, \quad (10)$$

$$N_{\psi\psi}^e = N_{\psi\psi}^a = d_{32} Y_p \phi^a, \quad (11)$$

$$M_{\phi\phi}^e = M_{\phi\phi}^a = r_\phi^a d_{31} Y_p \phi^a, \quad (12)$$

$$M_{\psi\psi}^e = M_{\psi\psi}^a = r_\psi^a d_{32} Y_p \phi^a, \quad (13)$$

where d_{3i} are the strain constants of the converse

piezoelectric effect; Y_p is Young's modulus of the distributed actuator; ϕ^a (in *italic*) is the control signal; r_i^a are the moment arms measured from the shell neutral surface to the mid-plane of the distributed actuator. The in-plane twisting (shear) effects are usually neglected, i.e., $N_{\phi\psi}^a \approx 0$ and $M_{\phi\psi}^a \approx 0$, for conventional hexagonal piezoelectric materials. Since the electric resistance on the actuator surface is negligible, the electric potential on the actuator surface is assumed constant and separated by patch divisions. Thus, the transverse control signal $\phi^a(\phi, \psi, t)$ is only confined on the segmented actuator patch electrode ranging from ϕ_1 to ϕ_2 and from ψ_1 to ψ_2 , Fig. 2.

$$\phi^a(\phi, \psi, t) = \phi^a(t) [u_s(\phi - \phi_1) - u_s(\phi - \phi_2)] \times [u_s(\psi - \psi_1) - u_s(\psi - \psi_2)], \quad (14)$$

where $u_s(\cdot)$ is the unit step function: $u_s(\phi - \phi_i) = 1$ when $\phi > \phi_i$, and $= 0$ when $\phi < \phi_i$. For uniform-thickness actuator and shell, $r_\phi^a = r_\psi^a$. Again, the in-plane twisting control effects are usually not considered, i.e., $N_{\phi\psi}^a \approx 0$ and $M_{\phi\psi}^a \approx 0$. Considering the resultant elastic and control effects (i.e., $N_{\phi\phi} = K(s_{\phi\phi}^0 + \mu s_{\psi\psi}^0) - N_{\phi\phi}^e$, $N_{\psi\psi} = K(s_{\psi\psi}^0 + \mu s_{\phi\phi}^0) - N_{\psi\psi}^e$, $N_{\phi\psi} = N_{\psi\phi} = [K(1-\mu)/2] s_{\phi\psi}^0 - N_{\phi\psi}^e$, $M_{\phi\phi} = D(k_{\phi\phi} + \mu k_{\psi\psi}) - M_{\phi\phi}^e$, $M_{\psi\psi} = D(k_{\psi\psi} + \mu k_{\phi\phi}) - M_{\psi\psi}^e$, and $M_{\phi\psi} = M_{\psi\phi} = [D(1-\mu)/2] k_{\phi\psi} - M_{\phi\psi}^e$, where s_{ij}^0 and k_{ij} are the membrane strains and bending strains respectively), one can re-define the total force/moment resultants, including control forces/moments and the elastic components. Accordingly, combining the inherent elastic components $N_{\phi\psi}^m$ and $N_{\psi\phi}^m$ and the control components induced by actuator patches ranging from ϕ_1 to ϕ_2 and from ψ_1 to ψ_2 yields

$$N_{\phi\phi} = \frac{K}{b} \left[\cos^3 \phi \frac{\partial u_\phi}{\partial \phi} + \mu \cot \phi \frac{\partial u_\psi}{\partial \psi} + \mu \frac{\cos^2 \phi}{\sin \phi} u_\phi + (\cos^3 \phi + \mu \cos \phi) u_3 \right] - d_{31} Y_p \phi^a, \quad (15)$$

$$N_{\psi\psi} = \frac{K}{b} \left[\cot \phi \frac{\partial u_\psi}{\partial \psi} + \frac{\cos^2 \phi}{\sin \phi} u_\phi + \mu \cos^3 \phi \frac{\partial u_\phi}{\partial \phi} + (\cos \phi + \mu \cos^3 \phi) u_3 \right] - d_{32} Y_p \phi^a; \quad (16)$$

$$M_{\phi\phi} = \frac{D}{b^2} \left\{ \cos^6 \phi \left(\frac{\partial u_\phi}{\partial \phi} - \frac{\partial^2 u_3}{\partial \phi^2} \right) - 3 \cos^5 \phi \sin \phi \left(u_\phi - \frac{\partial u_3}{\partial \phi} \right) + \mu \frac{\cos^2 \phi}{\sin \phi} \left(\frac{\partial u_\psi}{\partial \phi} - \frac{1}{\sin \phi} \frac{\partial^2 u_3}{\partial \psi^2} + u_\phi \cos^3 \phi \frac{\partial u_3}{\partial \phi} \right) \right\} - r_\phi^a d_{31} Y_p \phi^a, \quad (17)$$

$$M_{\psi\psi} = \frac{D}{b^2} \left\{ \frac{\cos^2 \phi}{\sin \phi} \left(\frac{\partial u_\psi}{\partial \psi} - \frac{1}{\sin \phi} \frac{\partial^2 u_3}{\partial \psi^2} \right) + u_\phi \cos^3 \phi - \cos^3 \phi \frac{\partial u_3}{\partial \phi} \right\} + \mu \left[\cos^6 \phi \left(\frac{\partial u_\phi}{\partial \phi} - \frac{\partial^2 u_3}{\partial \phi^2} \right) - 3 \cos^5 \phi \sin \phi \left(u_\phi - \frac{\partial u_3}{\partial \phi} \right) \right] - r_\psi^a d_{32} Y_p \phi^a. \quad (18)$$

Note that although the analytical system equations of paraboloidal shells are defined, evaluation of distributed micro-control effects is based on an assumed solution satisfying the given boundary conditions in later analyses, due to complicated analytical solution procedures of the paraboloidal shells of revolution.

3. Distributed Micro-Control Actions

Assume the shell dynamic response is composed of all participating modes, i.e., the modal expansion method: $u_i(\alpha_1, \alpha_2, t) = \sum_{m=1}^{\infty} \eta_m(t) U_{im}(\alpha_1, \alpha_2)$, where $\eta_m(t)$ is the m -th modal participating factor or the m -th modal coordinate and $U_{im}(\alpha_1, \alpha_2)$ is the mode shape function. Accordingly, the original distributed control equation can be transferred into the modal domain and individual modal control equations can be evaluated based on the mode shape functions and the distributed actuator patch defined from ϕ_1 to ϕ_2 and from ψ_1 to ψ_2 . The displacement and the moment on the boundary perimeter are assumed zero for a simply supported paraboloidal shell structure. Also, since the transverse axisymmetric oscillation dominates, the transverse mode shape function U_{3m} can be approximated by

$$U_{3m} = A_m \cos \frac{(2m-1)\pi}{2\phi^*} \phi = A_m \cos B_m \phi, \quad (19)$$

$$m=1, 2, \dots, \infty$$

where A_m is the m -th modal amplitude; $B_m = [(2m-1)\pi]/(2\phi^*)$; ϕ is the meridional angle measured from the pole ($\phi=0$) and ϕ is from 0 to the shell boundary rim at $\phi=\phi^*$. Accordingly, the modal control equation becomes

$$\ddot{\eta}_m + \frac{c}{\rho h} \dot{\eta}_m + \omega_m^2 \eta_m = \hat{F}_m, \quad (20)$$

where c is the damping constant; ω_m is the m -th natural frequency; and $\hat{F}_m$ is the distributed modal control force. Imposing the axisymmetric assumption, i.e., $\partial(\cdot)/\partial\psi=0$, assuming the uniform-thickness actuator and shell, i.e., $r_\phi^a = r_\psi^a = r^a$, the hexagonal piezoelectric materials, i.e., $d_{31} = d_{32}$, and substituting the control forces/moments, one can define the modal control force induced by the segmented distributed actuator defined by ϕ_1 to ϕ_2 and ψ_1 to ψ_2 .

$$\begin{aligned} \hat{F}_m = & \frac{Y_p d_{31} A_m \phi^a(t)}{\rho h N_m} \\ & \times \int_{\phi_1}^{\phi_2} \int_{\psi_1}^{\psi_2} \left\{ r^a \left[-\cos^5 \phi [u_s(\psi - \psi_1) \right. \right. \\ & \left. \left. - u_s(\psi - \psi_2)] \frac{\partial}{\partial \phi} [\delta(\phi - \phi_1) - \delta(\phi - \phi_2)] \right] \right. \\ & \left. + \frac{\cos^4 \phi (3 \sin^2 \phi - 2)}{\sin \phi} [\delta(\phi - \phi_1) \right. \\ & \left. - \delta(\phi - \phi_2)] [u_s(\psi - \psi_1) - u_s(\psi - \psi_2)] \right\} \end{aligned}$$

$$\begin{aligned} & + \frac{\cos^4 \phi}{\sin \phi} [\delta(\phi - \phi_1) - \delta(\phi - \phi_2)] \\ & \times [u_s(\psi - \psi_1) - u_s(\psi - \psi_2)] \Big\} + b \cos^2 \phi \\ & \times [u_s(\phi - \phi_1) - u_s(\phi - \phi_2)] [u_s(\psi - \psi_1) \\ & - u_s(\psi - \psi_2)] + b [u_s(\phi - \phi_1) - u_s(\phi - \phi_2)] \\ & \times [u_s(\psi - \psi_1) - u_s(\psi - \psi_2)] \Big\} \\ & \times \frac{\cos B_m \phi \sin \phi}{\cos^3 \phi} d\phi d\psi, \end{aligned} \quad (21)$$

where $N_m = \int_{\psi} \int_{\phi} U_{3m}^2 A_1 A_2 d\phi d\psi = \psi^* A_m^2 b^2 \int_0^{\phi} \cos^2 B_m \phi \times \frac{\sin \phi}{\cos^4 \phi} d\phi$, ψ^* is the circumferential boundary angle, $\psi^* = 2\pi$ for a circumferentially closed shell, $\delta(\cdot)$ is the Dirac delta function, and $u_s(\cdot)$ is the unit step function. Analytical solution of N_m is carried out using the Taylor series expansion and the surface integration defined by the distributed actuator is carried out to evaluate the patch modal control force. Furthermore, the detailed micro-control action can be divided into its contributing meridional/circumferential bending and membrane control components:

$$\begin{aligned} \hat{F}_m = & \frac{Y_p d_{31} A_m \phi^a(t)}{\rho} [(\hat{T}_m)_{\text{Total}}] \\ = & \frac{Y_p d_{31} A_m \phi^a(t)}{\rho} [(\hat{T}_m)_{\phi, \text{bend}} + (\hat{T}_m)_{\psi, \text{bend}} \\ & + (\hat{T}_m)_{\phi, \text{mem}} + (\hat{T}_m)_{\psi, \text{mem}}], \end{aligned} \quad (22)$$

where $(\hat{T}_m)_{\text{Total}}$ denotes the overall control effect, excluding the coefficients and modal parameters; $(\hat{T}_m)_{\phi, \text{bend}}$, $(\hat{T}_m)_{\psi, \text{bend}}$, $(\hat{T}_m)_{\phi, \text{mem}}$ and $(\hat{T}_m)_{\psi, \text{mem}}$ respectively denote the control effect resulting from $M_{\phi\phi}^c$ - the control moment in the ϕ direction, $M_{\psi\psi}^c$ - the control moment in the ψ direction, $N_{\phi\phi}^c$ - the control force in the ϕ direction, and $N_{\psi\psi}^c$ - the control force in the ψ direction. These individual contributing control effects are evaluated in case studies. Note that surface integrations in the modal force expression are difficult to carry out exactly. Thus, the Taylor series expansion technique is used in the solution procedures.

4. Case Study

To validate the assumed mode shape function, mode shapes of the first three natural modes are compared with experimental data first in a case study. Modal control forces and detailed micro-control actions including the meridional and circumferential contributing membrane/bending components, i.e., $(\hat{T}_m)_{\text{Total}}$, $(\hat{T}_m)_{\phi, \text{bend}}$, $(\hat{T}_m)_{\psi, \text{bend}}$, $(\hat{T}_m)_{\phi, \text{mem}}$ and $(\hat{T}_m)_{\psi, \text{mem}}$ induced by segmented distributed actuator patches at various locations are evaluated next.

4.1 Mode shapes of paraboloidal shells

To validate the assumed mode shape functions, analytical solutions are qualitatively compared with experimental data in this section. Figures 3~5

Fig 3 Paraboloidal shell, the first mode

Fig 4 Paraboloidal shell, the second mode

illustrate the first three axisymmetric natural mode shapes (three-dimensional (3D) and profiles) of a paraboloidal shell ($c=0.033\ 0\ \text{m}$, $a=0.190\ 5\ \text{m}$, $\phi^*=19.119\ 7^\circ=19^\circ 07'$, shell thickness $h=1.219\ 2\ \text{m}$), in which analytical solutions and experimental data points are plotted with respect to the original shell profile. The vertical axis denotes the mode amplitude superimposed on the original elastic shell (in dash line) and plotted along the experimental data points by Glockner and Tawardros⁽²⁾. The experimental modal analysis was based on a simply-supported paraboloidal shell fabricated from polyvinyl-chloride (P.V.C) sheet by a thermo-vacuum process using special molds and templates. Natural frequencies and mode shapes of axisymmetric oscillation were measured and presented⁽²⁾. These three figures demon-

Fig 5 Paraboloidal shell, the third mode

compared well with the experimental data for the first three modes. The errors could be introduced by the non-ideal simply supported boundary conditions on the experimental model. The non-ideal boundary condition tends to be relatively rigid at the base boundary and be more flexible at the pole, which can be observed in almost all experiment data. With the understanding of fundamental dynamic characteristics of paraboloidal shells, micro-control actions of distributed segmented actuators are investigated next.

4.2 Micro-control actions of distributed actuator patches

As discussed previously, a distributed actuator layer laminated on a deep paraboloidal shell is segmented into a number of actuator patches. Micro-control actions and control authorities of these actuator patches are evaluated. The deep thin paraboloidal shell is defined by meridian/radius $=c/a=2/1$, $\phi^*=1.325\ 8\ \text{radians}$, and thickness $h=0.002$; the actuator patch size is $\Delta\phi=\phi_2-\phi_1=0.1\ \text{radians}$ defined in the meridional direction with a "circumferential width" $\Delta\psi=\psi_2-\psi_1=0.2\ \text{radians}$. Note that these patch sizes (or the projected meridional and circumferential arc lengths) are not identical as they move from the pole to the rim, because of the curvature effect. Modal control force and micro-control actions, i.e., $(\hat{T}_m)_{\text{Total}}$, $(\hat{T}_m)_{\phi, \text{ bend}}$, $(\hat{T}_m)_{\psi, \text{ bend}}$, $(\hat{T}_m)_{\phi, \text{ mem}}$ and $(\hat{T}_m)_{\psi, \text{ mem}}$ for the first three modes are evaluated, respectively. Figures 6 - 8 show the first three modal control effects plotted with respect to the horizontal projected diameter of the deep paraboloidal shell. They show the variation of control forces along the shell surface in the meridional direction. (Note that T_m denotes $\hat{T}_m$ in these figures). Accordingly, each data point represents the actuator's micro-control effects at a specific patch location moving from the

Fig 6 Modal control forces at various actuator locations, 1st mode

Fig 8 Modal control forces at various actuator locations, 3rd mode

Fig 7 Modal control forces at various actuator locations, 2nd mode

Fig 9 Effective actuator sizes at various patch locations

top pole to the bottom rim in the meridional direction. The resultant control effects of actuator patches depend on the patch locations and the mode shape variations; these forces are symmetric due to the axisymmetric assumption of the mode shape functions. Note that there are positive and negative control forces induced by the actuator patch moving in the meridional direction on the shell surface. The positive forces induce positive control effects and the negative forces aggravate the vibrations when the positive control signals are used. However, since the sign of control signals can be manipulated in the control circuit, one should look at the “absolute” magnitudes to infer the “absolute” control effectiveness at these locations.

Detailed micro-control actions reveal that the circumferential and meridional membrane control forces dominate the overall control effects and the bending (both meridional and circumferential) control moments are relatively insignificant. These micro-control effects (meridional/circumferential membrane and bending control effects) are respective-

ly calculated based on the modal force formulation including the effects of actuator sizes. Since these actuator sizes are not constant, these control forces and their micro-control effects need to be normalized in order to evaluate the true control effect per unit actuator area. Considering the curvature variations influencing the projected arc lengths, one can calculate the “true” actuator size at a given location on the deep paraboloidal shell. Figure 9 indicates that the actuator size changes significantly, depending on the patch location of the deep paraboloidal shell, although they keep identical $\Delta\phi=0.1$ and $\Delta\psi=0.2$ radians. Thus, the control forces and the micro-control actions are normalized with respect to the actuator size and these normalized meridional/circumferential membrane and bending components of the first three paraboloidal shell modes are presented in Figs. 10–12. They show the variation of control forces per unit area along the shell meridional surface and the “size effect” is excluded. Higher nominal values imply that the induced control actions per unit area (or control authorities/actuator size) are higher, i.e., higher control effects at given control cost or power.

Fig 10 Normalized modal control forces at various locations, 1st mode

Fig 11 Normalized modal control forces at various locations, 2nd mode

Fig 12 Normalized modal control forces at various locations, 3rd mode

These data indicate that actuator patches located at the pole provide the best control effects for the first mode. There are three effective locations for the second mode and five locations for the third mode. In any case, centrally located actuator patch yields the most effective control effect for most natural modes of the simply-supported deep paraboloidal shell.

5. Conclusions

Deep paraboloidal shells are common components for nozzles, horns, rocket fairings, etc. in high-performance aerospace systems. This study is to evaluate the control effectiveness of distributed segmented actuator patches and to examine their microscopic control actions and authorities of these actuator patches laminated on a deep thin paraboloidal shell. Mathematical models of the paraboloidal shell and the distributed actuator patches were defined first, followed by governing system equations. Theoretical derivations of distributed control forces reveal that the micro-control actions of actuator patches can be divided into 1) the meridional membrane effect, 2) the circumferential membrane effect, 3) the meridional bending effect, and 4) the circumferential bending effect. These micro-control actions were analyzed in a case study. Analytical solutions based on the assumed mode shape function of a simply-supported (or knife-edge) deep paraboloidal shell were derived; these assumed mode shape functions were validated by experimental data.

Analysis of distributed control forces/moments suggests that these control actions are sensitive to actuator locations and natural modes; they generally decrease as the mode number increases. Among the four control actions, the meridional and circumferential membrane control force components dominate the overall control effect and the meridional control action is slightly higher than the circumferential control action. The bending control actions (both meridional and circumferential) are very small for lower shell modes and they gradually increase as the mode increases when the shell's bending behavior becomes observable. The bending control action is proportional to the shell thickness and it increases as the shell becomes thicker, while the membrane control action remains constant. Furthermore, analysis of micro-control actions, actuator locations, actuator-size variations related to mode shapes reveals a number of ideal patch locations introducing the maximal control effects at the minimal control cost or actuator size. These analysis data provide general design guidelines and optimal actuator locations for precision control of deep paraboloidal shell distributed systems.

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