Optimization Approach for Reducing Sound Power from a Vibrating Plate by Its Curvature Design*

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In many mechanical structures, structural design for noise reduction is becoming increasingly important. Noise reduction is often achieved through structural modifications. However, it is hard to predict the effectiveness of noise reduction by typical approaches. This paper presents an optimal design approach for reducing sound power from a vibrating plate by its curvature design. The method couples an optimization technique based on a genetic algorithm (GA) with the shape representation technique, vibration analysis and acoustic radiation analysis. It is shown that the curvature design of the plate obtained by using this method can achieve effective reductions in radiated sound power. Finally, the robustness of the optimized design candidates obtained is studied by using stochastic simulations in order to find those candidates which are least sensitive to changing design parameters.

**Key Words:** Computer Aided Analysis, Noise Control, Sound, Design Optimization, Curvature design, Genetic Algorithm

1. Introduction

In many mechanical structures such as vehicles, vessels and airplanes, structural design for reducing radiated noise is becoming increasingly important. Some of the noisy structures involve thin flat plates as covering devices for mechanical components. Noise is mostly generated by the vibrating plates excited by either a mechanical or an acoustic excitation, since the resonances of these plates may cause an increase in the directly radiated noise. For examples of the typical vibrating structures in automobiles, engine cylinder, gearbox cover, transmission system covers, body panels in the dash and floor pan area etc. are characterized by flat panels. Therefore it is very important to design the plates in order to reduce radiated noise from them.

Various structural modifications have been pursued to reduce radiated noise from vibrating structures by means of passive measures such as use of plate thickness, added masses, damping materials, and ribs or other stiffeners. Compared to the active noise control, they are still an attractive alternative for reason of economy, simplicity and stability. However, it is hard to predict the effectiveness of noise reduction through structural modifications because it often requires finding the optimal solution of complex multi-dimensional functions*. In addition to that, structural modifications often need to be designed to provide an optimal trade-off between performance and serious limitations on additional weight, material cost and formability which are most often the primarily consideration in the automotive industry. Therefore structural design for noise reduction is often achieved through trial and error by an experienced designer.

Another noise control measure which is often under utilized is the control of plate geometry**–***.
Whereas this approach is seldom used due to a lack of awareness regarding the advantages of such an approach, it is often the situation that the underlying geometry is the controlling factor in the system performance. There is infinite freedom in geometric changes which could be considered. And the dynamic performance of the structure can be altered by changes in geometry. Therefore it is possible to select from various alternative geometries, the one which meets our objectives. In this study, we investigated the effectiveness of noise control by improving acoustic characteristic by the curvature design approach. Very little work has been done on effects of curvature design on acoustic reduction, since there has been lack of consideration of them, moreover numerical analyses of them are not necessarily easy.

The design goal of this paper is to optimize plate geometry with respect to reduction of the sound power in a given frequency band. To achieve this goal, we use the calculation procedure which consists of four parts, the shape representation technique using B-splines, vibration analysis, acoustic radiation analysis, and the optimization technique using a modified genetic algorithm, micro genetic algorithm (µGA) which are performed in an iterative loop. The integrated program developed herein is utilized for the analysis of sound radiation from a simply supported plate in an infinite baffle. The results show the benefits and practicality of the geometric optimization through the use of plate curvature in reducing the sound power effectively. However, even if the optimized designs show better performance under nominal conditions, the performance may still be poor because of lack of robustness. Then, the robustness of optimized design candidates obtained is analyzed by using stochastic simulations, so that designs that are statistically less likely to be sensitive to parameter scatter are determined.

2. Determination of Vibro-Acoustic Behavior of a Vibrating Plate

The sound radiation of many box-like structures can be approximated by modeling the sides as flat rectangular plates lying in an infinite rigid baffle and having simply supported boundaries. Figure 1 shows the system considered in the present application. A square flat plate was chosen as a base design for improvement in its radiation characters. This case is relatively simple, but it gives a good idea of the approach that was used. The plate is assumed to be made of aluminum (\(E=70.3\) GPa, \(\nu=0.345\), \(\rho=2690\) kg/m³) with dimensions 0.2 m x 0.2 m x 0.0015 m. Modal damping ratio of 0.02 was considered throughout the analysis. At the point (0.05, 0.05, 0.0), the plate is excited by a harmonic point-force of 9.8 Newton uniformly over the entire frequency range of interest. The vibratory response as well as sound radiation in the half-space \(Z>0\) are considered, under the hypotheses of infinite baffle and negligible acoustic-structural coupling effects. To model the vibro-acoustic behavior of the plate requires both the vibration prediction and the acoustic radiation prediction. The required equations for calculation of the vibrational and acoustic responses are presented in the following sections.

2.1 Vibration analysis

To predict the acoustic behavior of the vibrating plate, it is first necessary to know the distribution of vibration velocity on the surface of the plate. Because the acoustic-structural coupling effects are neglected, vibration analysis can be treated independently from the acoustic radiation analysis. In the usual manner, the dynamic equation of motion for the forced harmonic response is expressed as

\[
[M][\ddot{w}]+[C][\dot{w}]+[K][w]=[f]e^{i\omega t}
\]  

(1)

where \([M]\), \([C]\) and \([K]\) denotes the mass, damping and stiffness matrix of the system, \([w]\) is the vector of structural displacements, \([f]\) is the vector of applied forces and \(\omega\) is the forcing frequency. Modal frequency response analysis for solving Eq.(1) is performed using the commercial FEM code MSC/NASTRAN. The plate is discretized into 64 shell elements. The frequency sweep of the harmonic force excitation is done uniformly from 10 to 1 200 Hz in steps of 10 Hz. The steady-state vibration response of the plate is investigated. The predicted velocities are used as boundary conditions and specified as input for the acoustic radiation analysis.

2.2 Acoustic radiation analysis

The normal components of the predicted velocities lead to the determination of the radiated sound pressures as described next. The acoustic pressure

Fig. 1 Description of the system considered in the present application
field generated by a vibrating structure can be written
in terms of the values of the pressure and normal velocity on the surface of the structure as
\[
p(r) = \int_S \{p(r) - \frac{\partial}{\partial r} G(r, r')\} G(r, r') + j\omega \rho v_s(r) G(r, r') dS(r')
\]  
(2)
where it is assumed \( n \) is the vector normal to the surface \( S \), \( p \) is the complex sound pressure, the field point \( r \) is outside the surface \( S \), \( v_s \) is the normal surface velocity at surface point \( r' \), \( G(r, r') = \exp(-i\omega |r-r'|) / 4\pi |r-r'| \) is the free space Green function, \( k = \omega / c \) is the acoustic wave number, \( c \) is the speed of sound in the ambient medium and \( \rho \) is the mass density of ambient medium. Equation (2) is called the Kirchhoff–Helmholz equation and its derivation can be found in the text by Junger and Feit\(^{(5)}\), among others. The boundary element method (BEM) has been formulated as the numerical calculation of Eq. (2) and successfully applied for solving the acoustic radiation problems. BEM has made it possible for investigating radiators of quite complex geometry. However, the use of BEM for acoustic radiation analysis is very time-consuming.

For systems having simple near-planar geometry, a Rayleigh integral approximation is known as a relatively inexpensive method to solve sound radiation problems and yields a good approximation\(^{(6)}\). With reference to Fig. 1 the acoustic pressure at position \( r \) by
\[
p(r) \approx 2j\omega c \int_S v_s(r') G(r, r') dS(r')
\]  
(3)
This integral is the reduced form of Eq. (2). It can only strictly be applied to distributions of normal velocity in an otherwise rigid plane. The calculation of the sound field based on Eq. (3) requires only surface integration, which makes the numerical effort considerably smaller than the use of BEM.

Radiation characteristics can be typically defined by radiated sound power, radiation efficiency and average sound mean square velocity of the plate. Of these, the most direct measure of sound radiation is the radiated sound power, evaluated over the entire frequency band of interest. In this study, the total sound power radiated from the plate is obtained by using ISO 3745-1977 rule\(^{(7)}\). It is calculated from the space average of the mean-square sound pressures at the receiver points. The locations of the receiver points are associated with equal areas on a hemispherical response surface around the model according to ISO 3745. And the sound power \( W \) is given by the following equation
\[
W = \frac{2\pi r^2}{\rho c} \sum_{i=1}^{N} \frac{p_i^2}{N}
\]  
(4)
where \( p_i \) is the sound pressure level at the \( i \)-th receiver point. \( 2\pi r^2 \) is the area of the hemisphere of radius of \( r \) and \( N \) is the number of receiver points.

A commonly used measure of the sound radiation characteristics to convert vibratory energy into sound energy is the radiation efficiency \( \sigma \), which is defined as
\[
\sigma = \frac{W}{W'} \quad W' = \rho c S \langle v_e^2 \rangle
\]  
(5)
where \( W' \) is the power which would be radiated by baffled piston motion of the plate whose all parts are vibrating in phase with velocity equal to the average mean square velocity of the structure in question. In this study, the corresponding dB-level are defined as
\[
LW = 10 \log(W/W_o) \text{ dB} \quad LV = 10 \log(\langle v_e^2 \rangle) \text{ dB}
\]
(6)
\[
La = 10 \log \sigma \text{ dB}
\]
where decibel values for the sound power levels are always calculated using a reference power level of \( W_o = 1.0 \times 10^{-21} \text{ W} \). The same elements of the plate are used for the acoustic radiation analysis and the vibration analysis.

3. Normal Engineering Approach by Adding Ribs

Herein, we are going to demonstrate the effect of a conventional technique to reduce noise. Vibration produces noise, so the best method to control noise is to reduce surface vibration response of the plate. Vibration response can be reduced by increasing intrinsic frequency as the results of increasing stiffness. Typical engineering approaches to changing plate’s stiffness and frequency are adding ribs to stiffen the top surface or applying damping material to absorb vibration energy, or changing the nominal thickness. Stiffening the plate by adding ribs is the most common approach adopted by product development engineers to provide a quick fix for a NVH problem. Since adding ribs does not increase much weight of the plate, it is an economical method to reduce vibration and noise. So in the following analyses, numerical simulations are performed to evaluate the effectiveness of improving the vibro-acoustic characteristics using this approach.

We consider two rib structures with four (four crosses) and six (nine crosses) additional ribs as shown in Fig. 2. Each rib has a cross section of 1.0 mm in width and 10.0 mm in depth. The surface mean square velocity of the plate with two rib structures is

![Fig. 2 Two rib structures](image-url)
shown in Fig. 3(a) in comparison to the performance of the original flat plate. According to the results, four ribs give a fundamental frequency raised from 120 Hz to 270 Hz and an average reduction of 7.3 dB in the whole frequency band, while six ribs give a fundamental frequency of 360 Hz and a reduction of 8.1 dB. It becomes obvious that raising fundamental frequency leads to increasing stiffness and an improvement of the vibrational characteristics in the frequency domain. Both of two rib structures come out better than the original, however, comparison between them shows that additional ribs provide a little reduction of only 0.8 dB. Then the corresponding sound power and radiation efficiency response are shown in Fig. 3(b) and (c) respectively. In this case, four ribs result in an increase of 0.5 dB in the sound power averaged over the whole frequency band, while six ribs result in an increase of 2.2 dB. Therefore they did not result in broad band reductions in radiated power. The major reason for the poor performance is that these structures had increased radiation efficiency.

This example illustrates that a typical rib stiffened plate design may have no effect or even a negative effect on an improvement of the radiation characteristics if not used properly in terms of structural and acoustical effects. High amount of additional ribs may substantially reduce the vibro-acoustic response, however, has adverse consequences in terms of additional weight, material cost, and formability problems. This leads to an over-designed and excessively conservative system under many circumstances. So it is necessary to find out a method that can effectively improve radiation properties without increasing much weight, size and cost of the structure.

4. Curvature Design Approach

The main degrees of freedom for the structural design correspond often to the shape of the structure itself. To the same plate, its dynamic characteristic often has much to do with its shape besides its material characteristic and thickness. An alternative approach to improve the radiation characteristics is to change the plate geometry. Since changes in plate geometry can potentially have an infinite variety of dynamic characteristics, it is possible to take many effects into account, thereby to change the radiation behavior in a complicated manner. Therefore, it may be possible to optimize the inherent performance without the need for any conventional NVH control measures.

Whereas structural design optimization for reducing vibro-acoustic response has been studied extensively in many articles, there has historically been very little consideration of effects of geometric layouts on the vibro-acoustic performance. A few authors performed geometric optimization with respect to different criteria which optimize the NVH performance using the geometric optimization features of some commercial design software packages. Marburg et al. optimized to minimize the sound pressure level at a driver's ear using the geometry based model of a vehicle body by the commercial code ANSYS. In their work, the sound power has not been considered. Steyer et al. found that the double curvature added to the plate geometry resulted in significant stiffness and dramatic increase in the fundamental frequency of the plate by using the commercial code IDEAS. However, increasing stiffness may not necessarily culminate in an effective improvement of the vibro-acoustic properties considering the frequency range of interest, as described in the Section 3.

We will extrapolate their study concerning the benefits of changes in the plate geometry to our problem. In this study, optimum design of a plate geometry for reducing sound power effectively was studied by introducing the addition of double curvature to it.

5. Numerical Shape Representation Technique for Curvature Design

The numerical calculation for curved surface structure design is discussed in this section. The appropriate selection of the numerical shape representation technique for this problem is necessary for
constructing finite element models for various curved plates. Numerically, the shape of any continuous surface is completely identified if the coordinates of all points on the surface are known. Blending functions typical of computer graphic methods are employed to determine the coordinates of any point inside the surface geometry. The B-spline technique is used in this study. Previous work\textsuperscript{[9,10]} have considered the shape optimal design using B-splines. The coordinates of each point (x, y, z) on a continuous surface in three-dimensional space are determined by satisfying a set of parametric equations:

\[ x(s, t) = \sum_{j=1}^{M} \sum_{i=1}^{N} a_{ij} B_i(s) B_j(t) \]

\[ y(s, t) = \sum_{j=1}^{M} \sum_{i=1}^{N} b_{ij} B_i(s) B_j(t) \]

\[ z(s, t) = \sum_{j=1}^{M} \sum_{i=1}^{N} c_{ij} B_i(s) B_j(t) \]  \hspace{1cm} (7)

for suitable ranges of the parameters, s and t, where \( B_i(s) \) is B-splines of degree \( k-1 \), \( a_{ij}, b_{ij}, c_{ij} \) are constants determined by the control points that control two families of curves defining the surface and \( M \) (or \( N \)) is the number of control points along one \( s \) (or \( t \)) curve. For instance, 25 control points of Fig. 4 show a shape represented by cubic splines.

Compared to other techniques, the main advantage of the B-splines is that local control of the curve shape can be achieved by using a set of blending functions that have local support only, so the location of each control point does not influence the shape of the whole curve. This is very desirable for maintaining an adequate finite element mesh during the geometric optimization process. Another advantage is that additional control points can be introduced without increasing the degree of the curves. Therefore B-splines offer the designer flexibility to select the degree as well as the multiplicities of control points. Consequently, various shapes may be represented by quadratic or cubic splines which are automatically pieced together to form the B-spline. In this study, cubic splines are used.

The nodal coordinates for the finite element model are obtained from the surface determined by Eq. (7). The shape design approaches using the independent node movement technique have generally
to resort to an automatic mesh generator for each updated geometry throughout the iterative process. We have developed a computer program which can automatically generate the full finite element model including boundary conditions from the geometric definition. So we are able to create several structural models easily only by choosing design constraints and design parameters.

6. Acoustic Behavior of Simple Curved Geometries

In the first example, simple curved geometries were modeled to illustrate the effect of curvature designs on the acoustic behavior. They were modeled as circular arcs symmetric about \( x=0 \) and \( y=0 \) with center heights 3.0 mm and 6.0 mm which are relatively small compared to the dimension of the plate. Figure 5 shows the finite element model of a curved plate with the center height 6.0 mm. The initial design, the finite element mesh, and the design model are the same as for the original flat square plate. Figure 6 shows the sound power response of the two curved plates compared to original flat plate. From this figure, it is seen that curved plates dramatically raised the frequency of the first peak for radiated sound power, so the number of the peaks in the whole frequency range was decreased. At the curvature height of 6.0 mm, the first peak was increased from 120 Hz to 720 Hz and then the number of the peaks in the domain was decreased from six to two. The above analyses demonstrate that the addition of curvature to
a flat plate has a remarkable influence on the acoustic behavior.

7. Curved Geometry Based Optimization

The study is extended to investigate the effect of more general curved geometry for optimal design of the plate. In this geometric optimization, we presume that the optimized plate must be symmetric only about \( y = 0 \) and the contour connecting the plate must not be changed. Herein, the vertical coordinates \( z_1 - z_6 \) of six control points shown in Fig. 7 were then adopted as the design variables within optimization run. The variational geometry defined with control points each will allow for a number of solutions. The frequency averaged sound power is treated as the optimization criterion.

7.1 Design space visualization for two variables problem

Acoustic radiation optimization problems may be highly non-linear ones, due to the inherent complexity of the relations involved. A prior knowledge of the behavior of the objective function is always helpful in the selection of an appropriate optimizer algorithm. While it is not feasible to directly search the design region in all the six variables, it is reasonable and instructive to evaluate the performance of the function in the problem behavior for one or a few variables in order to gain an appreciation for the complexity of the optimization. Figure 8 shows the sound power averaged over 10 - 1200 Hz band with respect to \( z_1 \) and \( z_2 \) set to within 0.0 and 6.0 mm while fixing all the other variables 0.0 mm. The complex response results in several valleys in the response surface. This two variables behavior leads one to expect the true multi-modality of the problem that is not easy to predict the optimal solution. This, however, is with the remaining design variables fixed while in the optimization process the response surface shown here would also vary as other variables are adjusted. The case of all the six variables would represent a much more difficult optimization problem, due to an exhaustible search over a large number of possible solutions in a complex multi-dimensional function.

7.2 Optimization technique

References (2), (3) discussed in Section 4 employ traditional optimization approaches using numerically determined gradients for their problems. Such classical optimization methods have advantage of converging on an optimal solution quite rapidly, especially if the objective function is well behaved. In designs where the search space is multi-modal and contains many local optima, these methods can converge to a local optimal solution as only the local neighboring search space is explored. For a such problem, success for the search process largely depends on the starting point of the design. Therefore, the application of these methods to this problem may be limited when a feasible starting point is unknown. In this case, an optimizer based on a different search algorithm is expected to have more success with this problem.

In the recent past, genetic algorithms (GA) optimization technique, which is one of non-deterministic methods, has been successfully implemented for a wide variety of optimization problems. In several references(11-13), it was applied for the vibro-acoustic optimization problem. GA was presented by Holland in the 1970s, and a detailed description can be referred to Mitchell(19) as an example, among many others. GA is well-suited to finding solutions to highly complex optimization problems. When applying GA to problems having a large number of locally optimal solutions, near-optimal solutions are obtained after a limited number of iterations and in most cases these outperform the existing design. Seeking the true globally optimum design is not a necessity since it often computationally expensive due to a relatively large number of iterations. The use of GA provides a reasonably efficient method of finding optimized design candidates.

Fig. 7 Plate model symmetric about \( y = 0 \) indicating the six parameters \( z_1 - z_6 \)

Fig. 8 Sound power averaged over 10 - 1200 Hz band as a function of \( z_1 \) and \( z_2 \)
In the case of the simplest form of a GA (SGA), the general choice of population size typically range over 50 individuals. One serious drawback of SGA is the time penalty involved in evaluating objective functions for large populations, generation after generation. As a contrast to the large population GA, schemes for using micro genetic algorithm (μGA) have been introduced by several authors and have been shown to be more effective (i.e., have fewer function evaluations)\(^{(12\text{a})}(13)\).

A population size fewer than 20 can be used successfully with μGA. It is a known fact that SGA generally does poor with a small population size due to insufficient information processing and early convergences to non-optimal results. The fundamental process with key to success with small population is outlined as follows:

1. Select a random, very small population. Quite often the population size of 5 is used.

2. The best individual is saved and the remaining population members compete for the parent selection. This is based on tournament selection.

3. The selected parent individuals are mated and crossed-over using the uniform crossover operation.

4. Perform above operation until the population converges. For this study, population convergence is considered achieved when less than 5% of the bits of the other individuals are different from the best individual.

5. If converged, the best individual is copied over to the next generation, and the remaining population is again randomly chosen.

6. Go to procedure (2) and repeat.

No mutation is used in μGA because with the rapid convergence cycles mutations do not have time to evolve before a new random population is introduced. SGA is known to reach premature convergence forcing the search process to rely entirely on the mutation operator to find the optimum. In the case of μGA, the “start and restart” procedure helps in avoiding the premature convergence and μGA is always looking for better individuals. Average populations fitness values are not meaningful with a μGA because of the start–restart nature of the μGA evolution process. Performance measure for μGA is based on the best-so-far individual, rather than on any average performance. In implementing μGA, our interest is purely to find the optimum as quickly as possible and not in the average behavior of the population. Krishnakumar and Carroll found that μGA demonstrated faster convergence to the near-optimal region than did SGA for the multi-modal problems they studied. For the current problem, μGA is applied to adjust the curved geometry to reduce its sound power.

### 7.3 The formulation of the optimal design

Using GA, the optimal design problem is specified in terms of a set of the design variables and the value of fitness function, i.e., the value of the objective function to be optimized. For this problem, the independent design variables selected were the six parameters z1 – z6. These variables were limited by lower and upper bounds to 0.0 – 6.0 mm. This restriction on the variables may not cause drastic distortion of the shape of a finite element during optimization, so the new mesh may be adequate for accurate finite element analysis. The sound power averaged over 10 - 1200 Hz band (in 10 Hz steps) is used as the fitness function. It is desirable to improve not one particular peak but the overall power level in the frequency band of interest in practical structural design for noise reduction. Thus, this optimal design problem can be represented as

\[
\text{minimize } W = \frac{1}{n} \sum_{i=1}^{n} W_i \\
\text{subject to } 0.0 \leq z_i \leq 6.0 \quad i=1,2,\ldots,6
\]

where \(n\) is the number of frequencies. Optimization was done to minimize the value of fitness function. There is no need to scale fitness, since tournament selection is used. The crossover probability for the μGA was set at 0.5 and population size at 5.

In the design analysis for this study, there are more than one computer programs involved. To model the complete system, coupling between these programs is required. We have developed a design tool composed of four steps, the shape representation technique, vibration analysis, acoustic radiation analysis, and the optimization technique which are performed in an iterative loop.

Herein, when the design variations remain small compared to acoustic wave length, the geometry of the model for the acoustic radiation analysis can be considered as the original one. Since a flat plate was chosen as a base design for this optimization, in the iteration process acoustic radiation is treated by means of the Rayleigh integral approximation as described in Section 2.2. This yields the computational efficiency in view of the many evaluations occurring in an iterative loop compared to the use of BEM. For example, Fig. 9 shows predicted sound power response of the curved plate with the center height 6.0 mm shown in Fig. 5 comparing by using the Rayleigh integral and BEM for the acoustic radiation analysis. The response indicates good agreement between the two results. As well, in the case that complex geometry is chosen as a base design, the same vibration–sound pressure transformation function as for the original one may be available in the iteration process if the design variations...
remain small.

In this design analysis, μGA has been integrated with the shape representation technique (B-splines), the vibration analysis (MSC/NASTRAN) and the acoustic radiation analysis (the Rayleigh integral approximation). We can view the analysis program flow for the optimization iteration process in Fig. 10. It automatically generates all the necessary analysis input files, launches the analysis runs and extracts the data from the analysis generated output files. No personal intervention is needed during the optimization run. Doing them manually is not very practical because of amount of time involved with them. This optimization process allows us to ask the computer to automatically make adjustments to produce the optimal design.

The iteration process is created until a convergence criterion is judged satisfactory. The criterion for this study was chosen as a maximum number of objective function evaluations. For rejecting unnecessary calculations, duplicate genetic codes are skipped over and a new code is used for the GA implementation.

8. Design Optimization Example

In the following section, the plate geometry will be optimized using the procedure described in the above sections. Figure 11 displays the evolution of the ensemble average of the best-so-far fitness. Each optimization is conducted for 10 different random starts and an ensemble average of the best-so-far fitness is calculated. In this optimization, each of 6 variables was presented by 8-bit number, so the total number of possible combinations are very large, $2,8 \times 10^{18}$. The progress and rapid convergence of the optimization is illustrated in this figure. The optimized geometry of the plate for a typical case is shown in Fig. 12. It has a sound power response as shown in Fig. 13(a). Although this case is not the globally optimal solution but one of near-optimal solutions, a substantial reduction of about 22 dB was achieved in the frequency averaged sound power.

In general, the reduction of the sound power can be attributed to acoustical effects as well as structural effects. The effect of this optimized geometry on surface mean square velocity and radiation efficiency is shown in Fig. 13(b) and Fig. 13(c) respectively. From these figures, it is seen that at lower frequencies...
the power reductions are achieved primarily by the decreased mean square velocity, while at higher frequencies by the decreased radiation efficiency. Of interest is an increase in the mean square velocity at the resonant frequency of 1150 Hz, but the decreased radiated sound power. To understand why the sound power has been reduced at this frequency, it is informative to examine the vibration shape of the optimized plate (Fig. 14). As can be seen, the vibration shape is mainly contributed by an asymmetrical mode and therefore the acoustic energy can erase each other. As a result, the plate design was optimized to effectively reduce the sound power.

The optimization process was successfully able to find out the optimal design. The results emphasize that relatively small changes in the geometry of the plate can result in significant reductions in radiated sound power. They highlight the advantages of using the underlying curved geometry to achieve improved acoustic performance of a structure. It is important to point out that in many instances, the increase in structural size required to accommodate such curvature designs may be no more than would be required to accommodate additional ribs. Moreover, more general curved geometry may include a good design which might lead to a better performance.

9. Investigation of Robustness

Industrial development should yield robust and reliable designs with desired performance levels in an uncertain environment. Robustness means finding an acceptable balance between the scatter of the system parameters and the scatter of the performance. Even if one finds the optimized designs for the computer simulation model, practical design implementations with exactly the required parameters may not be feasible. This is caused by the unavoidable scatter of the properties and the boundary conditions due to, for example, manufacturing and assembly tolerances, thermal expansion and contraction of real systems.

If the simulated case does not reflect uncertainties in the design parameters, i.e. scatter of input variables, it is simply one out of an infinite number of possible states of a system and it may never occur in reality. The performance of the optimized solution may still be poor because of the scatter in the system which can never be avoided in the real world. Therefore it is clear that if the optimal solution performs so only under nominal conditions, its practical application is limited.

In this case, it seems highly appropriate that the computations would not produce a single (nominal) result but rather assemble a statistical record of events by introducing scatter in simulation models\textsuperscript{145–146}. Knowing the statistical description of the result of analyses allows one to evaluate the robustness and to assess much better reliability of the results than single ones from nominal analysis. Therefore it is possible to reveal the system and its behavior.

9.1 Optimized design candidates

GA creates a rich database of design variables and their objective function values during the optimization process which may be of great value to the designer. Often in process sequence design, non-optimal solutions may be selected from a practical engineering point of view. This database helps in selecting "preferred non-optimal solutions".

As can be seen in the Section 8, the fact that relatively small geometric changes of the plate result in such an improvement indicates the great sensitivity of the objective function with respect to the given parameter set. Especially in complex systems, it can be very risky to rely on the results of the nominal case
only, since even minute changes in model parameters can often have significant changes in the results. In the light of above statements, it seems to make much more sense to focus on robustness of improved solutions than on nominal optimality. It is informative to define a somewhat sub-optimal design that performs well even in the presence of scatter. This section then considers the sensitivity of some optimal candidates to small changes in model parameters.

Herein, four optimized design candidates were chosen. The numerical results for the candidates are summarized in Table 1. The resulting designs are labelled with suffixes "A" to "D". The nominal value for each case shows a similar magnitude of performance improvement. Normally the best nominal value is taken to be the best candidate, however, it is not necessarily a robust one. On the other hand, another candidate, although having a slightly lower optimized performance under nominal conditions may be less sensitive to changing conditions, and thus would be a more practical choice. Therefore it is interesting to compare the four candidates in the presence of scatter. In next section, then, the robustness of them is investigated by introducing scatter.

9.2 Robustness of optimized design candidates

The robustness of the optimized performance of the four candidates to small design variations was briefly studied by applying small perturbations to model parameters. The same 200 sets of perturbations were applied to each candidate and the objective function re-evaluated and recorded. A random deviation at six design variables (uniformly distributed) was assumed to be ±0.05 mm. The choice of the maximum size of perturbations used was arbitrary to some extent. In addition to design parameters, real systems have many more parameters (which are called free stochastic variables), which change arbitrarily within their range of scatter. Frequently, only the design parameters are taken into account, but in some cases it is possible that free stochastic variables are even more important for the performance. Here, the plate thickness was considered as a free stochastic variable. This random deviation was assumed to be

![Fig. 15 Statistical distribution of the perturbed performance for the four design candidates](image)

±0.05 mm.

Figure 15 shows the distribution of the perturbed performance for each candidate, that is derived from a sample of 200 cases. The perturbed values are displayed using a histogram to indicate the statistical spread about the nominal value. For the histograms, the range is divided equally into 10 bars between the minimum and maximum values. The robustness of each candidate is then determined by the spread of each statistical distribution; the narrower it is, the more robust the design. The mean and worst values of the perturbed performance for each case are also included in Table 1. In all cases, the nominal values are too optimistic since they are better than the mean values. The worst values for all candidates are better than 10 dB for the original flat plate. In a fairly complex system, it would be important indeed to at least look at the worst case in advance. Only when the worst case is known, serious decisions are possible, although this does not have a great relevance since its probability is very low in practice. In this case, an overall measure of performance depends on both the mean and worst values in perturbed objective function value. If the worst case was used as the criterion, then design D is shown to have the best performance. However, its average performance is not as good as design A. Typically at this point, a trade-off has to be made between performance and tolerances.

In this case, it is informative to further examine the perturbed performance. Therefore in addition to histograms, graphical analysis was done by means of scatterplots. Figure 16 shows the character of the perturbed performance of design A versus z1 which was chosen as one design parameter. Each point in this figure corresponds to a single computer run. The basic global pattern that shapes the scatter and deter-
Fig. 16 Perturbed performance of design A versus z1

determines its density fluctuations shows the physics behind the phenomenon when physical uncertainties are considered. From Fig. 16, it is seen that a main cluster of results appeared, together with several outliers. These outliers mean sudden jumps. Moreover some of them have the distance of about 10 dB from the mean value. Such outliers hint at a possible substantial loss of the performance of the simulated system with the given scatter of input parameters. A system like this can be controlled by keeping the scatter of the system parameters low since obviously, by reducing the tolerances the perturbed performance comes closer to the nominal one. Reducing tolerances, however, normally means a thing that is expensive and labor-intensive and in some cases there are even technical limitations to it.

9.3 Design improvement

Another method for keeping the scatter low is to further improve the perturbed performance by changing design parameters. In complex systems, it is the connectivity of the components and their basic properties that establishes the global behavior. Therefore, what is required for improving the global behavior of the system is to design it considering collective changes of all the design variables, in particular their mean values, so that the system globally behaves in a different and acceptable manner.

A design improvement was undertaken using a stochastic method called stepping technique[14]. This stochastic improvement method uses a natural and simple iterative process. A small random sample (in this study 15 cases) is generated with minimal scatter around the nominal design as the starting point. Within this sample, the objective function of each case is being calculated. Then, the best case is chosen as the starting point for the next scramble of 15 cases and a new sample is generated around it. The idea of the method is to shift the average values of the design parameters to the value that the respective parameter came out to in the best case. This iterative procedure is repeated until no further improvement in the value of the objective function takes place, or rather until a substantially improved solution is obtained. In some circumstances, the results depend on the starting point, since this method approaches the problem step by step moving through the design space by iteratively evaluating subsets of it. This method is suitable for cases where relatively small improvement are necessary.

Six design variables were chosen as design parameters to improve the perturbed performance of design A by the stepping method. The convergence was achieved within 100 steps. Figure 17 is the improved results obtained after 100 steps. The mean value was slightly decreased from 76.78 to 76.11 dB, but much more important is that the number of distant outliers was decreased. The improved design had not one case exceeding 82 dB in a sample of 200 cases. This result outperforms design D with respect to the worst value. This example shows that the stepping method brought about major improvements, always keeping the scatter and uncertainties in mind.

10. Conclusion

This paper shows the advantages of using the underlying curved geometry to improve acoustic performance using the integrated program developed herein. Radiation characteristics of a curved plate were predicted using the shape representation technique, vibration analysis and acoustic radiation analysis. Although the resulting optimization problem involves highly nonlinear functions, the optimizer based on GA was successfully able to perform this optimization. It has been demonstrated that curved geometry-based optimization of the plate structure can lead to a remarkable improvement of the radiation characteristics. This approach is both more effective and economical of materials and weight than the try and error procedure using conventional approaches such as ribs.

The robustness of the optimized design candidates obtained was evaluated by applying a common ensemble of random perturbations to each candidate. The resulting statistical distribution of perturbed performance helped in the choice of the best optimized
candidate. And the stochastic improvement method using the stepping technique was successfully implemented for improving the perturbed performance. Such stochastic simulations enable engineers to evaluate the real response in complex systems that would be difficult to do only by nominal case. Therefore they are an important supplement to numerical optimization techniques.

In the future work, this application presented will be extended to control of the vibro-acoustic response of more general and various vibrating structures.

References


