Dual Mode Transmission Designs for Motorcycles*

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This paper presents two types of the dual mode transmission for use in a motorcycle. A motorcycle transmission typically using the discrete speed ratio transmission that works by alternating the gear drive, and the continuously variable transmission (CVT) that transmits power by using the rubber V-belt. The dual mode transmission described in this paper, utilizing a V-belt drive, a fixed ratio mechanism, and a planetary gear train, can provide two operation regimes over the full speed ratio range by means engaged clutches. One is the fixed gear ratio and differential transmission system; the other is the differential and continuously variable transmission system. The kinematics, power flow, and mechanical efficiency of each transmission system are analyzed. An example is employed to illustrate the design procedure. Furthermore, several available design concepts of the dual mode transmissions for motorcycle applications are proposed.

Key Words: Dual Mode Transmission, Differential Transmission, Continuously Variable Transmission (CVT), Motorcycle

1. Introduction

A mechanical-type continuously variable transmission (CVT) consists of a speed-sensing pulley as a driver and a torque-sensing pulley as a driven jointed by a rubber V-belt. Since such CVTs have the advantages of, over discrete speed ratio drives, simple in construction, smooth in operation, easy derivability, low cost, the operating conditions of the engine closed to the optimum fuel consumption line, etc. The mechanical-type CVT is widely used in motorcycles. However, the mechanical efficiency of the mechanical-type V-belt CVT is quite low, especially, at the instant of speed ratio change with frequent stops-and-goes⁽¹⁾.

Another design approach is the concept of the split-power that was developed to partially overcome

the poor efficiency characteristics of certain CVTs. A split-power CVT that consists of a continuously variable unit (CVU) connected in parallel with a differential gear may be namely a differential transmission. The basic idea is to send a part of the power though the CVT (with lower efficiency), with the remainder of the power going through a mechanical path (with higher efficiency). For the differential transmissions, in gaining higher efficiency, however, we suffer a reduction in ratio range. In order to extend the ratio range for overcome the reduction ratio range characteristic of the differential transmission and improve the mechanical efficiency, the concept of dual mode transmission systems, focused on automotive industries, have been studied extensively⁽²⁾⁻⁽¹¹⁾. Moreover, the configuration of many commercial dual mode transmissions can be found in various US patents⁽¹²⁾⁻⁽²¹⁾. An examination of the existing patents revealed that all of the transmissions are used in automobiles. For example, Moan⁽¹⁷⁾, by using a belt drive mechanism as the variable ratio unit, a compound planetary gear set as the differential gear, proposed a fixed ratio and continuously variable transmission system. Analyses of differential trans-

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missions have been numerous, including power flow analysis^{(22)–(28)} and mechanical efficiency analysis^{(29)–(35)}.

Our purpose here is regarding the design of dual mode transmissions that suitable use in a motorcycle. Four types of the dual mode transmission proposed by authors, including (1) The differential transmission system, (2) The differential and continuously variable transmission system, (3) The fixed gear ratio and differential transmission system, and (4) The fixed gear ratio and continuously variable transmission system⁽³⁶⁾. Here we will focus on two types of the dual mode transmission. One is the fixed gear ratio and differential transmission system; the other is the differential and continuously variable transmission system. This paper starts with the description of the dual mode transmission and goes on the kinematic analysis to obtain the valid range of the relative speed ratio of the dual mode transmissions. The power flow and mechanical efficiency of the differential transmission will be investigated and an example is used to illustrate the design procedure. Finally, we propose several available design concepts of the dual mode transmission for motorcycle applications.

Nomenclature

- B_a, B_b, B_v : brakes adjacent to input axis, output axis, continuously variable unit (CVU)
 C_a, C_b, C_v : clutches adjacent to input axis, output axis, CVU
 K_a, K_b : speed ratios of fixed ratio mechanism
 P_a, P_b, P_v : powers of differential gear adjacent to input axis, output axis, and CVU of the input-coupled system
 P_{cv} : power carried by CVU
 P_{in}, P_{out} : input and output powers of transmission
 τ : speed ratio of transmission
 τ_{max}, τ_{min} : maximum and minimum speed ratios of transmission
 R : relative speed ratio of planetary gear train (PGT)
 R_o : basic speed ratio of PGT
 $(R_o)_{max}, (R_o)_{min}$: maximum and minimum basic speed ratios of PGT
 T_a, T_b, T_v : torques of differential gear adjacent to input axis, output axis, and CVU of the input-coupled system
 T_r, T_s, T_c : torques of simple differential gear adjacent to ring gear, sun gear, and planet carrier
 V : speed ratio of CVU
 V_{max}, V_{min} : maximum and minimum speed ratios of CVU
 $\omega_a, \omega_b, \omega_v$: angular velocities of differential gear

adjacent to input axis, output axis, and CVU of the input-coupled system

$\omega_r, \omega_s, \omega_c$: angular velocities of simple differential gear adjacent to ring gear, sun gear, and planet carrier

η_{ha}, η_{hb} : efficiencies of fixed ratio mechanism

η_o : Overall efficiency of differential transmission

η_{sr}^c : efficiency of simple differential gear

η_v : efficiency of CVU

2. Dual Mode Transmission Systems

The differential transmission can be classified as an input-coupled system or output-coupled system. One shaft of the differential gear is linked to the power input side of an input-coupled system; while the other shaft of the differential gear is linked to the power output side for an output-coupled system. When such concepts are adopted for designing the dual mode transmissions, the block diagram can be arranged as shown in Fig. 1. Thus the dual mode transmission is made up of three elements: a CVU, a planetary gear train (PGT), and a fixed ratio mechanism (FR). By engaging different clutches and/or brakes on the sequence listed in Table 1, the fixed gear ratio and differential transmission system and the differential and continuously variable transmission system can be achieved. Here, $C_a (C_b, C_v)$ and $B_a (B_b, B_v)$ are the clutches and the brakes adjacent to input axis (output axis, CVU), respectively.

To obtain the benefits of low cost, small size, and manufacturing simplicity, a rubber V-belt CVT, a simple planetary gear train, and a chain drive are adopted in the following example as the CVU, the differential gear, and the fixed ratio mechanism of the dual mode transmission. Theoretically six connection arrangements for the input-coupled and output-coupled system are possible provided that all three

Table 1 Clutches and brakes engaged state

	Input-coupled				Output-coupled			
	C_a	C_v	B_a	B_v	C_v	C_b	B_a	B_v
Differential mode	x	x			x	x		
Fixed gear ratio mode	x			x		x	x	
CVT mode		x	x		x			x

x: engaged

Fig. 1 Block diagrams of dual mode transmission systems

members of a simple planetary gear train can change positions as shown in Fig. 2.

(a) Input-coupled

(b) Output-coupled

Fig. 2 Atlas of dual mode transmission systems

3. Kinematic Analysis

For the input-coupled system as shown in Fig. 1 (a), the transmission ratios of the single elements are:

$$V = \frac{\omega_v}{\omega_a}; K_a = \frac{\omega_a}{\omega_v}; R = \frac{\omega_b - \omega_v}{\omega_a - \omega_v}; r = \frac{\omega_b}{\omega_a} \quad (1)$$

From Eq.(1), the global transmission ratio can be expressed as:

$$r = V + (K_a - V)R \quad (2)$$

Where r , V , and K_a are the speed ratio of the output speed to the input speed of the transmission, the CVU, and the fixed ratio mechanism, respectively; and R is defined as the speed ratio of the output speed to the input speed with the remaining members being relatively fixed of the planetary gear train. For the example as shown in Fig. 2, its range of the relative speed ratio of the simple differential gear, Table 2, characterizes each transmission system. Here, the basic speed ratio of the differential gear R_o is defined as the ratio of the output speed to the input speed with the carrier being fixed. Subscripts s , r , and c denote the sun gear, the ring gear, and the carrier of the simple differential gear, respectively.

Equation (2) establishes the speed ratio of the transmission and the speed ratio of the CVU. A CVU has a speed ratio range from a minimum value and a maximum value. For simplicity, for example as a mechanical-type CVU, only the differential transmission speed ratio is increased as the speed ratio of the CVU is increased, i.e., $dV/dr > 0$. For hydraulic and electrical variable ratio drive, the differential transmission can operate with the speed ratio is increased as the speed ratio of the CVU is decreased, i.e., $dV/dr < 0$. If $dV/dr > 0$, $r = r_{\max}$ ($r = r_{\min}$) happens

Table 2 Range of the relative speed ratio of simple differential gear

Type of simple differential gear connections	Relative speed ratio	Range of relative speed ratio
1	$\frac{1}{1 - R_o}$	$0 < R < 0.5$
2	$\frac{R_o}{R_o - 1}$	$0.5 < R < 1$
3	$\frac{R_o - 1}{R_o}$	$1 < R < 2$
4	$1 - R_o$	$R > 2$
5	$\frac{1}{R_o}$	$-1 < R < 0$
6	R_o	$R < -1$

at $V = V_{\max}$ ($V = V_{\min}$). Once fixed the maximum and minimum speed ratio of the CVU ($V_{\max}$, $V_{\min}$) and the transmission ($r_{\max}$, $r_{\min}$), the speed ratio of the differential gear and the fixed ratio mechanism can be determined from Eq. (2) as follows.

$$R = \frac{V_{\max} r_{\min} - V_{\min} r_{\max}}{K_a (V_{\max} - V_{\min})} \quad (3)$$

If $dV/dr < 0$, $r = r_{\max}$ ($r = r_{\min}$) happens at $V = V_{\min}$ ($V = V_{\max}$), we have:

$$R = \frac{V_{\max} r_{\max} - V_{\min} r_{\min}}{K_a (V_{\max} - V_{\min})} \quad (4)$$

Similarly, for the output-coupled system, the speed ratio of the differential transmission can be expressed as:

$$r = \frac{K_b R V}{K_b R + V - K_b} \quad (5)$$

The speed ratio of the differential gear and the fixed ratio mechanism, analogous to Eqs. (3) and (4), can be obtained:

$$R = \frac{r_{\max} r_{\min} (V_{\max} - V_{\min})}{K_b (V_{\max} r_{\min} - V_{\min} r_{\max})} \quad \text{for } dV/dr > 0 \quad (6)$$

$$R = \frac{r_{\max} r_{\min} (V_{\max} - V_{\min})}{K_b (V_{\max} r_{\max} - V_{\min} r_{\min})} \quad \text{for } dV/dr < 0 \quad (7)$$

4. Power Flow Analysis

In general, it can be assumed that friction does not change the direction of power flow in the gear train; this assumption leads to a considerable simplification in analysis. For the input-coupled system as shown in Fig. 1(a), let subscripts $\bar{a}$, b , and v denote the input axis, the output axis, and the CVU of the differential gear, respectively; and $P_{\bar{a}}$ (P_b , P_v), $T_{\bar{a}}$ (T_b , T_v), and $\omega_{\bar{a}}$ (ω_b , ω_v) be the power, the torque, and the angular velocity adjacent to the input axis (output axis, CVU) of the differential gear, respectively. The relation between the external torques acting on the differential gear is:

$$T_{\bar{a}} + T_b + T_v = 0 \quad (8)$$

Furthermore, with no energy losses, the relation between powers is:

$$T_{\bar{a}} \omega_{\bar{a}} + T_b \omega_b + T_v \omega_v = 0 \quad (9)$$

By solving Eqs. (8) and (9), we have:

$$\frac{T_b}{T_{\bar{a}}} = \frac{(\omega_v - \omega_{\bar{a}})}{(\omega_b - \omega_v)} \quad (10)$$

$$\frac{T_v}{T_{\bar{a}}} = \frac{(\omega_b - \omega_{\bar{a}})}{(\omega_v - \omega_b)} \quad (11)$$

and

$$\frac{P_v}{P_{\bar{a}}} = \frac{T_v \omega_v}{T_{\bar{a}} \omega_{\bar{a}}} = \frac{(\omega_b - \omega_{\bar{a}}) \omega_v}{(\omega_v - \omega_b) \omega_{\bar{a}}} \quad (12)$$

Let P_{cv} and P_{in} (P_{out}) be the power carried by the CVU and the input (output) power of the transmission, respectively; and, let P_a and $P_{\bar{a}}$ be the input power and the output power of the fixed ratio mechanism, respectively.

Table 3 Types of power flow of differential transmissions

	Range of relative speed ratio	Types of power flow		
		$R > 1$	$0 < R < 1$	$R < 0$
Input-coupled	$R < \frac{r}{K_a}$		$0 < \frac{P_{cv}}{P_{in}} < 1$	$\frac{P_{cv}}{P_{in}} > 1$
	$\frac{r}{K_a} < R < \frac{2r}{K_a}$	$-1 < \frac{P_{cv}}{P_{in}} < 0$	$-1 < \frac{P_{cv}}{P_{in}} < 0$	
	$R > \frac{r}{K_a}$	$\frac{P_{cv}}{P_{in}} < -1$	$\frac{P_{cv}}{P_{in}} < -1$	
Output-coupled	$\frac{r}{2K_b} < R < \frac{r}{K_b}$		$-1 < \frac{P_{cv}}{P_{in}} < 0$	$\frac{P_{cv}}{P_{in}} > 1$
	$R < \frac{r}{2K_b}$		$\frac{P_{cv}}{P_{in}} < -1$	$\frac{P_{cv}}{P_{in}} > 1$
	$R > \frac{r}{K_b}$	$0 < \frac{P_{cv}}{P_{in}} < 1$	$0 < \frac{P_{cv}}{P_{in}} < 1$	

nism, respectively. Referring to Fig. 1(a), the input power divides into two paths such that $P_{in} = P_a + P_{cv}$. And, if the CVU and the fixed ratio mechanism have no energy losses, $P_a = P_{\bar{a}}$ and $P_{cv} = P_v$. Substituting $P_{\bar{a}}$ and P_v into Eq. (12), we obtain:

$$\frac{P_{cv}}{P_{in} - P_{cv}} = \frac{(\omega_b - \omega_{\bar{a}}) \omega_v}{(\omega_v - \omega_b) \omega_{\bar{a}}} \quad (13)$$

Based on Eqs. (1) and (13), the ratio of the power carried by the CVU to the input power of the transmission can be obtained as:

$$\frac{P_{cv}}{P_{in}} = 1 - \frac{K_a R}{r} \quad (14)$$

Similarly, for the output-coupled system, we have:

$$\frac{P_{cv}}{P_{in}} = 1 - \frac{r}{K_b R} \quad (15)$$

Conventional forms of differential gears can provide a range of values for the relative speed ratio R , and these values can be divided into three characteristic regions $R < 0$, $0 < R < 1$, and $R > 1$. Hence according to Eqs. (14) and (15), the three types of power flow of the differential transmission can be identified; that is, the true split system (i.e. $0 < P_{cv}/P_{in} < 1$), the negative recirculation system (i.e. $P_{cv}/P_{in} < 0$), and the positive recirculation system (i.e. $P_{cv}/P_{in} > 1$) as listed in Table 3.

For the example as shown in Fig. 2, according to Table 3 and the motion requirements for motorcycle applications including (1) during the range of overall speed ratio, the CVU should be operative in the same direction, i.e., $V > 0$, (2) the reverse speed gear is not necessary, i.e., $r_{\min} > 0$, the types of power flow can be identified for every valid range of the relative speed ratio of the input-coupled and output-coupled systems, respectively, Table 4.

5. Efficiency Analysis

The concept of relative power for computing the

Table 4 Types of power flow of dual mode transmissions

Type no.	Input-coupled			Output-coupled		
	Motion requirement	Valid range of relative speed ratio	Type of power flow	Motion requirement	Valid range of relative speed ratio	Type of power flow
1 2	$\frac{dV}{dr} > 0$	$0 < R < \frac{r_{\min}}{K_a}$	$0 < \frac{P_{cv}}{P_{in}} < 1$	$\frac{dV}{dr} < 0$	$0 < R < \frac{r_{\min}}{K_b}$	$\frac{P_{cv}}{P_{in}} < 0$
3 4	$\frac{dV}{dr} < 0$	$R > \frac{r_{\max}}{K_a}$	$\frac{P_{cv}}{P_{in}} < 0$	$\frac{dV}{dr} > 0$	$R > \frac{r_{\max}}{K_b}$	$0 < \frac{P_{cv}}{P_{in}} < 1$
5 6	$\frac{dV}{dr} > 0$	$R < 0$	$\frac{P_{cv}}{P_{in}} > 1$	$\frac{dV}{dr} > 0$	$R < 0$	$\frac{P_{cv}}{P_{in}} > 1$

mechanical efficiency of the differential gear is well known and is used here to derive the efficiency formulas of the differential transmission. Since the input power and the output power are defined as positive and negative, respectively, and the efficiency is a positive value, the mechanical efficiency η , therefore, can be expressed as:

$$\eta = -P_{out}/P_{in} \quad (16)$$

With the input-coupled systems, for the power flow of the true split system, referring to Fig.1(a), and based on Eq.(16), we have:

$$P_{in} = -P_a - P_{cv} \quad (17)$$

$$\eta_v P_{cv} = -P_v \quad (18)$$

$$\eta_{ka} P_a = -P_a \quad (19)$$

$$P_{out} = P_b \quad (20)$$

Here, η_v and η_{ka} are the efficiency of the CVU and the fixed ratio mechanism, respectively. Solving Eqs. (17) – (20), the overall mechanical efficiency of the differential transmission for the true split system can be expressed as:

$$\eta_o = \frac{-P_{out}}{P_{in}} = \frac{-P_b}{\frac{P_a}{\eta_{ka}} + \frac{P_v}{\eta_v}} \quad (21)$$

Similarly, for both the positive and the negative recirculation systems, the overall mechanical efficiency is:

$$\eta_o = \frac{-P_b}{\eta_{ka} P_a + \frac{P_v}{\eta_v}} \quad \text{for positive recirculation system} \quad (22)$$

and

$$\eta_o = \frac{-P_b}{\frac{P_a}{\eta_{ka}} + \eta_v P_v} \quad \text{for negative recirculation system} \quad (23)$$

In the same manner, the overall mechanical efficiency of the differential transmission for the output-coupled systems can be expressed as:

$$\eta_o = \frac{\eta_{kb} P_b + \eta_v P_v}{-P_a} \quad \text{for true split system system} \quad (24)$$

$$\eta_o = \frac{\frac{P_b}{\eta_{kb}} + \eta_v P_v}{-P_a} \quad \text{for positive recirculation system} \quad (25)$$

and

$$\eta_o = \frac{\frac{P_v}{\eta_v} + \eta_{kb} P_b}{-P_a} \quad \text{for negative recirculation system} \quad (26)$$

According to Eqs.(21) – (26), once the power ratios of the differential gear are obtained, the mechanical efficiency of the differential transmissions can be solved.

5.1 Directions of relative power of differential gear

The influence of the power lost in friction of the differential gear depends on the direction of the relative powers. For calculating the mechanical efficiency, the direction of the relative power must be identified. Let $T_i(T_j, T_k)$ and $\omega_i(\omega_j, \omega_k)$ be the torque and angular velocity adjacent to the input (the output, the carrier) of the differential gear. Based on the concept of the relative power and Eq.(16), the mechanical efficiency can be written as:

$$\eta_{ij}^k = \frac{-T_j(\omega_j - \omega_k)}{T_i(\omega_i - \omega_k)} \quad \text{for } T_i(\omega_i - \omega_k) > 0 \quad (27)$$

$$\eta_{ij}^k = \eta_{ji}^k = 1 \quad \text{for } T_i(\omega_i - \omega_k) = 0 \quad (28)$$

$$\eta_{ij}^k = \frac{-T_i(\omega_i - \omega_k)}{T_j(\omega_j - \omega_k)} \quad \text{for } T_i(\omega_i - \omega_k) < 0 \quad (29)$$

Here, $\eta_{ij}^k(\eta_{ji}^k)$ is the efficiency for the power flowing from the input to the output (the output to the input) with the carrier fixed.

Referring to Fig.1(a), for the input-coupled systems, the relative power of the differential gear is expressed as $T_a(\omega_a - \omega_b)$; then substituting $r = \omega_b/\omega_a$ and $K_a = \omega_a/\omega_b$ into above equation, we have:

$$T_a(\omega_a - \omega_b) = T_a \omega_a \left(1 - \frac{r}{K_a}\right) \quad (30)$$

Similarly, the directions of the relative power of the differential gear for the output-coupled systems can be expressed as:

$$T_b(\omega_a - \omega_b) = T_a \omega_a \left(1 - \frac{r}{K_b}\right) \quad (31)$$

Hence according to Eqs.(30) and (31), the directions of the relative power of the differential gear during the full transmission speed ratio range can be identified. For the example demonstrated in Fig. 2, let $T_s(T_r, T_c)$ and $\omega_s(\omega_r, \omega_c)$ be the torque and angular velocity adjacent to the sun gear (the ring gear, the carrier) of the simple differential gear. With type 1 of the input-coupled system, the carrier of the simple differential gear is adjacent to the output axis, i.e., $P_c = T_c \omega_c = T_b \omega_b < 0$; and, the sun gear (the ring gear) is adjacent to the input axis (the CVU). Since ω_c and ω_s rotate with the same direction and T_c and T_s have opposite algebraic signs, hence, $T_a \omega_a = T_s \omega_s > 0$. Thus based on Eq.(30), the directions of the relative power of the simple differential gear during the full transmission speed ratio range can be

Table 5 Directions of relative power of simple differential gear

Type no.	Input-coupled			Output-coupled		
	$r < K_a$	$r > K_a$	$r = K_a$	$r < K_b$	$r > K_b$	$r = K_b$
1	@	#	&	@	#	&
2	#	@	&	#	@	&
3	#	@	&	#	@	&
4	@	#	&	@	#	&
5	#	@	&	@	#	&
6	@	#	&	#	@	&

@: $T_s(\omega_s - \omega_c) > 0$ &: $T_s(\omega_s - \omega_c) = 0$

#: $T_s(\omega_s - \omega_c) < 0$

identified as:

$$T_s(\omega_s - \omega_c) > 0 \quad \text{for } r < K_a \quad (32)$$

$$T_s(\omega_s - \omega_c) = 0 \quad \text{for } r = K_a \quad (33)$$

$$T_s(\omega_s - \omega_c) < 0 \quad \text{for } r > K_a \quad (34)$$

From Eq.(34), similarly, the directions of the relative power of the simple differential gear, for the output-coupled system, can also be identified. The results for different arrangements are summarized in Table 5.

5.2 Torque ratios in differential gear

Once the directions of the relative power of the differential gear is known, through Eqs.(27) - (29) and Eqs.(30) and (31), the torque ratios of the differential gear can be determined.

In particular, for type 1 of the input-coupled system as shown in Fig. 2 (a), the planet carrier of the simple differential gear is adjacent to the output axis; while the sun gear and the ring gear are adjacent to the input axis and CVU, respectively. According to Eqs.(27) and (32), for $r < K_a$ (i.e., $T_a(\omega_a - \omega_b) = T_s(\omega_s - \omega_c) > 0$), the relation of the relative power of the simple differential gear can be rewritten as:

$$\eta_{sr}^c = \frac{-T_r(\omega_r - \omega_c)}{T_s(\omega_s - \omega_c)} = \frac{-T_v(\omega_v - \omega_b)}{T_a(\omega_a - \omega_b)} \quad (35)$$

Based on Eqs.(1) and (35), the torque ratio in the simple differential gear can be expressed as:

$$\frac{T_v}{T_a} = \frac{-\eta_{sr}^c(\omega_a - \omega_b)}{(\omega_v - \omega_b)} = \frac{\eta_{sr}^c(1-R)}{R} \quad (36)$$

From Eq.(8), the torque equilibrium requirement, and Eq.(36), we have:

$$\frac{T_a}{T_b} = \frac{R}{\eta_{sr}^c(R-1)-R} \quad (37)$$

and

$$\frac{T_v}{T_b} = \frac{\eta_{sr}^c(1-R)}{\eta_{sr}^c(R-1)-R} \quad (38)$$

For $r > K_a$ (i.e., $T_s(\omega_s - \omega_c) < 0$), based on Eq.(29), the relation of the relative power of the simple differential gear is:

$$\eta_{rs}^c = \frac{-T_s(\omega_s - \omega_c)}{T_r(\omega_r - \omega_c)} = \frac{-T_a(\omega_a - \omega_b)}{T_v(\omega_v - \omega_b)}$$

Table 6 Torque ratios in simple differential gear with $T_s(\omega_s - \omega_c) > 0$

Type no.	Torque ratios for input-coupled (output-coupled)		
	$\frac{T_a}{T_b}(\frac{T_a}{T_b})$	$\frac{T_v}{T_a}(\frac{T_v}{T_a})$	$\frac{T_v}{T_b}(\frac{T_v}{T_b})$
1	$\frac{R}{\eta_{sr}^c(R-1)-R}$	$\frac{\eta_{sr}^c(1-R)}{R}$	$\frac{\eta_{sr}^c(1-R)}{\eta_{sr}^c(R-1)-R}$
2	$\frac{\eta_{sr}^c R}{(R-1)-\eta_{sr}^c R}$	$\frac{1-R}{\eta_{sr}^c R}$	$\frac{1-R}{(R-1)-\eta_{sr}^c R}$
3	$\frac{(1-R)-\eta_{sr}^c}{\eta_{sr}^c}$	$\frac{R-1}{(1-R)-\eta_{sr}^c}$	$\frac{R-1}{\eta_{sr}^c}$
4	$\eta_{sr}^c(1-R)-1$	$\frac{\eta_{sr}^c(R-1)}{\eta_{sr}^c(1-R)-1}$	$\eta_{sr}^c(R-1)$
5	$\frac{-R}{\eta_{sr}^c}$	$\frac{\eta_{sr}^c - R}{R}$	$\frac{R - \eta_{sr}^c}{\eta_{sr}^c}$
6	$-\eta_{sr}^c R$	$\frac{1 - \eta_{sr}^c R}{\eta_{sr}^c R}$	$\eta_{sr}^c R - 1$

Hence, by substituting $1/\eta_{rs}^c$ into Eqs.(36) - (38) to replace η_{sr}^c , we obtain:

$$\frac{T_v}{T_a} = \frac{1-R}{\eta_{rs}^c R} \quad (39)$$

$$\frac{T_a}{T_b} = \frac{\eta_{rs}^c R}{(R-1)-\eta_{rs}^c R} \quad (40)$$

and

$$\frac{T_v}{T_b} = \frac{1-R}{(R-1)-\eta_{rs}^c R} \quad (41)$$

In the same manner, for $T_s(\omega_s - \omega_c) > 0$, the expressions for calculating the torque ratios in the simple differential gear, for all six configurations, are summarized in Table 6. Furthermore, when $T_s(\omega_s - \omega_c) < 0$, the efficiency η_{sr}^c in Table 6 can be replaced by $1/\eta_{rs}^c$ to obtain the torque ratios.

5.3 Equations of mechanical efficiency

Once the directions of the power flow in the differential gear is known, a second step is the determination of the torque ratios in the differential gear. Finally, if the torque ratios in the differential gear have been carried out, through Eqs.(21) - (26), the mechanical efficiency of the differential transmission can be obtained.

For instance, referring to Table 4, the type 1 of the input-coupled system as shown in Fig. 2 (a) is a true power split system. Hence Eq.(21) can be rewritten as:

$$\eta_o = \frac{-1}{\frac{1}{\eta_{ka}} \frac{T_a \omega_a}{T_b \omega_b} + \frac{1}{\eta_v} \frac{T_v \omega_v}{T_b \omega_b}} \quad (42)$$

For $r < K_a$ (i.e., $T_s(\omega_s - \omega_c) > 0$), substituting Eqs.(37), (38), and (1) into Eq.(42), we have:

Table 7 Efficiency of differential transmission for input-coupled systems

Type no.	Equations of mechanical efficiency	
	$r < K_a$	$r > K_a$
1	$\frac{\eta_v \eta_{ka} r (R + \eta_{sr}^c (1-R))}{K_a \eta_v R + \eta_{sr}^c \eta_{ka} (r - K_a R)}$	$\frac{\eta_v \eta_{ka} r (\eta_{rs}^c R + (1-R))}{K_a \eta_v \eta_{rs}^c R + \eta_{ka} (r - K_a R)}$
2	$\frac{\eta_v \eta_{ka} r (\eta_{rs}^c (1-R) + R)}{\eta_v K_a R + \eta_{ka} \eta_{rs}^c (r - K_a R)}$	$\frac{\eta_v \eta_{ka} r (\eta_{rs}^c R + (1-R))}{\eta_v \eta_{sr}^c K_a R + \eta_{ka} (r - K_a R)}$
3	$\frac{\eta_{ka} r}{K_a (\eta_{rs}^c (R-1) + 1) + \eta_v \eta_{ka} \eta_{rs}^c (r - K_a R)}$	$\frac{\eta_{ka} \eta_{sr}^c r}{K_a (\eta_{rs}^c + (R-1)) + \eta_v \eta_{ka} (r - K_a R)}$
4	$\frac{r \eta_{ka}}{K_a (\eta_{rs}^c (R-1) + 1) + \eta_v \eta_{ka} \eta_{rs}^c (r - K_a R)}$	$\frac{\eta_{ka} \eta_{rs}^c r}{K_a (\eta_{rs}^c + (R-1)) + \eta_v \eta_{ka} (r - K_a R)}$
5	$\frac{\eta_v r (1-R)}{K_a \eta_v \eta_{ka} \eta_{rs}^c R (1-R) + (1-\eta_{rs}^c) (r - K_a R)}$	$\frac{\eta_v \eta_{sr}^c r (1-R)}{K_a \eta_v \eta_{ka} R (1-R) + (\eta_{rs}^c - R) (r - K_a R)}$
6	$\frac{\eta_v r (1-R)}{\eta_v \eta_{ka} \eta_{sr}^c K_a R (1-R) + (1-\eta_{rs}^c) (r - K_a R)}$	$\frac{\eta_v \eta_{rs}^c r (1-R)}{\eta_v \eta_{ka} K_a R (1-R) + (\eta_{rs}^c - R) (r - K_a R)}$

Table 8 Efficiency of differential transmission for output-coupled systems

Type no.	Equations of mechanical efficiency	
	$r < K_b$	$r > K_b$
1	$\frac{\eta_v \eta_{kb} r [\eta_{sr}^c (1-R) + R] + \eta_{sr}^c (K_b R - r)}{\eta_v K_b R}$	$\frac{\eta_v \eta_{kb} r [(1-R) + \eta_{rs}^c R] + (K_b R - r)}{\eta_v \eta_{rs}^c K_b R}$
2	$\frac{\eta_v \eta_{kb} r [\eta_{rs}^c (1-R) + R] + \eta_{rs}^c (K_b R - r)}{\eta_v K_b R}$	$\frac{\eta_v \eta_{kb} r [(1-R) + \eta_{sr}^c R] + (K_b R - r)}{\eta_v \eta_{sr}^c K_b R}$
3	$\frac{\eta_{kb} r + \eta_v \eta_{rs}^c (K_b R - r)}{K_b [\eta_{rs}^c (R-1) + 1]}$	$\frac{\eta_{kb} \eta_{sr}^c r + \eta_v (K_b R - r)}{K_b [(R-1) + \eta_{rs}^c]}$
4	$\frac{\eta_{kb} r + \eta_v \eta_{sr}^c (K_b R - r)}{K_b [\eta_{sr}^c (R-1) + 1]}$	$\frac{\eta_{kb} \eta_{rs}^c r + \eta_v (K_b R - r)}{K_b [(R-1) + \eta_{rs}^c]}$
5	$\frac{\eta_{sr}^c r (R-1) + \eta_v \eta_{kb} (R - \eta_{rs}^c) (K_b R - r)}{\eta_{kb} K_b R (R-1)}$	$\frac{r (R-1) + \eta_v \eta_{kb} (\eta_{rs}^c R - 1) (K_b R - r)}{\eta_{kb} \eta_{rs}^c K_b R (R-1)}$
6	$\frac{\eta_{rs}^c r (R-1) + \eta_v \eta_{kb} (R - \eta_{sr}^c) (K_b R - r)}{\eta_{kb} K_b R (R-1)}$	$\frac{r (R-1) + \eta_v \eta_{kb} (\eta_{sr}^c R - 1) (K_b R - r)}{\eta_{kb} \eta_{sr}^c K_b R (R-1)}$

$$\eta_o = \frac{\eta_v \eta_{ka} r (R + \eta_{sr}^c (1-R))}{\eta_v K_a R + \eta_{sr}^c \eta_{ka} (r - K_a R)} \quad (43)$$

Similarly, for $r > K_a$ (i.e., $T_s(\omega_s - \omega_c) < 0$), substituting Eqs.(40), (41), and (1) into Eq.(42), we obtain:

$$\eta_o = \frac{\eta_v \eta_{ka} r (\eta_{rs}^c R + (1-R))}{K_a \eta_v \eta_{rs}^c R + \eta_{ka} (r - K_a R)} \quad (44)$$

Therefore, based on Tables 5 and 6, the directions of the power flow and the torque ratios of the simple differential gear can be obtained. Then according to Eqs.(21) – (23), the mechanical efficiency of the input-coupled differential transmission can be derived and summarized as in Table 7 with $r < K_a$, and $r > K_a$, respectively. When $r = K_a$, substituting $\eta_{sr}^c = \eta_{rs}^c = 1$ into the equation of each corresponding type gives the mechanical efficiency. Similarly, the mechanical efficiency of the output-coupled system is listed in Table 8. Figure 3 shows the mechanical efficiency of the differential transmission with $R_o = -3$. In this case, the efficiency of the CVU, the fixed ratio mechanism, and the differential gear are each considered as $\eta_v = 0.6$, $\eta_{ka} = \eta_{kb} = 0.95$, and $\eta_{sr}^c = \eta_{rs}^c = 0.95$, respectively, and $K_a = K_b = 1$.

(a) Input-coupled

(b) Output-coupled

Fig. 3 Efficiency of differential transmissions

5.4 Maximum mechanical efficiency

A further distinction between transmissions with different power flow modes is concerned with the point at which efficiency tends to be a maximum. Since power is transmitted through the differential gear more efficiently than the CVU, it is obvious that when the portion of the input power passing through the CVU tends to be a minimum the mechanical efficiency of the transmission tends to be a maximum. According to the efficiency analysis, the true split system and a portion range of the negative recirculation system are preferred.

For the example as shown in Fig. 2, with reference to Table 4 and Fig. 3, types 1, 2, 3, and 4 of the input-coupled and output-coupled systems, where the efficiency of the differential transmission is greater than the CVU, are appropriate for dual mode transmission systems. Now with the input-coupled systems, the ratio of the power carried by the CVU to the input power of the transmission is:

$$\frac{P_{cv}}{P_{in}} = 1 - \frac{K_a R}{r} \quad (14)$$

Table 9 Speed ratios for peak efficiency

Type no.	Relative speed ratio	Basic speed ratio	Speed ratio of fixed ratio mechanism
1	$\frac{1}{1-(R_o)_{\max}}$	$(R_o)_{\max}$	$(K_a)_{\max} ((K_b)_{\max})$
2	$\frac{(R_o)_{\min}}{(R_o)_{\min}-1}$	$(R_o)_{\min}$	$(K_a)_{\max} ((K_b)_{\max})$
3	$\frac{(R_o)_{\min}-1}{(R_o)_{\min}}$	$(R_o)_{\min}$	$(K_a)_{\min} ((K_b)_{\min})$
4	$1-(R_o)_{\max}$	$(R_o)_{\max}$	$(K_a)_{\min} ((K_b)_{\min})$

For the true split system, (i.e., types 1 and 2), from Table 4, the valid range of the relative speed ratio is $0 < R < r_{\min}/K_a$. Hence, based on Eq.(14), since $K_a R < r$ and $0 < R < 1$, if R tends to be a maximum, P_{cv}/P_{in} tends to be a minimum and the mechanical efficiency of the differential transmission tends to be a maximum. Similarly, for the negative recirculation system (i.e., types 3 and 4), the valid range of the relative speed ratio is $R > r_{\max}/K_a$. Since $K_a R > r$ and $R > 1$, if R tends to be a minimum, P_{cv}/P_{in} tends to be a minimum, and the mechanical efficiency of the differential transmission tends to be a maximum. In the same manner, based on Table 4 and Eq.(15), the maximum mechanical efficiency of the output-coupled system can be identified and listed as in Table 9.

6. Acceptable Design Concepts

After studying the analyses of power flow and mechanical efficiency, it becomes clear that the true split system and a portion range of the negative recirculation system of the differential transmission are suitable for the dual mode transmissions. If a mechanical-type CVT is adopted here as the CVU of the differential transmission, the differential transmission speed ratio is increased as the speed ratio of the CVU is increased, i.e., $dV/dr > 0$. And, as seen in Table 4, only the relative speed ratio $0 < R < r_{\min}/K_a$ of the input-coupled system and the relative speed ratio $R > r_{\max}/K_b$ of the output-coupled system corresponding to the type of the true split power flow can be used for motorcycle applications. Hence for the example as shown in Fig. 2, type 1 and type 2 of the input-coupled system and type 3 and type 4 of the output-coupled system can be used as the differential transmission and CVT system. The acceptable design concepts are rearranged as shown in Fig. 4. For type 1 of the differential transmission and CVT system, in start-up and low speed situations, one-way clutch C_1 is engaged and clutches C_2 is released. The input power is transmitted to the chain drive assembly while the power flow is transferred from the V-belt

Fig. 4 Differential transmission and CVT systems

Fig. 5 Fixed gear ratio and differential transmission systems

drive to the planetary gear train to act as a differential transmission mode. As vehicle speed increases to and passes a medium speed, clutches C_2 is engaged and one-way clutch C_1 is released; then the transmission system operates in a CVT mode with the planetary gear train locked at a 1:1 ratio.

For the fixed gear ratio and differential transmission system, since the motion requirement of the fixed gear ratio regime must be $0 < R < 1$, only type 1 and type 2 of the input-coupled system can be used. The acceptable design concepts are rearranged as shown in Fig. 5. As one-way clutch C_1 is engaged and clutch C_2 is released, the transmission first operates in a fixed gear ratio mode. Then, as clutch C_2 is activated while one-way clutch C_1 is released, the transmission operates in a differential transmission mode.

7. Conclusion

Two types of the dual mode transmission presented in this paper can be used in a motorcycle. One is the fixed gear ratio and differential transmission system; the other is the differential and continuously variable transmission system. The dual mode transmission, consisting a mechanical-type V-belt drive, a chain drive, and a planetary gear train, can provide two operation regimes over the full speed ratio range by means engaged clutches. The kinematics, power flow, and mechanical efficiency of each transmission system are analyzed. Three types of power flow are possible for the differential transmissions. Only the true split system and a portion range of the negative recirculation system of the differential transmission are suitable for the dual mode transmissions. Furthermore, in this paper the available design concepts of the dual mode transmissions for motorcycle applications, four differential and continuously variable transmission systems and two fixed gear ratio and differential transmission systems, are proposed.

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