Analysis of Spherical Form Errors to Coordinate Measuring Machine Data*

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Coordinates measuring machines (CMMs) are commonly utilized to take measurement data from manufactured surfaces for inspection purposes. The measurement data are then used to evaluate the geometric form errors associated with the surface. Traditionally, the evaluation of spherical form errors involves an optimization process of fitting a substitute sphere to the sampled points. This paper proposes the computational strategies for sphericity with respect to ASME Y14.5M-1994 standard. The proposed methods consider the trade-off between the accuracy of sphericity and the efficiency of inspection. Two approaches of computational metrology based on genetic algorithms (GAs) are proposed to explore the optimality of sphericity measurements and the sphericity feasibility analysis, respectively. The proposed algorithms are verified by using several CMM data sets. Observing from the computational results, the proposed algorithms are practical for on-line implementation to the sphericity evaluation. Using the GA-based computational techniques, the accuracy of sphericity assessment and the efficiency of sphericity feasibility analysis are agreeable.

Key Words: Form Error Assessment, Sphericity, Coordinate Measuring Machines, Genetic Algorithms

1. Introduction

Since manufacturers have realized the utilities and economic advantages of geometric dimensioning and tolerancing (GD & T), coordinate measuring machines (CMMs) are widely used in manufacturing to verify and control the dimensional accuracy of the manufactured parts. Through an assortment of mechanisms, CMMs collect the measurement data sets, and the form errors are then estimated. The precision of a CMM in measurement depends not only on its design and construction, but also the evaluation algorithms used to deal with the sampled data.

To produce better quality parts, manufacturers can adequately control the production process and accurately evaluate the form errors. The assessment algorithms for form errors are typically applied to meet the intent of the ASME Y14.5M standard⁽¹⁾. This standard identifies that a substitute feature (geometric form that best fits the measured points) must be established from the sampled data to minimize the maximum deviation between the substitute feature

and the actual feature concerned.

Sphericity is a type of form error, which is broadly used in the industry for the geometrical measurement of precision balls. The sphericity is defined as the minimum shell width (minimum zone) between the concentric circumscribed and inscribed spheres enclosing all the measurement data. The sphericity assessment involves the optimization step to find the center coordinates and radii of the substitute spheres. The evaluation of sphericity based on the minimum zone condition is a non-linear and non-convex problem that is difficult to solve mathematically. Therefore, the efficient and accurate assessment of sphericity is an important issue.

CMMs have emerged to be important inspection tools owing to the recent advancements in numerical control and precision machining. However, CMMs are slowed down by three difficult problems⁽²⁾:

1. Develop suitable measurement techniques to obtain the data points, which accurately represent the parts being inspected.
2. Correctly and unambiguously interpret the definition of tolerances given in ASME Y14.5M standard, and formulate the problems precisely and systematically as optimization problems.
3. Develop assessment algorithms which are con-

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sistent with Y14.5M standard, highly efficient, robust, and easy to use.

The measurement data sampled from CMMs are treated by an evaluation algorithm to accurately estimate the form errors. There exists no precise mathematical definition of the constraints implied by tolerance specifications in the ASME Y14.5M standard. Tolerance is not interpreted in agreement by different CMM systems, which leads to inconsistency between the requirements of tolerances and the results of CMM software. Current CMM verification algorithms are based on the least-squares solution, which minimizes the sum of the squared errors of the measured points from the substitute sphere. The least-squares solution is only capable of obtaining an approximate solution that does not guarantee to meet the minimum zone criterion. Therefore, the least-squares solution can result in a possible overestimation of the form error. While CMM algorithms successfully reject bad parts, they may also reject some good parts⁽³⁾. An approximate method for calculating the least-squares center and radius is given in ASME⁽⁴⁾. Chen and Papadoupoulos⁽⁵⁾ compared the linear and non-linear least-squares fitting algorithm and concluded that there is no practical difference between the two algorithms. The method of least-squares is not necessarily optimal if the irregularity significantly departs from a Gaussian distribution⁽⁶⁾.

With increasing accuracy of CMMs, the minimum zone criterion for determining a substitute element becomes more appropriate⁽³⁾. Gass et al.⁽³⁾ developed a linear programming approach for determining reference spheres and minimum zone sphericity based on CMM measurements of spherical parts. Kanada⁽⁷⁾ modeled the minimum zone sphere (MZS) as non-linear programs and applied simplex search to estimate the sphericity. Kanada compared the minimum zone method in which simplex search is applied to the iterative least-squares method. Observing from the results in Kanada, the minimum zone approach is comparably superior to the least-squares one. Fan and Lee⁽⁸⁾ evaluated the minimum zone sphericity based on a direct search method starting from the least-squares results. Using the 3D Voronoi diagram, Huang⁽⁹⁾ solved the minimum zone solution for sphericity problems. Chen and Liu⁽¹⁰⁾ evaluated sphericity errors by directly resolving the simultaneous linear algebraic equations.

Sphericity plays an important role in industry, and its design standard can be found in ASME⁽⁴⁾. The evaluation of sphericity based on the minimum zone criterion is a non-linear and non-convex problem that is hard to handle in mathematics⁽⁹⁾. Assessment algorithms apply linear approximation approaches can

give incorrect results to non-linear form-fitting problems⁽¹¹⁾. The objective of this paper is developing mathematical models in non-linear terms and assessment algorithms for sphericity analysis. By building the mathematical models for spherical form errors, genetic algorithms (GAs), which are robust non-linear optimization methods, can be introduced to assess sphericity. GAs are comparably simple; they do not require differential information for the objective function, and they have good convergence features.

The remainder of this paper is organized as follows. Section 2 introduces the background information for sphericity errors. Section 3 constructs the mathematical models for the sphericity measurement and sphericity analysis, respectively. Section 4 presents two GA-based inspection strategies for the sphericity measurement and sphericity feasibility analysis, respectively. In Section 5, measurement data of CMMs are used to demonstrate the viability of the proposed sphericity inspection strategies. Finally, conclusions are made in Section 6.

2. Background Information

The sphericity error is determined by selecting a center point for the reference sphere, with the value being difference between the largest and smallest radii to the measured points⁽⁴⁾. To allow for presumed differences in function, four criteria for the substitute sphere fitting are internationally defined⁽⁴⁾. They are: (1) least-squares sphere (LSS); (2) maximum inscribing sphere (MIS); (3) minimum circumscribing sphere (MCS); and (4) minimum zone sphere (MZS).

A LSS is the sphere chosen so that the sum of the squares of the radial distance of all data points from the fitting sphere is a minimum. Given a set of data points in 3-D, $P = \{p_i(x_i, y_i, z_i), i=1, 2, \dots, N\}$ which represent the surface of a workpiece, it is possible to find the sphere parameters by minimizing the least-squares errors between the given set of data points and the surface. The normal deviation (e_i) between a data point (x_i, y_i, z_i) and the sphere of radius R and center (x_c, y_c, z_c) is given by

$$e_i = [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} - R \quad (1.)$$

A shell is formed by the minimum inscribed sphere and the maximum circumscribed sphere having the least-squares center. Just about all CMMs compute the sphericity from LSS⁽³⁾. However, the deviations obtained using LSS may be large.

Spherical form fitting can be viewed as an optimization problem: find the parameters of reference sphere that optimize a particular fitting objective for a set of points. Form fitting algorithms are used to analyze the measurement data to create the reference

geometry of the feature being measured. Consequently, the fitting parameters are compared to the design intent to determine conformance.

For sphericity assessments, there exists a distinction between the specification of tolerances in design and the assessment methods in metrology community. By using the normal deviation defined in Eq. (1), the fitting methods for estimating substitute sphere of MIS, MCS and MZS involve nonlinear constraints. Provided that numerous measured points on the part surfaces are obtained, the fitting optimization models for MIS, MCS and MZS are non-linear programs with a huge set of constraints. These mathematical models hence have a high degree of computational complexity.

The operation cost in precision manufacturing is generally high, and engineers thus intend to design tolerance sizes as large as possible. The acceptance/rejection decision is then made by checking the conformance of measured features and design specifications. Traditionally, acceptance/rejection decision is usually made by initially finding an optimal reference. The optimal reference and the associated feature deviation are then tested against the design specification⁽¹²⁾. Through reformulating the geometric conditions and adding further constraints to the sphericity measurement problems, the acceptance/rejection decision of sphericity inspection can be made by only investigating the feasibility of the constrained non-linear programs, namely sphericity feasibility analysis, instead of advancing to the optimality. Therefore, it is not necessary to find the reference geometry for an sphericity feasibility analysis.

The form fitting algorithms that convert measurement data to the reference geometry can contribute significantly to the measurement in the coordinate metrology. CMMs require sizeable capital investment, both for the tools themselves and for the infrastructure needed to keep up proper environmental conditions for operation. To produce convincing measurement results within reasonable time, it is necessary to optimize the throughput of CMMs by developing an effective assessment algorithm that will cater the needs of sphericity assessments. This research concentrates upon the issue of developing the robust form-fitting algorithms for sphericity assessments to reduce computational errors. In this paper, the fitting optimization models of MIS, MCS and MZS for the sphericity measurement and sphericity feasibility analysis are formulated as constrained non-linear programs. In order to solve these complicated non-linear programs, two methods of computational metrology based on genetic algorithms (GAs) are proposed to explore the optimality of sphericity

measurement and the feasibility of sphericity inspection, respectively. Genetic algorithms (GAs) are heuristic search techniques, which are full of promise to deal with complex optimization problems^{(13),(14)}. GAs can solve optimization problems possessing arbitrary degrees of non-linearity, discontinuity and stochasticity.

3. Mathematical Models for Sphericity

To evaluate the sphericity error, a substitute sphere has to be established from the collected data. The mathematical models of MIS, MCS and MZS for sphericity assessments are formulated in the following subsections.

3.1 Sphericity measurements

3.1.1 Maximum inscribed sphere The MIS attempts to find a sphere which is the largest sphere being inscribed within the set of measured points, $\mathbf{P} = \{p_i(x_i, y_i, z_i), i=1, 2, \dots, N\}$. The center coordinates (x_c, y_c, z_c) and radius R are then can be estimated. Using the center coordinates and radius of MIS, this approach can find the minimum shell width, which all the measured points are exterior to this sphere. Common practice seeks to maximize the reference radius R while maintaining the condition that the profile is disclose, that is, subject to the geometrical constraints. The constrained non-linear program of MIS takes the form as

$$\begin{aligned} & \text{Maximize } R \\ & \text{Subject to:} \\ & [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} \geq R \\ & i=1, 2, \dots, N \\ & R_L \leq R \leq R_U; \quad x_L \leq x_c \leq x_U; \quad y_L \leq y_c \leq y_U; \\ & z_L \leq z_c \leq z_U \end{aligned} \quad (2)$$

where R_L, x_L, y_L and z_L are the lower bounds of R, x_c, y_c and z_c ; R_U, x_U, y_U and z_U are the upper bounds.

The optimization algorithm resolves the MIS model to find the reference sphere. The sphericity error for MIS, e_{MIS} , is then estimated by measuring the maximal peak-to-valley deviation between the measurement data and the reference sphere. It can be expressed as

$$e_{\text{MIS}} = \max_{i=1,2,\dots,N} \{e_i = [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} - R\} \quad (3)$$

3.1.2 Minimum circumscribed sphere The MCS attempts to find a sphere, which circumscribes the given set of measured points, using a smallest sphere. Using the center coordinates and radius of MCS, we can obtain the minimum shell width, which encloses all the measured points. Mathematically, the MCS model tries to minimize the reference radius R while maintaining the condition that the profile is enclosed. The mathematical programming model of

MCS can be formulated as

Minimize R

Subject to :

$$\begin{aligned} [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} &\leq R \\ i &= 1, 2, \dots, N \\ R_L &\leq R \leq R_U; x_L \leq x_c \leq x_U; y_L \leq y_c \leq y_U; \\ z_L &\leq z_c \leq z_U \end{aligned} \quad (4)$$

Similarly, the sphericity error for MCS, e_{MCS} , can be defined as

$$e_{MCS} = \max_{i=1,2,\dots,N} \{e_i = R - [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2}\} \quad (5)$$

3.1.3 Minimum zone sphere The MZS method is proposed to comply with the ASME tolerancing definition⁽¹⁾ for form tolerances. The MZS defines a pair of concentric spheres, a circumscribed sphere S_1 and an inscribed sphere S_2 that just contains the measured points with a minimum shell width. The mathematical problem of finding the minimum zone can be formulated as a constrained non-linear program. This is a rather complicated non-linear problem, and exact algorithms for solving it are not readily available⁽³⁾. The center of this pair of concentric spheres is positioned at a point (x_c, y_c, z_c) . The mathematical model can be formulated as

Minimize $R_1 - R_2$

Subject to :

$$\begin{aligned} [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} &\leq R_1; \\ i &= 1, 2, \dots, N \\ [(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} &\geq R_2, \\ i &= 1, 2, \dots, N \\ R_1 - R_2 &\geq 0; R_L \leq R_1, R_2 \leq R_U; \\ x_L &\leq x_c \leq x_U; y_L \leq y_c \leq y_U; z_L \leq z_c \leq z_U \end{aligned} \quad (6)$$

where R_1 and R_2 are the radii of S_1 and S_2 , respectively.

In the case of measurement, the shell zone needs to be minimal. The sphericity error for MZS, e_{MZS} , is the minimum shell zone. It can be simply expressed as

$$e_{MZS} = R_1 - R_2 \quad (7)$$

After the sphericity error has been obtained, one can check the conformance of measured features and design specifications, and then make the acceptance/rejection decision for the inspected part. If the sphericity error is within the tolerance, the acceptance decision is made. Otherwise, the sphericity error is larger than the specification, and the rejection decision is made.

3.2 Sphericity feasibility analysis

Provided that the specified sphericity tolerance is known *a priori*, one can try a pair of concentric spheres in which the shell zone between the circumscribed sphere and inscribed sphere is equal to or less than the specified tolerance. The sphericity from the measured surface is within the specified tolerance if

such a pair of concentric spheres is found. This inspection strategy can be achieved through reformulating the geometric conditions and adding further constraints to the sphericity measurement models presented in Subsection 3.1. The acceptance/rejection decision of this sphericity inspection strategy (sphericity feasibility analysis) can be made by only investigating the feasibility of the constrained non-linear programs instead of advancing to the optimality. If there is at least one feasible solution (a pair of concentric spheres enclosing the measured profile is found), the sphericity error is within the tolerance, and the acceptance decision is made. Otherwise, the sphericity error is larger than the specification (not any pair of concentric spheres enclosing the measured profile can be found), and the rejection decision is made. The modified mathematical models for sphericity feasibility analysis are described as follows.

3.2.1 Feasibility analysis for MIS Analyzing the sphericity feasibility for MIS reference, the design specification for sphericity tolerance actually imposes another set of constraints. The deviation of any measured point p_i from the maximum inscribed sphere must be less than the design tolerance t_z . This can be written as

$$[(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} \leq R + t_z, \quad i = 1, 2, \dots, N \quad (8)$$

In this case, the constraints defined in Eqs. (2) and (8) must be simultaneously satisfied, but the radius need not be maximal. The association of this constraint set expressed in Eq. (8) may or may not define a region of feasible solutions. The feasible solution, if any, provides an acceptable solution to the inspection problem because it implies that there are two concentric spheres centered at (x_c, y_c, z_c) and with radii R and $R + t_z$, enclosing the measured points. It is not necessary to optimize the fitting parameters for the mathematical model of sphericity feasibility analysis. If there is at least one feasible solution, the sphericity error is within the tolerance, and the acceptance decision is made (workpiece passes the inspection). Otherwise, the sphericity error is larger than the specification, and the rejection decision is made (workpiece fails the inspection).

3.2.2 Feasibility analysis for MCS For the sphericity feasibility analysis of MCS, the specified sphericity tolerance imposes another set of constraints: the deviation of any measured point p_i from the minimum circumscribed sphere must be less than the design tolerance t_z . This can be written as

$$[(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2]^{1/2} \geq R - t_z, \quad i = 1, 2, \dots, N \quad (9)$$

Provided that a pair of concentric spheres centered at

(x_c, y_c, z_c) and with radii R and $R - t_z$ which enclose the measured points can be found, it indicates that there is at least one feasible solution. It implies that the sphericity error is within the tolerance (acceptance decision). Otherwise, the sphericity error is larger than the tolerance size (rejection decision). In this situation, the constraints expressed in Eqs. (4) and (9) must be satisfied, but the fitting parameters need not be optimal.

3.2.3 Feasibility analysis for MZS Since the MZS reference model is to find two spheres centered at (x_c, y_c, z_c) and with radii R_1 and R_2 , enclosing the measured points, it only requires appending a tolerance related constraint to divide the feasible and infeasible regions for the sphericity feasibility analysis. The additionally imposed constraint for the tolerance specified in design t_z can be expressed as

$$R_1 - R_2 \leq t_z \quad (10)$$

In the case of sphericity feasibility analysis for MZS, the constraints expressed in Eqs. (6) and (10) must be satisfied, but the shell width need not be minimal. Any feasible solution provides an acceptable solution to the inspection problem of MZS because it implies that there are two spheres centered at (x_c, y_c, z_c) and with radii R_1 and R_2 , enclosing the measurement data, and the sphericity error (shell width) is less than the tolerance size (acceptance decision). Otherwise, the sphericity error is larger than the specification, and the workpiece fails the inspection (rejection decision).

4. The Sphericity Assessment Algorithms

The mathematical models of sphericity assessments formulated in Section 3 are constrained non-linear programs with a large set of constraints. Two assessment algorithms based on genetic approach for the sphericity measurement and sphericity feasibility analysis are proposed herein. The genetic approaches are motivating here because they are effective and operate without explicit information of concept boundaries.

4.1 Genetic algorithms

Genetic algorithms (GAs) have several distinctive features borrowed from the abstract information of natural selection and biological evolutionary process. They were firstly introduced by Holland⁽¹⁵⁾ in the 1970s. GAs combine the notion of survival of the fittest, random and yet structured search, and parallel evaluation of points in the solution space. In such situation, relatively high fitness structures have a larger chance to survive and to produce higher fitness offspring. This evolutionary process can be applied to the real-world problems where heuristic solutions generally lead to unsatisfactory results. GAs have been used successfully for dealing with various optim-

ization problems. For a more applied orientation and applications of the GA to various problems, refer Goldberg⁽¹³⁾, Gen and Cheng⁽¹⁶⁾ and other references cited in these two books.

GAs consist of a string representation (chromosome) of solutions in the search space. Within the evolutionary generation to optimal solutions, chromosomes compete to survive, and genetic operators are utilized to alter the population. Simulating this natural process of GAs on computers premises:

1. A representation of problem solutions maps the solution space into the chromosome space, i.e. a genetic encoding.
2. A fitness measure for each solution (chromosome) is defined to estimate the likelihood of leaving surviving offspring in the next generation.
3. The encoding can be manipulated through some sort of genetic operators such as reproduction, crossover and mutation, to yield other candidate solutions to the problem.

In the basic genetic operators, selection is a biased reproduction process to determine which chromosomes will be reproduced in the new population. Chromosomes with higher fitness values (better solutions) have larger chances of being selected for reproduction. The crossover operator is the "mating" operator that allows production of new chromosomes through partial information exchange between pairs of chromosomes. The mutation operator significantly changes the structure of a chromosome and hence produces albinos. Mutation serves as a guaranteed procedure to prevent solutions from being trapped in local optima. The procedure of GAs are illustrated as follows:

begin

set parameters

$t := 0$

initialize $P(0)$ (initial population)

evaluate $P(0)$

while termination condition is false **do**

begin

$t := t + 1$

select parents from $P(t-1)$ for reproduction
crossover to generate $P(t)$

mutate some of the members of $P(t)$

cull some members to maintain the same population size

if no improvement in $P(t)$

then $t_c := t_c + 1$

else $t_c := 0$

end

end

GAs are useful where the search space is large, non-linear and noisy, and solutions are ill-defined

a priori. Several characteristics of GAs suggest that they may have advantages relative to alternative heuristic solution algorithms for optimization problems. They are^{(13),(14)}: (1) dealing with coding of the parameter set, not the parameters themselves; (2) simultaneous search from a population of points, rather than from a point; (3) use of objective information, not derivative or other knowledge; (4) use of stochastic rather than deterministic transition rule in the search process; and (5) ease of implementation through efficient encoding of solution alternatives.

4.2 The GA-based sphericity assessment algorithms

In this study, two GA-based methods are developed for addressing the problems of the sphericity measurement and sphericity feasibility analysis described in Section 3. For the proposed GA-based optimization method, a real encoding scheme is used to represent a solution. The genetic operators of *roulette wheel selection*⁽¹³⁾, *arithmetic crossover* and *non-uniform mutation*^{(16),(17)} are adopted herein. Constraints are augmented to the objective function using penalty functions. This subsection describes in more detail how these operations are performed.

Solution representation: The encoding scheme is problem dependent and not unique. The solution structure is the vector of decision variables, $S = \{s_1, s_2, \dots, s_m\}$. A real value encoding is used herein so that each solution of decision variables is encoded as a vector of real value coefficients. For the sphericity assessment problems addressed here, the vectors of decision variables can be constructed as $S = \{x_c, y_c, z_c, R\}$ for MIS and MCS, and $S = \{x_c, y_c, z_c, R_1, R_2\}$ for MZS.

Data pre-processing: The bounds of decision variables (refer to Eqs. (2), (4) and (6)) are determined by manipulating the input measurement data. The lower and upper bounds for estimated center coordinates and radius, (x_c, y_c, z_c, R) , are determined by the least-squares sphere⁽¹⁸⁾, (x_q, y_q, z_q, R_q) .

$$\begin{aligned} x_U &= 1.1 \times x_q; x_L = 0.9 \times x_q \\ y_U &= 1.1 \times y_q; y_L = 0.9 \times y_q \\ z_U &= 1.1 \times z_q; z_L = 0.9 \times z_q \\ R_U &= 1.1 \times R_q; R_L = 0.9 \times R_q \end{aligned} \quad (11)$$

Initial population: An initial population is formed by randomly choosing several values for each of the parameters and assembling them into a given number (population size) of real strings. Consecutive populations will then be obtained through the evolving process, but the population size is kept constant.

Fitness measure: The real values in S are then inserted into the mathematical models of MIS, MCS and MZS to obtain the relative objective function

value, and then transform to fitness value. In general, one searches for the solution with highest fitness in the case of solving a maximization problem. The developed mathematical models of MIS, MCS and MZS are constrained non-linear programs. Constraints are added to the objective function using penalty functions.

GA-specific parameters: The algorithmic parameters include population size (*pop_size*), number of generation (*max_gen*), number of consecutive generations with no improvement (*con_gen*), crossover probability (P_c), mutation probability (P_m) and degree of non-uniformity (n).

Selection: In most practice, a *roulette wheel* approach⁽¹³⁾ is adopted as the selection procedure. It belongs to the fitness-proportional selection and can select a new population with respect to the probability distribution based on fitness values. For the *roulette wheel selection*, each current chromosome in the population has a roulette wheel slot sized in proportion to its fitness.

Crossover: The crossover operator applied here is the *arithmetic crossover*^{(16),(17)}. The *arithmetic crossover* is defined as the combination of two chromosomes S_1 and S_2 as follows:

$$S_1' = rS_1 + (1-r)S_2, S_2' = rS_2 + (1-r)S_1 \quad (12)$$

where $r \in (0, 1)$. The probability of crossover is set as p_c , i.e. on average, $p_c \times 100\%$ of chromosomes undergo crossover.

Mutation: The *non-uniform mutation*^{(16),(17)} is utilized in the proposed algorithm. For a given parent S , if the element s_k is selected for mutation, the resulting offspring is $[s_1, \dots, s_k', \dots, s_m]$, where s_k' is randomly selected from the two possible choices:

$$s_k' = s_k + \Delta(t, s_k^U - s_k) \text{ or } s_k' = s_k - \Delta(t, s_k - s_k^L) \quad (13)$$

where s_k^U and s_k^L are the upper and lower bounds for s_k . The function $\Delta(t, y)$ returns a value in the range in $[0, y]$ such that the value of $\Delta(t, y)$ approaches to 0 as t increases (t is the number of current generation). This function takes the form as

$$\Delta(t, y) = y \times r \times \left(1 - \frac{t}{\max_gen}\right)^n \quad (14)$$

where r is a random number from $[0, 1]$ and n is a parameter determining the degree of non-uniformity. The probability of mutation is set as p_m , i.e. on average, $p_m \times 100\%$ of total elements in population would undergo mutation. Every element in the selected chromosome has an equal chance to be mutated.

Termination criteria: The search will normally converge to some nearly optimal solutions under two conditions: (1) a predefined number of generations (*max_gen*) has been reached; and (2) a certain number of consecutive generations with no improve-

Table 1 Sample results of the four data sets for measurements

Data Set	1	2	3	4
R	9.0000	1.0018	0.9959	50.0301
MIS (x_c, y_c, z_c)	(0.0009, 0.0000, 0.0011)	(0.0018, -0.0003, 0.0000)	(0.0038, 0.0026, 0.0046)	(0.0041, -0.0027, -0.0004)
e	1.0003	0.0079	0.0083	0.0098
R	10.0000	1.0092	1.0035	50.0390
MCS (x_c, y_c, z_c)	(0.0000, 0.0000, 0.0000)	(0.0018, -0.0002, 0.0007)	(0.0041, 0.0025, 0.0038)	(0.0024, -0.0031, 0.0002)
e	1.0004	0.0080	0.0083	0.0099
R_1	10.0000	1.0093	1.0042	50.0394
R_2	9.0000	1.0016	0.9960	50.0297
MZS (x_c, y_c, z_c)	(0.0000, 0.0000, 0.0000)	(0.0025, -0.0001, 0.0005)	(0.0039, 0.0025, 0.0046)	(0.0038, -0.0031, -0.0001)
e	1.0000	0.0077	0.0082	0.0097

ment (*con_gen*) has been reached. For the measurement problem (advancing to the optimality), the solution procedure terminates after the first criterion mentioned above is satisfied. After the sphericity has been obtained, one can check the conformance of measured features and design specifications, and then make the acceptance/rejection decision for the inspected part. If the sphericity is within the tolerance, the acceptance decision is made. Otherwise, the sphericity is larger than the specification, and the rejection decision is made. For the sphericity feasibility analysis (advancing to the feasibility), the above two termination criteria are also applied. In the case of sphericity feasibility analysis, provided that the terminating criteria are achieved without finding any feasible solution, the rejection decision for inspection is then made. Additionally, the solution procedure for the sphericity feasibility analysis terminates once any feasible solution has been obtained, and the acceptance decision for inspection is made. It is possible that the specified tolerance may be overly tight or the process capability is unstable, and hence no part can pass the inspection.

5. The Experimental Study

The proposed GA-based assessment algorithms for sphericity measurement and sphericity feasibility analysis are tested using CMM measurement data sets taken from the sphericity assessment literature⁽⁸⁾⁻⁽¹⁰⁾. All the experiments are implemented on a PC with a Pentium III 800 MHz processor. The GA-specific parameters (refer to Subsection 4.2) are listed as follows: *pop_size*=150, *max_gen*=400, *con_gen*=100, *P_c*=0.95, *P_m*=0.1 and *n*=2.

5.1 Sphericity measurements

The data sets are listed in Appendix. The sample results of these four data sets are summarized in Table 1. As shown in Table 2, the GA-based MZS is equal or comparably better than the LSS and MZS reported in the previous studies.

The final solution of each run may vary due to the stochastic nature of the GA approach. It has been decided that a set of 50 runs will be sufficient to obtain

Table 2 Comparison of the MZS values obtained by GA-based method with the previous results

	Data Set 1	Data Set 2	Data Set 3	Data Set 4
LSS	1.1522	0.0085	0.0090	0.0132
Previous MZS	1.0000 ⁽⁹⁾	0.0077 ⁽⁸⁾	0.0083 ⁽¹⁰⁾	0.0097 ⁽¹⁰⁾
GA-based MZS	1.0000	0.0077	0.0082	0.0097

Table 3 The statistics of 50 runs of four data sets for measurements

Data Set	1	2	3	4
$\bar{e}$	1.0043	0.0079	0.0084	0.0099
σ_e	0.0022	0.0006	0.0001	0.0001
MIS A	60000	60000	60000	60000
T	1.32	0.70	0.55	0.53
$\bar{e}$	1.0036	0.0082	0.0084	0.0109
σ_e	0.0017	0.0005	0.0001	0.0006
MCS A	60000	60000	60000	60000
T	1.48	0.76	0.62	0.57
$\bar{e}$	1.0010	0.0077	0.0083	0.0097
σ_e	0.0009	0.0004	0.0000	0.0001
MZS A	60000	60000	60000	60000
T	1.98	1.01	0.83	0.77

* A and T represent the average number of search points and CPU time (seconds/run), respectively.

** All the CPU times are based on a personal computer with a Pentium III processor.

the meaningful output statistics for each data set. The statistics of the 50 runs include average sphericity $\bar{e}$, standard deviation of sphericity σ_e , average number of search points A and average CPU time T (seconds/run). From the statistics shown in Table 3, the solution consistency has been satisfactory since the standard deviations of sphericity error are small. The required computational time indicates the propriety of the proposed GA-based approach for MIS and MCS and MZS. The CPU time is between 0.5 - 2.0 seconds for these examples. Based on the experimental results described above, the proposed GA-based optimization algorithm is effective for sphericity measurements in CMMs. The assessment algorithms developed by previous studies⁽⁸⁾⁻⁽¹⁰⁾ are only suited to MZS, and not to MIS and MCS. However, the GA-based approach is a general method for MIS, MCS and MZS.

Table 4 The statistics of 50 runs of Data Set 1 for feasibility analysis

Tolerance		MIS	MCS	MZS
1.0100	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	3182.8	3089.4	37481.6
	<i>T</i>	0.12	0.09	1.26
1.0500	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	1538.2	1672.3	12132.9
	<i>T</i>	0.06	0.05	0.41
1.1000	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	1333.4	946.2	4553.3
	<i>T</i>	0.05	0.03	0.15

Table 5 The statistics of 50 runs of Data Set 2 for feasibility analysis

Tolerance		MIS	MCS	MZS
0.0080	acceptance	50	48	48
	rejection	0	2	2
	<i>A</i>	3515.1	5778.0	38327.5
	<i>T</i>	0.06	0.08	0.65
0.0082	acceptance	50	49	50
	rejection	0	1	0
	<i>A</i>	2039.0	1994.9	37190.8
	<i>T</i>	0.03	0.03	0.63
0.0084	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	1374.1	1155.0	16640.5
	<i>T</i>	0.02	0.02	0.28

5.2 Sphericity feasibility analysis

The data sets aforementioned are also used to investigate the viability of the proposed GA-based algorithm for the sphericity feasibility analysis. For these data sets, examples of tolerance sizes for the sphericity feasibility analysis are set as shown in Tables 4–7. Three situations are examined which include likely rejection (tolerance is smaller than LSS by a large amount and extremely close to MZS), hard acceptance (tolerance is close to MZS) and easy acceptance (tolerance is close to LSS).

The measurement data and tolerance values are fed to the algorithm that returns an acceptance/rejection flag. The average search iterations, average CPU time and acceptance/rejection analysis are tabulated in Tables 4–7. From the computational results, it confirms that the number of iterations and required CPU time are generally small in the case of easy acceptance (0.02–0.36 seconds/run). It takes longer CPU time to investigate the cases of hard acceptance (0.02–0.63 seconds/run). The CPU time of likely rejection case is much larger than that of easy acceptance (0.03–1.26 seconds/run). For the MZS, it usually takes larger search points and longer CPU time. The results of the sphericity feasibility analysis (Tables 4–7) are consistent to that of the measurements reported in Tables 1 and 3. However, the required CPU time of the sphericity feasibility analysis (for acceptance case) is usually much shorter than

Table 6 The statistics of 50 runs of Data Set 3 for feasibility analysis

Tolerance		MIS	MCS	MZS
0.0085	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	2591.8	2600.2	43442.9
	<i>T</i>	0.04	0.03	0.59
0.0087	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	2582.5	2491.5	29089.6
	<i>T</i>	0.04	0.03	0.39
0.0089	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	2204.5	2400.4	24722.4
	<i>T</i>	0.03	0.03	0.33

Table 7 The statistics of 50 runs of Data Set 4 for feasibility analysis

Tolerance		MIS	MCS	MZS
0.0100	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	3497.9	4028.7	52041.9
	<i>T</i>	0.04	0.04	0.63
0.0110	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	2144.5	2172.9	38851.0
	<i>T</i>	0.03	0.02	0.47
0.0120	acceptance	50	50	50
	rejection	0	0	0
	<i>A</i>	1935.6	1812.8	29988.5
	<i>T</i>	0.03	0.02	0.36

that of measurements (0.02–1.26 vs. 0.5–2.0 seconds/run). In addition, provided that least-squares solutions are used, rejection decisions will be made (reject good parts) since LSS overestimates the form errors.

6. Conclusions

Sphericity assessment algorithms have become increasingly imperative in modern dimensional measurement systems due to the coordinate measuring systems can potentially generate a large set of profile coordinates. To produce convincing measurement results within reasonable time, it is necessary to optimize the throughput of CMMs by developing an effective assessment algorithm. In this paper, the constrained non-linear models of the sphericity measurement (refer to Subsection 3.1) and sphericity feasibility analysis (refer to Subsection 3.2) with respect to MIS, MCS and MZS have been developed. The acceptance/rejection decision is made by checking the conformance of measured features and design intents. For the sphericity feasibility analysis, it is not necessary to find the reference sphere for checking conformance. Two genetic algorithms (GAs) based approaches are developed to explore the optimality of the sphericity measurement and the sphericity feasibility analysis, respectively. According to the above observations, the proposed GA-based optimization algorithm is effective for sphericity measurements in CMMs. Additionally, it is not necessary to find the

reference geometry for the sphericity feasibility analysis. Provided that the process capability is stable and the tolerance size is adequately specified, the required CPU time for sphericity feasibility analysis is comparably shorter than that of measurements.

Appendix : The Data Ses

Table A1 Data set 1⁽⁹⁾

No.	X	Y	Z	No.	X	Y	Z
1	8.42055	3.46726	1.19323	51	4.80000	6.40000	6.00000
2	-2.12100	-8.61568	2.36505	52	-1.36655	-2.55289	8.57942
3	0.86853	5.11876	8.40479	53	1.89727	-4.47838	-8.04041
4	3.17803	1.82449	9.29814	54	-7.94820	4.58889	-1.09601
5	-4.75718	8.23602	-0.01232	55	-2.73783	1.44044	8.67116
6	-0.00288	-0.00001	-9.22020	56	7.26417	2.57809	-5.19624
7	8.02534	2.43064	-3.34796	57	6.73556	3.02284	-0.53863
8	-0.77363	-1.36294	9.73536	58	-3.23473	7.59841	-4.09855
9	-0.42047	-0.05775	9.89921	59	-7.91122	-3.08604	-3.61944
10	-2.48279	-1.09659	9.20123	60	-0.94722	1.19895	8.94372
11	-1.32236	-0.33044	8.94394	61	4.53577	-5.59322	-5.70760
12	-8.65863	-2.71179	2.06558	62	-4.80000	6.40000	6.00000
13	6.94724	4.54788	4.56109	63	-6.29337	4.51725	5.89396
14	1.23285	-4.10410	8.21792	64	-1.84698	-0.03353	9.58379
15	-4.98020	-8.66566	-0.00066	65	2.31123	-2.20250	8.46912
16	9.55234	0.02726	-0.01951	66	6.79260	5.21214	-2.80259
17	0.01829	0.00006	9.63208	67	1.47478	5.43919	7.04760
18	2.18897	8.46413	3.92633	68	0.16864	-5.92638	7.31883
19	-3.91959	8.01503	-1.25551	69	8.77527	2.10675	1.40999
20	2.35189	-6.47936	-5.90072	70	0.00000	-9.00000	0.00000
21	-4.93565	-6.79583	3.45841	71	-3.76405	-3.05133	-7.83327
22	6.91250	2.39367	5.84698	72	0.49092	9.03545	0.68276
23	7.54898	3.28552	4.06809	73	0.75328	-6.19756	6.88526
24	3.21574	8.02367	4.36928	74	-6.04510	7.58341	-0.86041
25	5.43679	-5.10013	5.28983	75	0.32479	9.39478	-2.96664
26	2.93158	2.45466	-8.24089	76	8.91620	1.36278	-0.15665
27	-3.06343	4.81048	-7.94254	77	-8.40008	1.03735	3.90549
28	7.24483	4.85365	-4.62014	78	-1.19949	-9.61640	-2.36150
29	-3.57018	1.37046	8.96134	79	0.00000	-10.00000	0.00000
30	-0.00414	0.11571	9.21327	80	3.05448	3.63890	7.67445
31	-5.50164	7.07481	3.52308	81	0.23151	-0.47424	9.12475
32	3.59470	3.35799	-8.69633	82	-5.67083	-7.29763	-3.60701
33	5.77727	7.19318	3.81492	83	-5.54879	2.48093	6.93936
34	1.09155	5.21130	-7.36348	84	-1.67709	-6.77174	5.92368
35	1.05627	9.86747	-0.86997	85	-4.66332	1.19213	8.29668
36	1.69428	6.09696	7.46172	86	1.82616	8.61360	3.44446
37	0.33632	-6.18998	-6.86310	87	-0.98304	-3.28498	-9.05523
38	-1.36144	-7.06321	5.74638	88	-6.80116	5.56668	3.16158
39	1.21537	-3.61139	9.02052	89	2.66568	-3.94312	7.86528
40	2.67713	-4.21685	-8.13177	90	-1.78909	2.91725	-8.51074
41	-8.50578	-2.75549	-2.48880	91	1.94884	2.94550	-9.05638
42	4.36823	-3.35307	7.41081	92	-0.67512	0.22825	9.94209
43	-3.40167	-6.65031	-5.24305	93	6.10308	5.99957	-3.17171
44	0.77619	1.66759	9.25988	94	-5.38958	3.65589	6.78410
45	6.40027	-6.67380	-3.49152	95	0.00000	9.00000	0.00000
46	-4.47850	-7.55166	-2.62989	96	2.06263	-8.17541	3.89862
47	-0.01253	0.03167	9.51814	97	-0.04665	-0.34500	9.67874
48	-4.24576	-8.30408	0.54593	98	7.15160	6.27399	2.33940
49	9.21459	0.78511	-0.27609	99	-5.54207	6.67783	2.70456
50	4.23950	-8.49940	1.74151	100	3.74134	-7.56337	3.36648

Table A2 Data set 2⁽⁸⁾

No.	X	Y	Z	No.	X	Y	Z
1	0.81814	-0.43615	-0.38525	26	0.36179	-0.07902	0.93410
2	-0.80885	-0.53045	0.27905	27	-0.78624	0.20326	-0.59063
3	0.03749	0.90643	-0.42737	28	-0.46529	0.69616	-0.55167
4	0.86935	-0.27403	-0.42518	29	-0.93402	-0.02267	-0.35813
5	-0.50786	-0.07441	0.86060	30	0.43016	0.49319	-0.76122
6	0.75348	-0.67012	-0.00610	31	-0.46929	0.11501	0.88119
7	0.05867	-1.00773	-0.00198	32	0.60788	-0.41130	0.68580
8	-0.17486	0.60442	0.78297	33	0.70712	0.46598	0.53998
9	0.71121	-0.50123	-0.50042	34	0.67398	-0.38068	-0.63956
10	-0.47493	-0.51466	0.71992	35	-0.06245	-0.50544	0.86696
11	0.06016	-0.74129	-0.67432	36	-0.09577	0.96216	0.26433
12	0.05337	0.02006	1.00515	37	0.29318	0.79379	0.54237
13	0.29869	0.87920	0.38988	38	-0.39873	-0.68009	0.61867
14	-0.15488	-0.86697	0.48585	39	-0.30833	-0.07805	0.94937
15	0.38994	-0.00060	-0.92592	40	-0.70424	-0.64359	0.31476
16	0.00776	0.73922	0.68741	41	0.67010	-0.17187	-0.72841
17	0.62090	0.52006	-0.59341	42	-0.12359	-0.70688	0.70559
18	0.00952	-0.03493	-1.00376	43	-0.09694	0.64820	-0.75729
19	-0.44592	0.76584	-0.47827	44	-0.13014	0.04876	0.99815
20	0.01069	0.99511	0.11265	45	-0.28485	0.86262	-0.43728
21	-0.93371	-0.17351	-0.33160	46	0.16745	-0.11142	-0.98603
22	-0.48121	-0.69836	0.53717	47	0.00946	-0.00746	1.00873
23	0.50394	0.86933	0.03322	48	0.15374	-0.33634	-0.93141
24	0.80924	0.07332	-0.59398	49	-0.44828	0.85926	-0.25853
25	0.84569	0.06876	-0.53570	50	-0.61673	-0.28843	0.73859

Table A3 Data set 3⁽¹⁰⁾

No.	X	Y	Z	No.	X	Y	Z
1	0.7826	-0.5434	0.3128	21	-0.7594	-0.2135	-0.6030
2	-0.7480	-0.5102	0.4158	22	0.4029	0.1826	-0.8993
3	-0.0632	-0.0273	1.0046	23	-0.3379	0.2205	-0.9125
4	0.0128	0.5284	-0.8454	24	0.1728	-0.5212	0.8349
5	-0.0166	0.3702	-0.9262	25	0.8387	0.5194	0.2075
6	0.2190	0.5347	-0.8167	26	0.9989	0.0085	0.0553
7	-0.2689	-0.3490	-0.8942	27	0.7089	0.3048	-0.6392
8	-0.6484	-0.3669	-0.6513	28	-0.0595	0.1537	0.9886
9	0.0425	-0.2701	-0.9558	29	-0.0451	-0.0381	-0.9977
10	-0.8319	0.5366	-0.1474	30	0.7196	0.2112	-0.6629
11	0.5364	0.3270	0.7831	31	0.1935	-0.0631	-0.9724
12	0.7894	0.6230	0.0312	32	-0.7160	0.2111	0.6617
13	-0.1405	0.8314	0.5453	33	-0.3249	0.5776	0.7506
14	0.0755	-0.4434	-0.8831	34	-0.0325	0.0040	-0.9980
15	-0.2803	-0.9239	0.2617	35	0.1292	0.0986	-0.9788
16	0.2170	0.1673	0.9634	36	0.5826	-0.8136	0.0115
17	0.7252	-0.6338	0.2723	37	-0.0392	-0.1252	-0.9835
18	-0.7934	0.2614	-0.5367	38	0.0199	0.1022	-0.9932
19	0.4571	0.8672	0.2103	39	0.8254	0.2545	-0.5065
20	0.8288	-0.4416	-0.3450	40	-0.5822	0.7861	0.2104

Table A4 Data set 4⁽¹⁰⁾

No.	X	Y	Z	No.	X	Y	Z
1	12.7529	2.2487	48.3286	19	-44.6577	-20.8242	8.6884
2	10.6073	7.4273	48.3267	20	-31.6748	-37.7486	8.6889
3	6.4751	11.2152	48.3306	21	-12.7544	-47.6001	8.6893
4	-4.4291	12.1689	48.3296	22	8.5569	-48.5287	8.6889
5	-9.1564	9.1564	48.3267	23	28.2626	-40.3632	8.6884
6	-12.1674	4.4286	48.3238	24	42.6737	-24.6377	3.6886
7	-12.9006	-1.1287	48.3296	25	19.8699	7.2320	-45.3458
8	-11.2156	-6.4753	48.3325	26	14.9512	14.9512	-45.3439
9	-7.4285	-10.6090	48.3344	27	7.2323	19.8707	-45.3476
10	-2.2489	-12.7539	48.3325	28	-1.8430	21.0654	-45.3476
11	3.3517	-12.5086	48.3296	29	-10.5727	18.3125	-45.3467
12	8.3242	-9.9204	48.3306	30	-17.3203	12.1278	-45.3439
13	42.6720	24.6367	8.6882	31	-20.8217	3.6714	-45.3412
14	28.2609	40.3608	8.6879	32	-20.4250	-5.4729	-45.3467
15	-12.7534	47.5963	8.6886	33	-16.1994	-13.5929	-45.3494
16	-31.6730	37.7464	8.6884	34	-8.9374	-19.1662	-45.3512
17	-44.6551	20.8230	8.6879	35	8.9365	-19.1634	-45.3467
18	-49.2685	0.0000	8.6874	36	16.1987	-13.5923	-45.3476

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