Integrated Study of Time-Frequency Representations and Their Applications in Source Identification of Mechanical Noise*

Min-chun PAN**

Three computation schemes of time-frequency representations have been developed and implemented to identify different components of mechanical noise emitted from the transmission system of electrical vehicles. This study explores the close relationships between three time-frequency representations, i.e. the spectrogram based on windowed Fourier transform, the Wigner-Ville distribution, and the smoothed Wigner-Ville distribution. One main purpose is to pursue the efficiency of computing the smoothed Wigner-Ville distribution of a dynamic signature. The revised scheme can tremendously reduce the computation time to a scale of around 1/90, compared with the original scheme. To assess the validation of these time-frequency representation schemes, firstly, four synthetic signals are designed and processed. Secondly, the developed time-frequency representations are applied to distinguish different spectral components of transmission noise, and identify their sources. This study takes an electrical scooter with a continuous variable transmission system as a test bench. The continuous-variable-transmission-belt noise, helical-gear whine noise, and fan noise can be clearly identified via the processing of the time-frequency representations. These obtained conclusions can be used as references for machine element modification to reduce annoying noise.

Key Words: Time-Frequency Representation, Smoothed Wigner-Ville Distribution, Nonstationary signals, Mechanical Noise Identification

1. Introduction

Conventional spectral analysis provides average spectra, which offer no time information. From some studies, one can find two intrinsically different time-waveforms with an identical spectrum. For instance, both the response of a linear system to stationary white noise excitation, and the impulse response of the same system offer the same power spectral density⁽¹⁾. One can also design two synthetic signatures, i.e., a chirp signal and a modulated form of $\frac{\sin(af)}{af}$ signal, with the same spectrum⁽²⁾. The afore-mentioned examples give expressions about shortcomings to analyze transient or nonstationary signals using conventional spectral analysis. For nonstationary

signatures, the frequency contents vary with each time-localization. The analysis resulting from taking the Fourier transform for a whole period of waveform, however, offers an overall average spectrum, but no time information indicated. It is not appropriate to process those signals with the conventional spectral analysis. In past two decades, it has been drawn extensive attention to develop and apply the time-frequency (TF)⁽³⁾⁻⁽⁵⁾ and time-scale^{(6),(7)} analysis schemes in processing transient or nonstationary signatures. It is well noticed that the spectral information of a signal processed via the window Fourier transform (WFT) based spectrogram can be spread in a TF plane. As a result of the confinement of the Heisenberg uncertainty principle, the product of the time- and frequency-resolution will retain a constant value as soon as the type of the time weighting window is decided. In another word, one cannot obtain both good time- and frequency-resolution simultaneously. The Wigner-Ville distribution (WVD) of a signal can yield extremely good TF

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** Assistant Professor, Department of Mechanical Engineering, National Central University, Chung-Li 320, Taiwan, R.O.C. E-mail: pan_minc@cc.ncu.edu.tw

resolution, but there exist interferences in-between each two mono-components. For dynamic signatures from real-world measurement, those interferences with negative energy content will smear meaningful spectral components and affect their physical interpretations. The development of the smoothed WVD (SWVD) offers the possibility of adjusting the time- and frequency-resolution independently. One can make a trade-off between TF resolution and interferences by using the SWVD.

All time-frequency representations (TFRs) of an analyzed signal can be expressed as the two-dimensional (2D), i.e., time and frequency, convolution of its WVD and the WVD of a weighting function⁽³⁾. As the WVD of the weighting function can be decoupled separately in time and frequency function, the constructed SWVD can yield arbitrary time- and frequency-resolution by tuning their smoothing parameters. In the study, the close relationships between the spectrogram, the WVD and the SWVD are discovered, i.e., the SWVD is exactly a compromise between the spectrogram and the WVD. By carefully choosing smoothing parameters, the SWVD can be identical to the spectrogram. Besides, better time- and frequency-resolution can be expected by tuning appropriate parameters of smoothing windows. The SWVD can suppress and partially eliminate interferences in the WVD by two times of smoothing scheme, however, it spends more time to yield an SWVD than that needed for a spectrogram and SWV. To increase the computation efficiency, the 2D-convolution form can be transferred as one time-convolution and one time-multiplication, and the FFT scheme can thus be applied. In this paper, the effort has been paid to achieve the best TF resolution, the minimum interferences, and the maximum computation efficiency for the SWVD of a dynamic signature.

In the research, an electrical scooter with continuously variable transmission (CVT) mechanism has been taken as a test bench. It demonstrates the TFRs, especially the SWVD, in processing annoying mechanical noise can be applied as effective tools for the tasks of machine fault diagnosis and machine element modification. After both the specifications of transmission system of a vehicle have been known and transmission noise has been measured, the TFRs can identify different mechanical noise resulting from CVT mechanism (e.g., belt noise and fan noise), and speed reduction gears. These analysis conclusions can be drawn as design references.

2. Theoretical Basis

The TFRs of a time waveform (or time function) can be expressed in a generalized form (the so-called

Cohen class), $C_f(t, \omega; \Phi)$. As shown in Eq.(1), it can also be rewritten in other formats.

$$\begin{aligned} C_f(t, \omega; \Phi) &= \frac{1}{2\pi} \iiint e^{j(\xi t - \tau \omega - \xi u)} \Phi(\xi, \tau) f\left(u + \frac{\tau}{2}\right) \\ &\times f^*\left(u - \frac{\tau}{2}\right) du d\tau d\xi, \\ &= \iint e^{-j\tau \omega} \varphi(t - u, \tau) f\left(u + \frac{\tau}{2}\right) f^*\left(u - \frac{\tau}{2}\right) du d\tau \end{aligned} \quad (1)$$

where $f(u)$ is the analyzed signal; $\Phi(\xi, \tau)$ denotes the kernel function in the specific TFR. In Eq.(1), $\varphi(t, \tau) = \frac{1}{2\pi} \int e^{j\xi t} \Phi(\xi, \tau) d\xi$. From the above definition and derivation, one can obtain a variant TFR with a specific kernel function.

2.1 WFT-based spectrogram

For a nonstationary signal, the Fourier transform of a whole segment of waveform can only reserve spectral information. But, the frequency contents at a certain time instant are not available. Sliding weighting window function along the time axis and multiplying time waveform can yield the windowed Fourier transform. Thus, both time and frequency information can be yielded. Let $f(t)$ be the analyzed signal, and $g(t)$ the weighting window. If the kernel function $\Phi(\xi, \tau)$ is chosen as the ambiguity function of $g(t)$, i.e.,

$$\Phi_{spect}(\xi, \tau) \equiv \int_{-\infty}^{\infty} g\left(t + \frac{\tau}{2}\right) g^*\left(t - \frac{\tau}{2}\right) e^{-j\xi t} dt, \quad (2)$$

then the WFT-based spectrogram of $f(t)$ can be deduced in the form

$$SPE_f^{(g)}(t, \omega) = \left| \int_{t'} Z_f(t') g^*(t' - t) e^{-j\omega t'} dt' \right|^2. \quad (3)$$

As a matter of fact, it is the squared magnitude of the analyzed signal's WFT. In reason that only positive-frequency range is considered, the analytical form, $Z_f(t)$, of the signal $f(t)$ is used to construct the spectrogram. $Z_f(t)$ is in a complex form, whose real part is the original analyzed signal $f(t)$ and imaginary part is the Hilbert transform of $f(t)$. Consequently, the discrete form of the WFT-based spectrogram can be expressed as

$$SPE_f^{(g)}(n, m) = \left| \sum_{k=0}^{M-1} Z_f(k) g^*(k - n) e^{-jm k \frac{2\pi}{M}} \right|^2, \quad (4)$$

where M denotes the length of the weighting window $g(t)$; n and m represent the index of discrete time and discrete frequency, respectively. Due to the confinement of the uncertainty principle one cannot obtain both extremely good time- and frequency-resolution from a WFT-based spectrogram. A measure of TF resolution as an ensemble can be illustrated as Fig.1, where σ_t and σ_ω denote time- and frequency-resolution, respectively. An excellent candidate in all weighting windows is the family of Gaussian func-

Fig. 1 An illustration of the Heisenberg uncertainty principle

tions, which fulfils, $\sigma_t \sigma_\omega = \frac{1}{2}$, i.e., a minimum uncertainty wave packet or best TF resolution compared with other weighting functions⁽⁸⁾.

2.2 Wigner-Ville distribution (WVD)

When the kernel function equals unity, i.e., $\Phi_{WVD}(\xi, \tau) = 1$, one can obtain the TFR in a WVD form,

$$C_f(t, \omega; \Phi) = \int e^{-j\tau\omega} f\left(u + \frac{\tau}{2}\right) f^*\left(u - \frac{\tau}{2}\right) d\tau \quad (5)$$

$$\equiv WV_f(t, \omega).$$

Substitute Eq. (5) into Eq. (1), the generalized TFR, $C_f(t, \omega; \Phi)$ shown as in Eq. (1), can be rewritten as

$$C_f(t, \omega; \Phi) = \frac{1}{2\pi} \iint \phi(t - \tau, \omega - \xi) \times WV_f(\tau, \xi) d\tau d\xi, \quad (6)$$

where $\phi(t, \omega) = \frac{1}{2\pi} \iint e^{j(\xi t - \omega \tau)} \Phi(\xi, \tau) d\xi d\tau$. From Eq. (6), it concludes that the generalized form, $C_f(t, \omega; \Phi)$, can be expressed as a 2D-TF convolution of the WVD of the analyzed signal, and the transformed kernel function, $\phi(t, \omega)$, i.e., imposing FT for τ and inverse FT (IFT) for ξ upon $\Phi(\xi, \tau)$. To prevent aliasing, the analytical form, $Z_f(t)$, of $f(t)$ has to be used when implementing the WVD⁽⁹⁾. This concept can be shown as the illustration of Fig. 2. Therefore, Eq. (5) will be

$$WV_{Z_f}(t, \omega) = \int_{-\infty}^{\infty} Z_f\left(t + \frac{\tau}{2}\right) Z_f^*\left(t - \frac{\tau}{2}\right) e^{-j\omega\tau} d\tau. \quad (7)$$

To evaluate the WVD at a time instant t , one can apply an FFT computation upon τ .

2.3 Smoothed Wigner-Ville distribution (SWVD)

Equation (6) indicates all TFRs can be formulated as the 2D-TF convolution of the WVD of the analyzed signal, and the transferred form, $\phi(t, \omega)$, of the chosen kernel function, $\Phi(\xi, \tau)$. If $\phi(t, \omega)$ is a time and frequency decoupled function, i.e.,

$$\phi(t, \omega) = g_1(t) G_2(\omega), \quad (8)$$

then the SWVD of the analyzed signal can be derived from Eq. (6).

Fig. 2 The difference of using the analyzed signal, $f(t)$, and its analytical form, $Z_f(t)$, for the evaluation of a WVD

$$SWV_{Z_f}(t, \omega) = \int \left[\int Z_f\left(t - t' + \frac{\tau}{2}\right) \times Z_f^*\left(t - t' - \frac{\tau}{2}\right) g_1(t') dt' \right] g_2(\tau) e^{-j\omega\tau} d\tau \quad (9)$$

where $g_2(\tau)$ is the IFT of $G_2(\omega)$ and called the frequency smoothing window, and $g_1(t')$ is called the time smoothing window. If $g_2(\tau)$ is chosen from a real and even function, then we can express $g_2(\tau) = \left| \chi\left(\frac{\tau}{2}\right) \right|^2 = \chi\left(\frac{\tau}{2}\right) \chi^*\left(\frac{-\tau}{2}\right)$. Equation (9) becomes

$$SWV_{Z_f}(t, \omega) = \int \left[\int Z_f\left(t - t' + \frac{\tau}{2}\right) \times Z_f^*\left(t - t' - \frac{\tau}{2}\right) g_1(t') dt' \right] \chi\left(\frac{\tau}{2}\right) \chi^*\left(\frac{-\tau}{2}\right) e^{-j\omega\tau} d\tau, \quad (10)$$

and its discrete form can be illustrated as

$$SWV_{Z_f}(n, m) = \sum_{l=-L+1}^{L-1} \left[\sum_{k=-L+1}^{L-1} Z_f(n-k+l) \times Z_f^*(n-k-l) g_1(k) \right] \chi(l) \chi^*(-l) e^{-jm(2l)\pi/N}, \quad (11)$$

$$n=0, 1, 2, \dots, N-1.$$

When implementing Eq. (11) in a microcomputer, one can apply an FFT scheme after the computation in the bracket $[\bullet]$ has been finished. The evaluation of $\sum_{k=-L+1}^{L-1} \bullet$ in the bracket $[\ ]$, however, is rather time-consuming via loop-computation. As a matter of fact, the computation of $[\ ]$ in Eq. (10) is exactly $\left[\left(Z_f\left(t + \frac{\tau}{2}\right) Z_f^*\left(t - \frac{\tau}{2}\right) \right) \otimes g_1(t) \right]_\tau$, where $\otimes$ in front of $g_1(t)$ denotes convolution. In this study, we propose applying and implementing the property of “the convolution in time domain for two signals equivalent to their multiplication in frequency domain then taking IFT”. In section 3, we shall see its computation efficiency through the comparison of different schemes for synthetic signals.

2.4 The connection between spectrogram, WVD and SWVD

When the WVD is evaluated via Eq. (7), there exist interferences in-between each two mono-compo-

Fig. 3 The relationship of WFT-based spectrogram, WVD and SWVD

nents in the TF plane⁽¹⁰⁾. This makes the interpretation of an analyzed dynamic signal difficult although the WVD offers extremely good TF resolution. At the other end, the spectrogram via the evaluation of Eq. (3) yields no interferences, but it is confined to the resolution problem. In the study, we choose

$$g(t) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\sigma_t}} e^{-\frac{t^2}{2\sigma_t^2}} \quad (12)$$

to evaluate the $SPE_{Z_f}^{(g)}(t, \omega)$ in Eq. (3) for a spectrogram, where σ_t relates to the time-resolution of the spectrogram. In an evaluation of a spectrogram, using a smaller value of σ_t can yield better time-resolution, but worse frequency-resolution, and vice versa. In addition, the $WV_g(t, \omega)$ is a decoupled function of t and ω . Consider both Eqs. (6) and (8) as well as the Gaussian type function $g(t)$ as a weighting window, Eq. (6) can then be rewritten as

$$\begin{aligned} S\tilde{W}V_{Z_f}(t, \omega) &= \frac{1}{2\pi} \iint WV_g(t - \tau, \omega - \xi) \\ &\times WV_{Z_f}(\tau, \xi) d\tau d\xi. \end{aligned} \quad (13)$$

As mentioned above, the $WV_g(t, \omega)$ is a decoupled function, i.e., $g_1(t)G_2(\omega)$, of t and ω . Furthermore, both $g_1(t)$ and $G_2(\omega)$, i.e., time- and frequency-smoothing windows, are Gaussian type of functions with parameters σ (time-resolution) and σ_ω (frequency-resolution), respectively. When $\sigma_t\sigma_\omega = 1/2$, which fulfils the condition of minimum wave packet, it can be proved that $S\tilde{W}V_{Z_f}(t, \omega) = SPE_{Z_f}^{(g)}(t, \omega)$, i.e., the SWVD is identical to the spectrogram with no interferences, but at a sacrifice of resolution. To pursue better TF resolution, one can flexibly tune the value of σ_t and σ_ω . Based upon this principle, as $\sigma_t\sigma_\omega < 1/2$ which violates the uncertainty principle, the resulting SWVD yields better TF resolution, but accompanies interferences. An illustration in Fig. 3 shows the relationship between the spectrogram, the WVD and the SWVD.

3. The Evaluation of TFRs with Synthetic Signals

For the developed TFRs, we first design four synthetic signals to check their validity as well as discuss their properties and differences. The purpose is to compare objectively the smearing degree of a TFR caused by interference terms, the TF resolution, and their individual computation efficiency. Four signals are described as follows. They are $f_a(t)$: four pieces of short-term sinusoids together with white

Fig. 4 TFRs of four synthetic signals (1st column: time waveform; 2nd column: spectrogram; 3rd column: WVD, and 4th column: SWVD)

noise; $f_b(t)$: two impulses appended to a frequency modulation (FM) signal; $f_c(t)$: one impulse appended to two sinusoids; $f_d(t)$: two FM signatures (one with frequency increasing, and the other with frequency decreasing) together with one impulse and two short terms of sinusoids. Figure 4 illustrates the TFRs of four synthetic signals in gray levels from 4 to 10 dB, where the 1st column: time waveform, the 2nd column: spectrogram, the 3rd column: WVD, and the 4th column: SWVD. To properly handle the negative values of WVDs and SWVDs, i.e. interference terms, in decibel expressions, they have been set to be a small positive number, e.g., 5×10^{-3} in the case. Table 1 summarizes the computation CPU time for four synthetic signals with three TFR schemes. Besides, the revised SWVD computation techniques are included as well. The Pentium II-350 desktop PC was used in numerical evaluation. In the analysis, both the spectrogram and the WVD utilize the Gaussian window as Eq. (12) shows. To formulate the Gaussian weighting function, we take $\sigma_t = 0.04$ except that $\sigma_t = 0.02$ for the

Table 1 A list of computation CPU time for four synthetic signals with different TFR schemes. (In the "SWVD" column, "A" denotes original computation scheme, "R1" denotes applying FFT technique in the computation of convolution integral, and "R2" denotes some improved numerical procedures adopted.)

Schemes of TFRs CPU time (sec)	Spectrogram	WVD	SWVD		
			A	R1	R2
$f_a(t)$	0.66	1.10	61.79	2.8	0.77
$f_b(t)$	0.16	0.27	6.81	0.6	0.22
$f_c(t)$	0.17	0.28	15.93	0.77	0.22
$f_d(t)$	0.27	0.44	26.3	1.26	0.38

synthetic signal $f_b(t)$. In the computation of the SWVD, $\sigma_t\sigma_\omega=1/4$ has been set. More specifically, the frequency-resolution of computed SWVDs is as two-times better as that of spectrograms at a condition of the same time-resolution. The overall TF resolution of SWVDs is better than that of spectrograms. From the results, some conclusions can be made as following.

- All three TFR techniques can characterize a dynamic signature in a TF plane to interpret their physical meanings.

- As the WVDs show in Fig.4, the values of the interferences between each two mono-components go up and down in the TF plane (x - y plane) illustrated by black-and-white color. These terms oscillate along the TF plane with positive and negative energy. They sometimes confuse the signature interpretation and affect correct recognition on each significant spectral component.

- In the aspect of TF resolution and interferences, both of them regard the validity of signature interpretation. The spectrograms offer no interferences but a fixed TF resolution, i.e., $\sigma_t\sigma_\omega=1/2$ due to a Gaussian function being applied. Although the WVDs offer good TF resolution, interferences appear in-between each two mono-components, which can affect signal interpretation. Compared with the other two TFRs, the SWVDs propose a compromised option between the resolution and interference. In the numerical evaluation here, $\sigma_t\sigma_\omega=1/4$ has been set to obtain better TF resolution with a sacrifice of slight interferences.

- In the aspect of computation efficiency, it regards TFRs' practical applications. The spectrograms need least CPU time. Based upon the time spent for evaluating the spectrograms of four synthetic signals, the WVDs spend around 0.66 times longer, and the SWVDs take tremendous CPU time (90 times more) for original computation scheme. Applying the FFT techniques in the convolution integral of Eq.(10), the

Fig. 5 Illustration of a comparison of the original scheme and the improved numerical procedure for the convolution of the analyzed signal and a time-smoothing window, where M_w denotes the length of a time-smoothing window, and NUM_{max} the length of the analyzed signal

time to evaluate a SWVD decreases to be 4.5 times of the spectrogram (see 'R1' column of Table 1). Moreover, when some reasoning approaches are adopted, the computation time can be saved further (see 'R2' column of Table 1). When evaluating the bracket, $[\bullet]$, of Eq.(9), i.e. a computation of the convolution of the analyzed signal and the time-smoothing window at each specific τ , the multiplication of both frequency spectra of the analyzed signal and a time-smoothing window is computed, and an inverse FFT (IFFT) is taken to yield the time convolution. It should be noted a phase-compensation computation is to be applied before the IFFT is taken, like illustrated in the left-hand-side column of Fig. 5. As the time-smoothing window is arranged like the right-hand-side column of Fig. 5, the phase-compensation step can be got rid of. Therefore, the computation time of evaluating an SWVD can be saved further. As to the SWVDs computed by the original two schemes (i.e., (i) convolution integral through time-domain loop computation, and (ii) convolution integral through multiplication in frequency domain) and by the improved time-saving scheme, all SWVD figures are the same.

4. The Application of TFRs in the Source Identification of Mechanical Noise

This study has taken an electrical scooter with a CVT system as a test bench to apply the developed TFRs in the source identification of mechanical noise. In addition to TFRs as analysis tools, speed informa-

Table 2 The specifications of transmission elements of the electrical scooter

Drive face of CVT (as a cooling fan to CVT mechanism)	No. of blades: 18	
Speed-reduction gear sets	(G11) Helical gear	No. of teeth: 13 (30°)
	(G21) Helical gear	No. of teeth: 48 (30°)
	(G22) Spur gear	No. of teeth: 12
	(G31) Spur gear	No. of teeth: 36
Cog number of CVT belt	79	

Fig. 6 The sketch of transmission system of the electrical scooter

tion, including motor and rear-wheel, and specification of the transmission system assist to identify different noise emitting from corresponding transmission element. The vehicle utilizes a DC motor as the power source. The transmission system consists of a CVT mechanism and two pairs of speed-reduction gear set. A sketch of the transmission system is illustrated in Fig. 6. Table 2 lists the specification of transmission elements. The total speed-reduction ratio from motor-shaft to rear wheel is not a fixed number, which depends upon motor-speed and road resistance.

The experimentation was conducted at a general-purpose laboratory, but its background noise can still keep below 50 dB. Since the transmission annoying noise can be clearly perceptible, it is acceptable the experiments, regarding noise source identification and comparative measurement between different designs, have been conducted at this laboratory. Figure 7 illustrates the set-up of annoying noise measurement. Some test conditions during experimentation can be summarized as follows.

- The vehicle runs up with the condition of wide-open-throttle till around 7200 rpm of motor-speed, then it coasts down with the throttle fully closed.
- The noise-pickup microphone is placed 30 cm

Fig. 7 Set-up of annoying noise measurement of the electrical scooter

Fig. 8 Some possible annoying frequencies of transmission components (from top to down-Belt RPS* Cog-number; Motor RPS* blade-number of Fan; Counter-Shaft RPS* Tooth-No. of Counter-Gear; Final-Shaft RPS* Tooth-number of Final-Shaft)

high, and 30 cm far from CVT case.

- An accelerometer is mounted on the L-case to pick up the vibration signatures of gear transmission.
- The sampling frequency is 18 kHz, and then spectral components below 9 kHz can be effectively processed. The anti-aliasing low-pass filter is tuned at 10 kHz.

Both the encoder pulse data of the motor, and rear-wheel have been recorded to compute their rotating speed. In addition, the revolution speed for the other rotary components of the transmission system can also be computed applying the kinematic equations⁽¹¹⁾, i.e., the geometric relationship of the CVT mechanism, together with two stages of gear speed-reduction. Figure 8 plots the rotating speeds of some transmission elements. More precisely, the multiples, i.e., the number of belt-cog, fan-leaf, and gear-tooth etc., of the revolution speed for those transmission elements are illustrated in the unit of round per second (RPS, or Hz). Figure 9 illustrate the TFRs of measured annoying noise. Quite a few significant spectral components are characterized via TFR processing. In addition, the vibration of L-case accommodating speed-reduction gear sets is proces-

(a) WFT-based spectrogram

(b) WVD

(c) SWVD

Fig. 9 TFRs of measured annoying noise

sed and shown in Fig. 10. In the computations, a Gaussian weighing window with $\sigma_t = 0.03$ sec ($\sigma_\omega = 16.7$ rad/sec, or 2.65 Hz) has been applied in both evaluating spectrograms and WVDs. In addition, a time-smoothing window and a frequency-smoothing window with Gaussian-type functions have been used to make $\sigma_t \sigma_\omega = 1/2$. More specifically, σ_t remained to be 0.03 sec, and σ_ω half-decreased 8.4 rad/sec (or 1.33 Hz) make SWVDs result in better frequency-resolution, and keep the same time-resolution as spectro-

(a) WFT-based spectrogram

(b) WVD

(c) SWVD

Fig. 10 TFRs of vibration signature picked up on the L-case

grams. To properly deal with the negative values, i.e. interferences of WVDs mostly and SWVDs partially, in illustrations, those values have been set to be a small positive number (e.g., 5×10^{-3} in the case) for decibel expressions. The differences between three TFRs are compared objectively through the same decibel range, i.e., 25 to 70 dB in Fig. 9, and 50 to 100 dB in Fig. 10. The SWVDs with the best performance to characterize certain spectral components can be

recognized by two-folds besides theoretical deduction. One is the spectral components highlighted by the SWVDs remain sharpest contrast in gray-level figures compared to the other two TFRs. The other is the characterized components in the illustration of the spectrograms and the WVDs are smeared more severely than the SWVDs due to worse TF resolution, and more interference terms, respectively. Compare the characterized noise/vibration components with the possible annoying frequency curves in Fig. 8, some remarks can be concluded as follows.

- The dominant annoying noise components are CVT-belt noise, helical-gear whine noise (G11-G21 pair), and first several harmonics of electrical motor speed. Although the annoying frequency curves of fan and helical-gears are much close, not a problem with fan noise can be concluded from the evaluated TFR results.

- After properly dealing with the negative terms of WVDs and SWVDs, the influences of interference terms, which affect signal interpretations, have been minimized to a great extent. Besides, a better contrast gray-level illustration is obtained from the evaluation of SWVDs due to better frequency-resolution.

- A location sensitive sensor like accelerometer can pick up the vibration originated from transmission gears, but not CVT-belt operation, as illustrated in Fig.10. It is another supplement to the aforementioned remark that some annoying noise emitting from meshing transmission-gears.

5. Conclusions

This research conducts an integrated study on TFRs. Some efforts focus on exploring the close relationship of the WFT-based spectrogram, WVD, and SWVD. If the spectrogram uses a Gaussian function as the weighting window, and the SWVD's time-smoothing window and frequency-smoothing window result from the Gaussian function's WVD, then the SWVD will be exactly a compromise of the spectrogram and WVD. Some synthetic signals are designed and processed to demonstrate three TFRs' properties. An electrical scooter with CVT mechanism and speed-reduction gears has been taken as a test bench. TFRs can effectively characterize different spectral components and identify their sources. These obtained conclusions can be referred

for machine element modification to cope with the annoying noise problem.

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